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Top Properties from the W-Decay

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Abstract

… weak production of the top quark in $\bar{p}p$-collisions is discussed with a particular emphasis on the distributions of the leptons from semileptonic decay of the top. We show that the near-in-future experimental information on these distributions can be used to determine the chiral nature of the top quark as well as the amount of depolarization of the top during its hadronization.

1. Introduction

The present standard theory of weak interactions by Glashow, Salam and Weinberg [1] is based on the assumption of left-handed structure for charged weak interactions. Since there is no natural explanation for this feature of the theory it is of great importance to test this structure for all observed charged weak currents of all quarks and leptons. The left-handed nature has been verified fairly accurately for the first two generations of leptons [2] and the dominant charged weak interaction of the first quark generation [3]. Furthermore, some less strict data prefer the left-handed nature for the third lepton [4] and the second quark family [5].

In this work we study the possibility of determining the Lorentz structure of the dominant top quark current coupling to W (in the standard theory it is the left-handed $t\bar{b}$-current). We assume here that the t quark is light enough to be produced in W-decay, and take its charged current to be of the general vector-axial vector form:

$$j_\mu = g_W \bar{t} \gamma_\mu (1 - \Lambda \gamma_5) b \tag{1}$$

where

$$g_W = \frac{e}{2\sqrt{2} \sin \theta_W} \tag{2}$$

This current will appear both in the weak production and (semileptonic) decay of the top quark, thus enhancing the effects of a possible nontrivial structure ($\Lambda \neq 1$).

Before decaying, the top quark will hadronize to $T(s = 0)$ and $T^*(s = 1)$ hadrons. Since the weak decay rate of massive vector bosons T^* exceeds their electromagnetic decay rate [7], we will have anisotropic distributions of leptons from the weak decays of longitudinally polarized T^*'s. However, since the hadronization is a non perturbative process and up to date still uncalculable, we cannot say for sure what is the relative production ratio of these hadrons, i.e. to what extent the initial top quark polarization will prevail. Naive state counting arguments [8] suggest this fraction to be one half. Our approach is to keep this figure (we call it the polarization prevailance ratio R_p) variable and to study the differences of the leptonic distributions for the two extreme cases, $R_p = 1$ (no depolarization) and $R_p = 0$ (total depolarization). As we will discuss, the future experiments will yield valuable information on R_p and thus on the fragmentation of the top quark.

Our main emphasis in this work is somewhat different from the former papers on the weak production of top [9, 10]. We will make here a quantitative study of the distributions of leptons coming from the semileptonic decay of the top, and question the sensitivity of these distributions to the modifications of the weak (Λ) and hadronic (R_p) properties of the top quark. We will show that large enough changes to certain leptonic distributions are induced in order to allow for the experimental determination of these properties.

We will not discuss here the hadronic production of top quarks, first of all since no information on R_p is obtained from the single top decay and secondly, since any effect of the modified nature of the weak couplings of the top will remain small due to the initial nonpolarization of the t quark. However, one could obtain information on R_p and enhance the weak modifications by studying the correlations of the decay products coming from the two decaying top quarks. Unfortunately, the cross-section for both t quarks decaying semileptonically will remain even in the near future too low for a useful analysis. Anyway, if it turned out that the top quark mass is heavier than M_W, such correlations might be the best way of studying these questions, but the necessary cross-sections could only be reached at the Jura/Tevatron energies.

2. Production and decay of a polarized heavy quark

We start by calculating the cross-section for producing a heavy quark Q_1 with a definite spin state in the weak process $q_1\bar{q}_2 \rightarrow W \rightarrow Q_1\bar{Q}_2$. The momenta of the initial and final quarks are

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called q_1, q_2 and k_1, k_2, respectively. The amplitude using the current of Eq. (1) is given by:

$$M = -ig_W^2 \frac{1}{2q_1\cdot q_2 - M_W^2 + i\Gamma_W M_W}[\bar{v}(q_2)\gamma_\mu(1-\gamma_5)u(q_1)] \times [\bar{u}(k_1)\gamma^\mu(1-\Lambda\gamma_5)v(k_2)] \quad (3)$$

