# Jet Variable Distributions in Antiproton-Proton Interactions (*). 

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#### Abstract

Summary. - The event shape in momentum space of $\overline{\mathrm{p}} \mathrm{p}$ interactions at $22.4 \mathrm{GeV} / \mathrm{c}$ has been investigated by means of the variables thrust, triplicity and dithrust. Events with clear multijet shape corresponding to the dual topological unitarization scheme predictions are not observed. The events have mainly collinear shape. A fast algorithm for the calculation of the event shape variables is proposed.


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## 1. - Introduction.

Investigations of hadron interactions with high $p_{\mathrm{T}}$ and $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation into hadrons have revealed jet production in these processes ( ${ }^{1}$ ). In the case of hadron interactions with low $p_{T}$ the existence of hadron jets and their appearance is up to now not completely clear. The dual topological unitarization (DTU) scheme is known as a promising approach to describe hadron production at high energies ( ${ }^{2}$ ). According to the DTU scheme particles are produced by hadronization of a number of coloured objects and they form subsets which resemble jets in momentum space. The event shape depends on the number of subsets and on correlations between them.

A successful attempt has been made to describe the properties of pp interactions on the basis of a two-jet fragmentation model $\left(^{3}\right)$. It would be interesting to study the jet structure of antinucleon-nucleon interactions, since there is in addition the annihilation channel. Particularly, according to the DTU scheme, the secondary particles in $\bar{p} p$ annihilation should form three rapidity chains $\left({ }^{4}\right)$, which may have been observed in the multiplicity distributions ( ${ }^{5}$ ) and in the event shape.

[^1]It has been noticed earlier $\left({ }^{6}\right)$ that momentum configurations have multijet features. Methods of event shape analysis in terms of jet variables are used and developed in this paper. Main attention is given to the search of structures suggested by DTU predictions.

The presented results have been obtained by studying about 27100 events from the two meter hydrogen bubble chamber Ludmila irradiated with a separated $22.4 \mathrm{GeV} / \mathrm{c}$ antiproton beam. For experimental details see $\left(^{7}\right)$ and references therein. Slow protons with $p_{\text {lab }}<1.5 \mathrm{GeV} / \mathrm{c}$ were identified by ionization. Altogether 11000 events with slow protons were found.

DTU predicts different structures for annihilation and nonannihilation processes and, therefore, events with and without an identified proton in the final state were studied separately. By this separation the fraction of annihilation events becomes larger in the sample without identified protons.

The paper is organized as follows. In the next section the possible configurations of antiproton-proton events are discussed. In the third section the methods of event shape analysis are presented and in the next one the algorithm for jet variable determination is described. The paper ends with the presentation and discussion of the results.

## 2. - Structure of $\mathrm{p} \overline{\mathrm{p}}$ interactions.

In the framework of DTU different dual diagrams correspond to nonannihilation (fig. $1 a$ )) and annihilation (fig. 1b)) processes ( ${ }^{3,4}$ ). If there would

a)

b)

Fig. 1. - Dual diagrams in the framework of DTU for nonannihilation (a)) and annihilation (b)) processes.

[^2]be equal energies in the chains and no correlations and if the centres of mass of the chains would coincide with the centre of the event, then the nonannihilation events would have a 4 -jet-like or a «crosslike» structure (fig. 2a)),


Fig. 2. - A four-jet-like or a "crosslike» (for nonannihilation) (a)) and a 6-jet-like (for annihilation) (b)) structure.
and annihilation events a 6 -jet-like structure (fig. $2 b$ )). If the centre of mass of each chain moves in the c.m. system of the event, configurations shown in fig. $3 a$ ) and $b$ ) may appear.


Fig. 3. - Configurations of events when the centre of mass of each chain moves. A two-jet (a)) and a three-jet structure (b)).

Unequal energy partition between the chains may lead to some kind of degeneration of the jet structure. A similar modification may be caused by limited $p_{\mathrm{T}}$. The DTU approach itself does not give unique predictions on the event shape in momentum space. In principle, the annihilation and nonannihilation events may look quite similar. In the present study we investigate the possibility of the appearance of the configurations presented in fig. $2 a$ ), $3 a$ ), and $b$ ).

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## 3. - Methods of jet analysis.