We will neglect the small masses of the initial quarks but keep the masses m_1 and m_2 of both outgoing quarks. Furthermore, we assume that Λ is real (no CP violation). The spin s_1 of the quark Q_1 is projected out in the normal way through the spin projection operator $1/2(1+\gamma_5 s_1)$. When squaring the above matrix element and summing over the spins of other quarks one finds the following trace for the final state current:

$$\mathrm{Tr}\left[\gamma^\mu(1-\Lambda\gamma_5)(\not{k}_2 - m_2)\gamma^\nu(1-\Lambda\gamma_5)(\not{k}_1+m_1) \times \frac{(1+\gamma_5\not{s}_1)}{2}\right] \quad (4)$$

One easily sees that the terms proportional to m_2 include a factor $1-\Lambda^2$, thus vanishing for both left- and right-handed coupling. Our final expression for the spin averaged matrix element squared is as follows:

$$\bar{\Sigma}|M|^2 = \frac{8g_W^4}{(2q_1\cdot q_2 - M_W^2)^2 + \Gamma_W^2 M_W^2}\{(1+\Lambda)^2 q_1\cdot k_2(k_1\cdot q_2 - m_1 s_1\cdot q_2) + (1-\Lambda)^2 q_2\cdot k_2(k_1\cdot q_1 + m_1 s_1\cdot q_1) + (1-\Lambda^2)m_2[m_1 q_1\cdot q_2 - k_1\cdot q_1 s_1\cdot q_2 + k_1\cdot q_2 s_1\cdot q_1]\} \quad (5)$$

So, for $m_2 = 0$ and $\Lambda = \pm 1$ one obtains the polarized case from unpolarized just by substituting k_1 for $1/2\,(k_1 - \mathrm{sign}(\Lambda)m_1 s_1)$.

The implications of this result are clearly seen in the CM frame of the process (W rest frame) where, taking the initial q_1 quark to come in z-direction and the final Q_1 quark to determine the xz-plane, the vectors can be written in the following form:

$$\begin{aligned} k_1 &= (E_1, p\sin\theta, 0, p\cos\theta) \\ k_2 &= (E_2, -p\sin\theta, 0, -p\cos\theta) \quad (6) \\ s_1 &= \frac{\lambda_1}{m_1}(p, E_1\sin\theta, 0, E_1\cos\theta) \end{aligned}$$

where $\lambda_1 = \pm 1$ stands for the helicity of the quark Q_1. If m_2 is neglected the two remaining terms of Eq. (5) attain particularly simple forms:

$$\bar{\Sigma}|M|^2 \sim (1+\Lambda)^2(E_1 - \lambda_1 p)(1-\lambda_1\cos\theta)(1+\cos\theta) + (1-\Lambda)^2(E_1+\lambda_1 p)(1-\lambda_1\cos\theta)(1-\cos\theta) \quad (7)$$

Thus for $\Lambda = 1$ ($\Lambda = -1$) the quark Q_1 is mostly left-handed (right-handed) with an angular distribution $(1+\cos\theta)^2$ peaking in the forward direction ($(1-\cos\theta)^2$ and backward dominance). The unfavoured right-handed (left-handed) quarks are produced with a forward-backward symmetric distribution $\sin^2\theta$. The production rate of quarks with an unfavoured helicity compared to that with a favoured helicity is given by

$$\frac{\sigma(\lambda_1 = \text{unf.})}{\sigma(\lambda_1 = \text{fav.})} = \frac{1}{2}\frac{E_1 - p}{E_1 + p} \sim \frac{1}{2}\left(\frac{m_1}{M_W}\right)^2 \quad (8)$$

Thus e.g. for $m_1 = 40$ GeV this ratio is as big as 1/8. In Fig. 1a we have plotted the helicity averaged angular distribution of the Q_1 quark at the W mass with $\Lambda = 1$ for a few values of m_1 (for $\Lambda = -1$ just reverse $\theta \to \pi - \theta$).