To describe the two-jet structure (fig. $3 a)$ ), the variables sphericity $S$ and thrust $T$ were used, which have been defined by ( ${ }^{8}$ )

$$
\begin{align*}
& S=\min \left(3 \sum_{i=1}^{N} \boldsymbol{p}_{\mathbf{T} i}^{2} / 2 \sum_{i=1}^{N} \boldsymbol{p}_{i}^{2}\right),  \tag{1}\\
& T=\max \left[\left(\sum_{i \in C_{1}} \boldsymbol{p}_{i} \cdot \boldsymbol{n}_{\mathrm{T}}-\sum_{i \in \sigma_{\mathbf{2}}} \boldsymbol{p}_{i} \cdot \boldsymbol{n}_{\mathrm{T}}\right)\right] / \sum_{i=1}^{N}\left|\boldsymbol{p}_{i}\right|,
\end{align*}
$$

where $\boldsymbol{p}_{\mathbf{T} i}$ is the transverse momentum of particle $i$ with respect to the sphericity axis and $n_{\mathrm{T}}$ is a unit vector along the thrust axis. $C_{1,2}$ is the subset of particles $i \in C_{1}\left(C_{2}\right)$, for which $\boldsymbol{p}_{i} \cdot \boldsymbol{n}_{\mathbf{T}}>0(<0)$. Because of momentum conservation in the c.m.s. the relation

$$
\begin{equation*}
\sum_{i=1}^{N} \boldsymbol{p}_{i}=0 \tag{3}
\end{equation*}
$$

is valid. In most experimental events some momentum is missing, due to neutral particles. Also measurement errors have an influence on the momentum. To fulfil condition (3) this was accounted for by introducing a «missing» particle.

The thrust axis coincides with the direction of the largest total momentum of all possible subsets of particles. By counting the sums of momenta of all possible subsets, the total number of combinations required is

$$
\begin{equation*}
K_{N}^{(2)}=2^{N-1}-1 \tag{4}
\end{equation*}
$$

To extend the thrust concept to find out if an event is of the three-jet-type triplicity TR is introduced:

$$
\begin{equation*}
\mathrm{TR}=\max \left(\sum_{j=1}^{3} \sum_{i \in \sigma_{j}} \boldsymbol{p}_{i} \cdot \boldsymbol{n}_{j}\right) / \sum_{i=1}^{N}\left|\boldsymbol{p}_{i}\right| \tag{5}
\end{equation*}
$$

where each particle $i$ belongs to only one of the three subsets $C_{1}, C_{2}$ and $C_{3}$ and $i \in C_{j}$, if $\boldsymbol{p}_{i} \cdot \boldsymbol{n}_{j} \geqslant \boldsymbol{p}_{i} \cdot \boldsymbol{n}_{k}(j \not \equiv k)$. The amount of combinations of $N$ particle momenta divided into three subsets is

$$
\begin{equation*}
K_{N}^{(3)}=\frac{1}{2}\left(3^{N-1}+1\right)-2^{N-1} . \tag{6}
\end{equation*}
$$

[^4]This is a rapidly increasing number, e.g. $K_{10}^{(3)}=9330$ and $K_{12}^{(3)}=86526$, whereas in the thrust case formula (4) gives $K_{10}^{(2)}=511$ and $K_{12}^{(2)}=2047$.

To study in more detail the events whose configurations are nearly three-jet-like, two further variables $\chi$ and $\eta$ are introduced:

$$
\begin{align*}
& \chi=\min \left(\left|\boldsymbol{q}_{i}\right| / \sum_{i=1}^{3}\left|\boldsymbol{q}_{i}\right|\right)  \tag{7}\\
& \eta=\max \left(\cos \left(\boldsymbol{n}_{i} \cdot \boldsymbol{n}_{j}\right)\right) \tag{8}
\end{align*}
$$

where $\boldsymbol{q}_{i}$ is the total momentum of particles belonging to the subset $C_{i}$. These subsets are determined by calculating the triplicity (see eq. (5)). Thus $\chi$ characterizes the momentum distribution into the subsets $C_{i}$. The variable $\eta$ describes the orientation of the triplicity axes.