From Eq. (7) one can also calculate the average helicity $\bar{\lambda}_1(\theta)$ of the Q_1 quark emitted at a definite angle θ with, say, $\Lambda = 1$ (the standard V–A coupling):

$$\bar{\lambda}_1^{V-A}(\theta) = \frac{d\sigma(\lambda_1 = 1) - d\sigma(\lambda_1 = -1)}{d\sigma(\lambda_1 = 1) + d\sigma(\lambda_1 = -1)} = -\frac{(1+\cos\theta)^2 - \frac{E_1 - p}{E_1 + p}\sin^2\theta}{(1+\cos\theta)^2 + \frac{E_1 - p}{E_1 + p}\sin^2\theta} \quad (9)$$

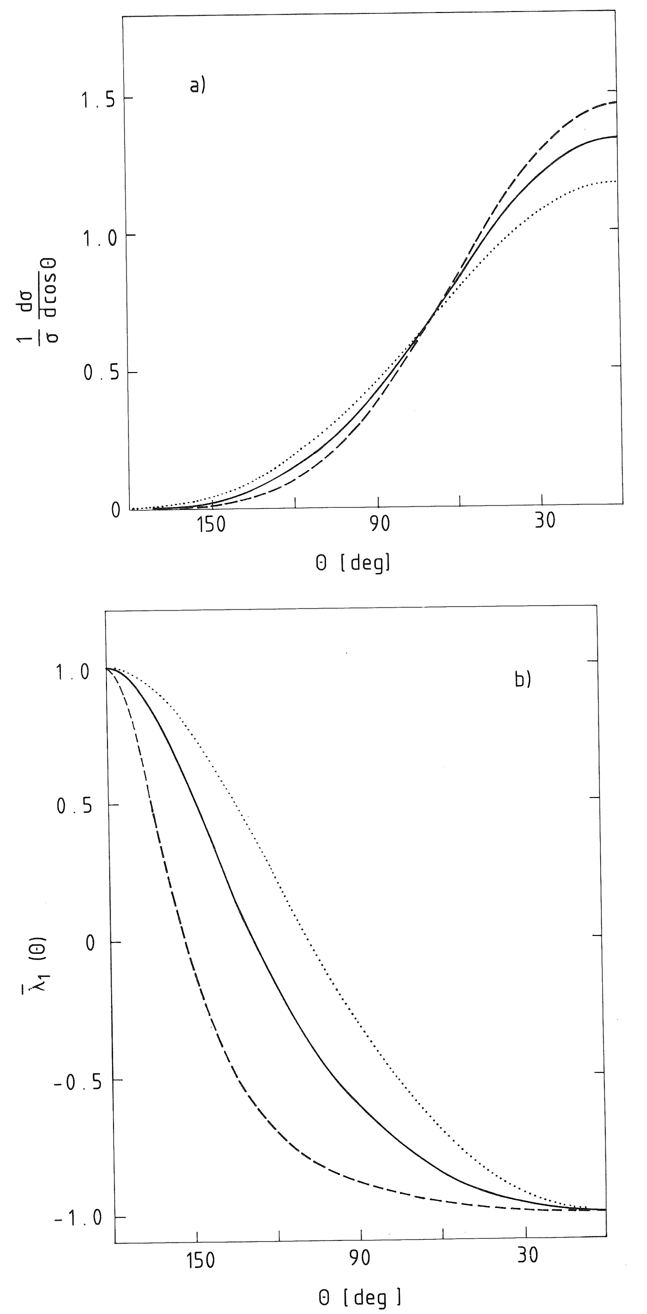

Fig. 1. a) The normalized, helicity averaged angular distribution of the Q_1 (top) quark from W decay (in W rest frame), b) The averaged helicity of the Q_1 quark from W decay as a function of the Q_1 emission angle (in W rest frame). The dashed, solid and dotted curves correspond to the masses m_1 = 20, 40 and 60 GeV. The forward direction is opposite to the polarization of the W.

Figure 1b presents this expression as a function of θ. For $\Lambda = -1$ (right-handed V + A coupling) we could have

$$\bar{\lambda}_1^{V+A}(\theta) = -\bar{\lambda}_1^{V-A}(\pi - \theta) \tag{10}$$

Next we will turn to discuss the weak decay of a polarized heavy quark Q_1 to Q_2 and a lepton-antilepton (or generally fermion-antifermion) pair l_1, $\bar{l}_2$ which we take massless and left-handed. Calling the momenta of leptons p_1 and p_2 and those of the quarks again k_1 and k_2, we find that the respective decay amplitude can be obtained from above by simply crossing the momenta:

$$\begin{aligned} &k_2 \to -k_2 \\ &q_1 \to p_1 \\ &q_2 \to p_2 \end{aligned} \tag{11}$$

Thus our result for the decay $Q_1 \to Q_2 l_1 \bar{l}_2$ is the following:

$$\begin{aligned} \Sigma |M|^2 = &\frac{32 g_W^4}{(2p_1 \cdot p_2 - M_W^2)^2 + \Gamma_W^2 M_W^2} \{(1+\Lambda)^2 p_1 \cdot k_2 (k_1 \cdot p_2 \\ &- m_1 s_1 \cdot p_2) + (1-\Lambda)^2 p_2 \cdot k_2 (k_1 \cdot p_1 + m_1 s_1 \cdot p_1) \\ &- (1-\Lambda^2) m_2 [m_1 p_1 \cdot p_2 - k_1 \cdot p_1 s_1 \cdot p_2 \\ &+ k_1 \cdot p_2 s_1 \cdot p_1]\} \end{aligned} \tag{12}$$

Due to the more complicated three-particle kinematics the interpretation of these formulae even in the CM system is less direct than above. However, some results are straightforward. Substituting the CM vectors

$$\begin{aligned} k_1 &= (m_1, \vec{0}) \\ p_2 &= E_2(1, \sin\phi_2, 0, \cos\phi_2) \\ s_1 &= |P_1|(0,0,0,1) \end{aligned} \tag{13}$$

one finds that the angular distribution of $\bar{l}_2$ for $\Lambda = 1$ with respect to the spin direction is proportional to

$$1 + |P_1| \cos\phi_2 \tag{14}$$

Here the initial polarization $|P_1|$ of Q_1 has been taken to consist of both its "natural" average helicity $\bar{\lambda}_1(\theta)$ from the production, and the depolarization factor R_p, coming from hadronization:

$$P_1 = \bar{\lambda}_1(\theta) R_p \tag{15}$$

(Of course, for $\Lambda = -1$, the angular distribution of l_1 would similarly be $1 - |P_1| \cos\phi_1$). Neglecting m_2, the angular distribution of $\bar{l}_2$ for general Λ is readily calculated to be of the form

$$1 + |P_1| \xi(\Lambda) \cos\phi_2 \tag{16}$$

where

$$\xi = \frac{2(1+\Lambda+\Lambda^2)}{3(1+\Lambda^2)} \tag{17}$$

This implies that the $\bar{l}_2$ distribution is always prefering the spin direction. The maximum of $\xi(\Lambda)$ and thus the maximal forward peaking is obtained for $\Lambda = 1$ whence $\xi(1) = 1$, while the minimum corresponds to $\Lambda = -1$ and then $\xi(-1) = 1/3$.

The l_1 distribution with respect to the spin of Q_1 also obeys the form of Eq. (16) once the substitution $\xi(\Lambda) \to -\xi(-\Lambda)$ (and, of course, $\phi_2 \to \phi_1$) is made. Thus the lepton l_1 always goes preferably to the backward direction and the maximal (minimal) peaking is obtained for $\Lambda = -1$ ($\Lambda = 1$).

One should notice that the corresponding distributions of the lepton $\bar{l}_2$ (as well as l_1) with respect to the (say, W rest frame) momentum of the decaying quark Q_1 can simply be obtained by taking P_1 instead of $|P_1|$ in Eqs (14) and (16). For example, if $\Lambda = 1$, Q_1 is mostly produced with negative average helicity (see Figs 1), the $\bar{l}_2$ distribution will prefer the direction opposite to the Q_1 momentum.