To describe the crosslike configurations (see fig. $2 a$ )) new variables are introduced. An obvious generalization of thrust, called dithrust, is defined as follows:

$$
\begin{equation*}
\mathrm{DT}=\max \left(\sum_{j=1}^{2} \sum_{i \in \tilde{\sigma}_{j}}\left|p_{i} \ldots n_{j}\right|\right) / \sum_{i=1}^{N}\left|p_{i}\right| \tag{9}
\end{equation*}
$$

where $\boldsymbol{n}_{1}$ and $\boldsymbol{n}_{2}$ are unit vectors along the dithrust axes. Particle $i \in \tilde{C}_{i}$, if $\left|\boldsymbol{p}_{i} \cdot \boldsymbol{n}_{1}\right|>\left|\boldsymbol{p}_{i} \cdot \boldsymbol{n}_{2}\right|$. Furthermore, two variables analogical to $\chi$ and $\eta$ (eqs. (7) and (8)) are introduced:

$$
\begin{align*}
& \chi^{\prime}=\min \left(\left|\boldsymbol{q}_{i}\right| / \sum_{i=1}^{2}\left|\boldsymbol{q}_{i}\right|\right)  \tag{10}\\
& \eta^{\prime}=\left|\cos \left(\boldsymbol{n}_{1} \cdot \boldsymbol{n}_{2}\right)\right| \tag{11}
\end{align*}
$$

## 4. - Algorithm for computing the jet variables.

The calculation of the jet variables for all combinations of possible momentum subsets requires an enormous amount of computing time. Therefore, a simpler and faster algorithm is worked out. The main idea of the algorithm is to divide the particles a priori into subsets with maximum total momentum by the following procedure:

1) particle $k$, which has the maximum momentum (call it $\boldsymbol{p}_{\mathbf{1}}^{0}=\boldsymbol{p}_{k}$ ) is chosen as the basic particle;
2) subsequently to $\boldsymbol{p}_{1}^{0}$ all the remaining particle momenta $\boldsymbol{p}_{i}^{(1)}=\boldsymbol{p}_{1}^{0}+\boldsymbol{p}_{i}$ $(i \neq k)$ are added;
3) all $\left|\boldsymbol{p}_{i}^{(1)}\right|$ are compared with each other and $\left|\boldsymbol{p}_{\mathbf{1}}^{0}\right|$ and the maximum momentum is chosen:

$$
\begin{equation*}
\left|\boldsymbol{p}_{1}^{1}\right|=\max \left(\left|\boldsymbol{p}_{i}^{(i)}\right|,\left|\boldsymbol{p}_{1}^{\mathbf{0}}\right|\right) . \tag{12}
\end{equation*}
$$

Now, if $\boldsymbol{p}_{1}^{1}=\boldsymbol{p}_{1}^{0}$, then the procedure is finished and only particle $k$ belongs to $C_{1}$, all other particles belong to $C_{2}$. But, if $\boldsymbol{p}_{1}^{1}=\boldsymbol{p}_{j}^{(1)}$ for some $j$, then $\boldsymbol{p}_{1}^{1}$ is the chosen basic momentum and the procedure is repeated until the addition of no further $\boldsymbol{p}_{i}$ increases the size of the sum of momenta $\boldsymbol{p}_{1}^{\boldsymbol{m}}$. Those particles whose momenta construct $\boldsymbol{p}_{1}^{m}$ belong to $C_{1}$, the others to $C_{2}$.

For collinear events ( $T \geqslant 0.75$ ), we have seen that the thrust values and axes computed by this and traditional algorithms almost coincide. The mean values of thrust differ by less than $1 \%$. For events which strongly deviate from the two-jet configuration, our algorithm gives a smaller value of thrust. Consequently, the presented algorithm is quite suitable for selecting the twojet events.

For computing dithrust and triplicity, the described algorithm can be extended; then the amount of combinations, which need to be computed, is

$$
\begin{equation*}
\tilde{K}_{N}^{(3)}=N^{2}(N-2)(N-1) \tag{13}
\end{equation*}
$$

For thrust this amount is

$$
\begin{equation*}
\tilde{K}_{v}^{(2)}=N(N-2) \tag{14}
\end{equation*}
$$

$E . g .$, for $N=10,12$, we get $\tilde{K}_{10}^{(2)}=80, \tilde{K}_{12}^{(2)}=120$ and $\tilde{K}_{10}^{(3)}=7200, \widetilde{K}_{12}^{(3)}=$ $=15840$, which are significantly smaller than those which follow from eqs. (4) and (6).

## 5. - Results and discussion.

To investigate the jet structure the two two-dimensional diagrams, thrust $v s$. triplicity and thrust $v s$. dithrust are plotted. For the ideal two-jet



Fig. 4. - Thrust $T$ vs. triplicity TR for events with $n_{\mathrm{ch}}=6$ without (a)) and with (b)) an identified proton.
structure all of the variables thrust $T$, triplicity TR and dithrust DT reach their maxima at 1 , but for ideal crosslike and three-jet conflgurations the following relations hold, respectively: $T<\mathrm{DT}=1, T<\mathrm{TR}=1$.