3. Observable distributions of leptons from top decay

We now apply the above results to the specific case of $Q_1 = t$, $Q_2 = b$ and $l_1 = \nu_e$, $\bar{l}_2 = e^+$, that is the weak production and semileptonic decay of the top quark. Since the rest frame of the decaying t quark is difficult to reconstruct e.g. due to our ignorance of the total momentum of the neutrino, we will have to present the results in the $p\bar{p}$-rest frame. We use a simple Monte Carlo program with the non-scaling parametrization of Glück, Hoffmann and Reya [11] for the quark distribution functions. Only the Drell-Yan subprocess was included because none of our estimates is very sensitive to the small transverse momentum of W obtained from first or higher order QCD corrections. In order to scale our total cross-section for these effects we used the K-factor $K = 1.5$.

We will restrict our numerical examples only to the extreme cases to find the maximal changes to the leptonic distributions from changing Λ and R_p. Accordingly, we will only plot the curves for $\Lambda = 1$ and -1 and $R_p = 0$ and 1. Furthermore, since the overall change of the partonic cross-section for the chosen extreme values of the mass m_1 was only about 20% (see Fig. 1a), we will, for definiteness, fix the mass m_1 to 40 GeV in what follows. None of our final results are more sensitive to the change of this mass within the range considered.

Figure 2 shows the angular distribution of the position relative to the proton direction. This is obviously the best way to

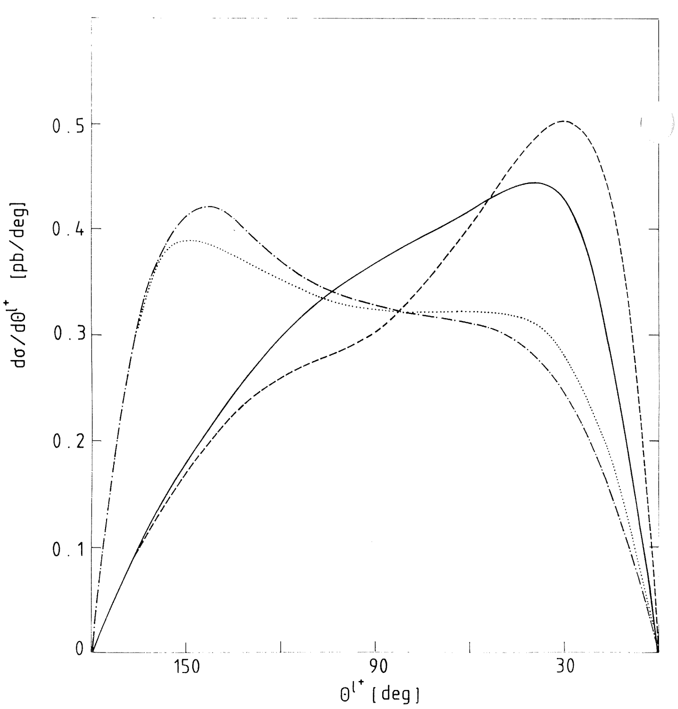

Fig. 2. Angular distributions of the positron from top decay with respect to the proton direction. The solid and dashed lines correspond to $\Lambda = 1$, $R_p = 1$ and $R_p = 0$, the dotted to $\Lambda = -1$, $R_p = 1$ and the dash-dotted to $\Lambda = -1$, $R_p = 1$ with $d(x) = 1/2\,u(x)$.

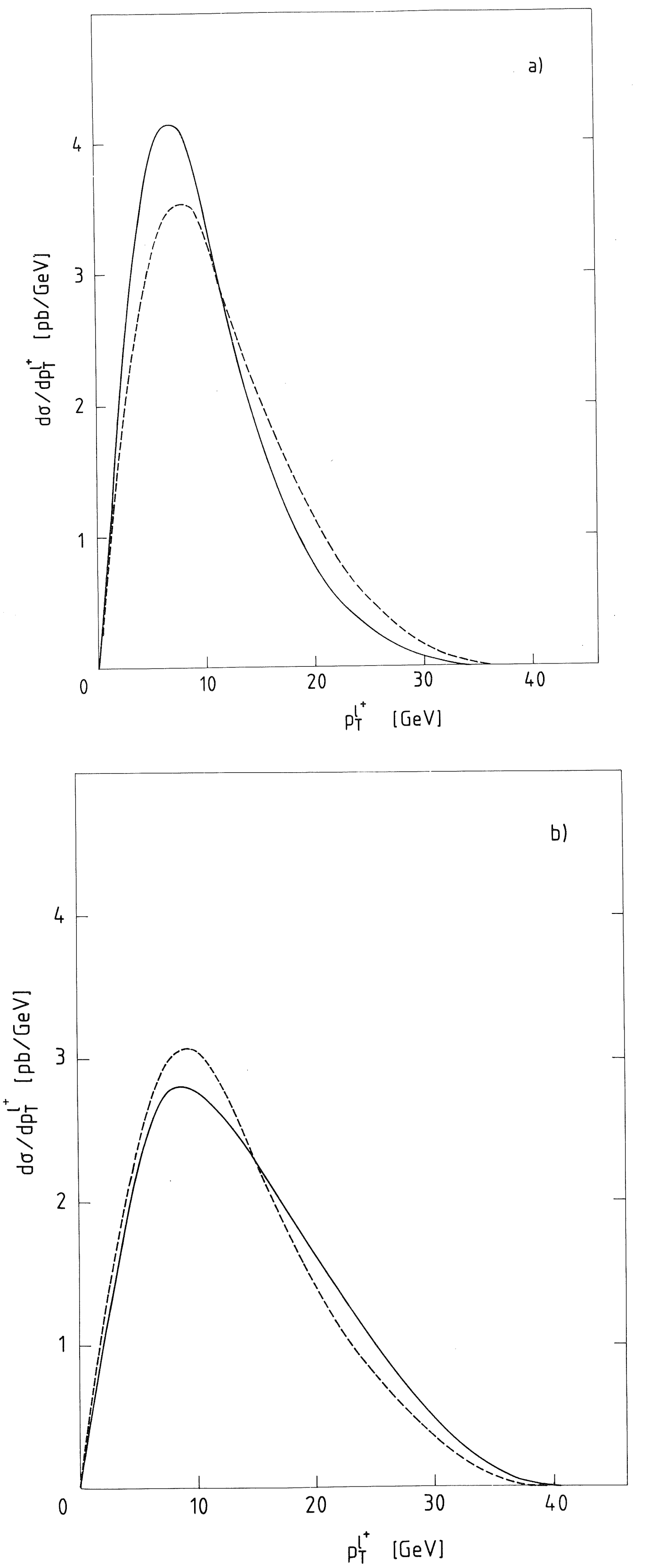

Fig. 3. The p_T distribution of the positron for a) $\Lambda = 1$, b) $\Lambda = -1$. The solid and dashed curves correspond to $R_p = 1$ and $R_p = 0$.

determine the nature of the top weak coupling (Λ). For $\Lambda = 1$ the distribution is strongly peaking forwards although the top polarization tends to flatten this angular asymmetry out. For $\Lambda = -1$ the situation is quite opposite: polarization makes the distribution peak even more to the backward direction, but this effect is only minor due to the more isotropic angular distribution and we can neglect it. Clearly, the two cases $\Lambda = 1$ and $\Lambda = -1$ are so different that they can be distinguished *independently of the value of* R_p with a foreseeable future luminosity. It may even be possible to obtain a fairly strict bound on Λ.

Another interesting aspect about Fig. 2 comes from the (u-d-asymmetric) quark distributions whence the W^+ has an average longitudinal momentum in the p direction. This shifts all the distributions somewhat into the forward direction. For clarity, we show the relative size of this effect by plotting the curve $\Lambda = -1$, $R_p = 1$ taking $d(x) = 1/2u(x)$. Due to this dependence on the quark distributions, it may be difficult to obtain a limit on R_p, even for $\Lambda = 1$.