Thrust vs. triplicity for events of charged multiplicity $n_{\mathrm{ch}}=6$ without and with an identified proton is shown in fig. 4a) and $b$ ). One sees the more collinear character of the events with the identified proton. This difference persists for all the multiplicities, which is seen in fig. 5, upper part, where


Fig. 5. - The relative amount of events without and with an identified proton in regions of different values of thrust $T$ and triplicity TR.


Fig. 6. - Thrust $T$ vs. dithrust DT for events with $n_{\text {ch }}=6$ without (a)) and with (b)) an identified proton.
the relative amounts of events are shown for different multiplicities in different regions of $T$ and $T R$. The density maxima on the plots move from the twojet region to the more spherical configurations with increasing multiplicity. Figure 6 illustrates the corresponding distributions in terms of $T$ and DT.


Fig. 7. - The relative amount of events without and with an identified proton in regions of different values of thrust $T$ and dithrust DT.

The relative densities for various regions of the plots and for different multiplicities are shown in fig. 7.

The above distributions demonstrate the mostly collinear character of the experimental results, more pronounced for the events with identified protons. This phenomenon is in agreement with the more probable existence of the configurations shown in fig. $2 a$ ) and $3 a$ ) in nonannihilation events. Clear crosslike and three-jet configurations were not observed.

The two-dimensional plot $\chi$ vs. $\eta$ (defined by eqs. (7) and (8)) allows us to distinguish specific configurations of momentum distributions between subsets (jets) and mutual orientation of jet axes. For events with identified protons the region of isotropical momentum space distribution, $\chi>0.16$,

| TR | no $p_{\text {id }}$ |  |  |  | with $p_{\text {id }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $n_{\text {ch }}$ | 4 | 6 | 8 | 10 | 4 | 6 | 8 | 10 |
| $\begin{array}{ll} 0 & N \\ 0 & N \\ i & 0 \\ & \underset{\sim}{2} \end{array}$ | 77 |  |  |  | 2 |  |  | $54$ |
|  | 27 | 22 |  |  |  |  |  |  |
|  | 72 |  |  |  |  |  |  |  |
| $\begin{array}{ll} \underline{\varphi} & \stackrel{n}{N} \\ \stackrel{0}{0} \\ \underset{\sim}{V} & \stackrel{V}{v} \\ \hline \end{array}$ | 7 | $170$ | 78 |  | $\mathbb{D}$ | 8 |  |  |

Fig. 8. - The relative amount of events without and with an identified proton in regions of different values of the triplicity variables $\chi$ and $\eta$.
$\eta<0.25$ (see fig. 8) has a lower density than for the events without identified protons. This is in agreement with the collinear character of these events. With increasing multiplicity the density in this region increases, which is due to the more spherical event shape at high multiplicities.


Fig. 9. - The relative amount of events without and with an identified proton in regions of different values of the dithrust variables $\chi^{\prime}$ and $\eta^{\prime}$.

The relative densities in various configuration regions of $\chi^{\prime}$ and $\eta^{\prime}$ (defined by eqs. (10) and (11)) are shown in fig. 9. They are very similar with the ones in fig. 8, which supports the conclusion of the absence of clear crosslike and three-jet configurations.

The dominance of collinear configurations can be understood within the dual topological unitarization scheme, as a result of strong inequality in the energy partition between dual chains, when one of the chains determines the event configuration and/or as a result of the dominance of low $p_{T}$. The more collinear configuration of the events with identified protons can be due to the leading-baryon effects.

## - RIASSUNTO (*)

E stata studiata la forma degli eventi nello spazio dei momenti nelle interazioni $\bar{p} p$ a $22.4 \mathrm{GeV} / \mathrm{c}$ per mezzo delle variabili spinta, triplicità e doppia spinta. Non si osservano eventi con chiara forma di multigetto che corrispondono alle previsioni dello schema di unitarizzazione duale topologica. Gli eventi hanno soprattutto una forma collineare. Si propone un algoritmo veloce per calcolare le variabili di forma degli eventi.
(*) Traduzione a cura della Redazione.

Рвспределения переменных ливней в антипротон-протонных взаимодействиях.

Резюме (*). - Исследуется форма событий в импульсном пространстве для $\bar{p} p$ взаимодействий при 22.4 ГэВ/с. Не наблюдаются события с многоструйной формой, соответствующей предсказаниям схемы дуальной топологической унитаризации. События, в основном, имеют коллинеарную форму. Предлагается быстрый алгоритм для вычисления переменных формы событий.
(") Переведено редакцией.


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