Figures 3a and 3b give the p_T-distributions of the positron for $\Lambda = 1$ and $\Lambda = -1$, respectively. (The same figures also represent the p_T-distributions of the neutrino for $\Lambda = -1$ and $\Lambda = +1$, respectively.) One can again clearly see the effect of the top polarization, making the soft (hard) $\Lambda = 1$ ($\Lambda = -1$) distribution even softer (harder). If $R_p = 1$ is assumed, the value of Λ can be fairly well restricted. However, for any fixed Λ, it may be difficult to estimate R_p. We can conclude that although affected by the polarization effects, the p_T-distributions are not particularly suitable for an independent determination of the two parameters Λ and R_p.

A good way to determine the value of R_p *independently of the value of* Λ is the $\psi_{e^+\nu}$-distribution, presented in Fig. 4, where $\psi_{e^+\nu}$ is the angle between the positron and neutrino momenta in the transverse momentum plane. The distribution is the same for both $\Lambda = 1$ and $\Lambda = -1$. Since there is a difference of about 10–20% over a large range of $\psi_{e^+\nu}$-values between the curves for $R_p = 0$ and $R_p = 1$ one could obtain an independent estimate on R_p from fairly poor statistics. Furthermore, for this distribution only the transverse direction,

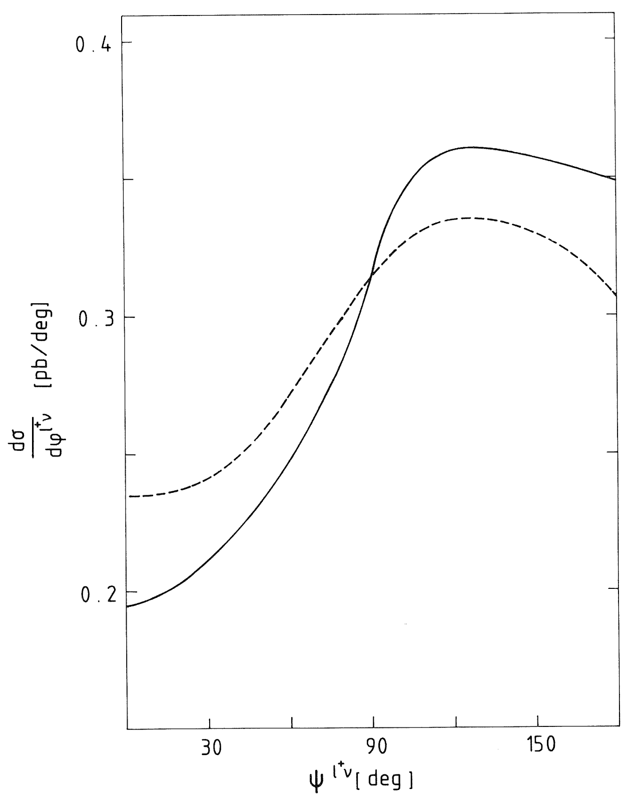

Fig. 4. Distribution of the relative angle of the positron and neutrino in the transverse plane. The solid and dashed curves correspond to $R_p = 1$ and $R_p = 0$.

not its magnitude, has to be determined, increasing the experimental accuracy.

4. Conclusions

We have studied the possibility of having information on the weak and hadronic properties of the top quark produced in $p\bar{p} \to W \to tb$ with $t \to b\bar{l}\nu_e$. Questioning the chirality nature of the dominant charged weak coupling of the top quark, we modified this to be a general combination of vector and axial vector couplings. Furthermore, in order to study the hadronization of the top quark we introduced the factor R_p, measuring the prevailance of the helicity of the initially polarized top quark and thus giving information on the relative abundance of scalar and vector mesons including the top quark.

After presenting general results for the production and decay of a polarized heavy quark and discussing their qualitative features, we calculated several experimentally observable distributions for leptons coming from the top decay. We found that the chiral nature of the top is best, and independently of R_p, measured from the distribution of the polar angle between the positron and proton. Also the modified p_T-distributions were calculated and shown to be useful with high experimental accuracy. The best way to determine R_p was shown to be the distribution of the transverse angle between the positron and the neutrino. This distribution is the same for both left- and right-handed top. The differences between the calculated distributions are large enough to allow experimental separation between the extreme choices for both variables already with the luminosity attainable within the next run of $p\bar{p}$-experiments.

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