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LOW-ENERGY EFFECTS OF MIRROR LEPTONS

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Abstract

We study the effects of mixing of leptons and mirror leptons in charged current and neutral current processes at low energy. We also discuss the effects of the new phenomenon of neutrino-mirror neutrino oscillations which may occur with light mirror neutrinos. Using a recent data set we determine the allowed ranges of the mixing angles and of $\sin^2\theta_W$ in several mixing models, distinguished by different assumptions on the mirror neutrino masses. We conclude that sizeable mixing is possible and that a light electron-like mirror neutrino may be the explanation for the electron deficiency observed in beam dump experiments.

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1. INTRODUCTION

Mirror leptons are leptons with V+A interactions, all other quantum numbers being the same as for ordinary leptons. The charged mirror leptons must be heavier than about 20 GeV since they have not been pair-produced at PETRA [1]. One also expects that they are lighter than a few hundred GeV, since their mass originates in the breaking of $SU(2)_W \times U(1)_Y$ and also because they can affect radiatively the value of the parameter $= (M_W/M_Z \cos\theta_W)^2$ which is known to be close to 1.0 [2].

Mirror leptons appear in many grand unified theories based on symmetry groups such as $SO(n > 10)$ 3,4 , $SU(n < 5)$ [5] and E_8 [6]. They can also appear in composite models [7], and may also be needed in globally super-symmetric theories with an extra U(1)-group à la Fayet [8] to cancel axial anomalies connected with that group [9].

When mirror leptons mix with ordinary leptons, either via explicit Yukawa terms or radiatively [3a], they change the chiral structure of weak currents. Charged currents acquire a small V+A component, while for neutral currents only the axial vector part will be modified. These changes might be detectable already at present energies. We call this scheme "the mirror mixing model".

In the above mentioned theories there naturally appear mirror quarks as well as mirror leptons. However, the possible quark-mirror quark mixing angles must be so small that they have no observable effect on the weak processes because the mirror quark mixing would otherwise spoil the subtle cancellation of flavor changing neutral currents (FCNC). In the most general case FCNC arise already at the tree level. According to a recent general analysis [10] of the first two generations, the observed absence of FCNC sets an upper bound of the order of 10^{-3} to the mirror mixing angles of the quark sector. A more stringent limit is obtained from the fact that FCNC are alos suppressed at the one-loop level [15]. Therefore in the following we will neglect all mirror mixings in the quark sector and thus the quark weak interactions are those of the standard GWS model.

In contrast, the effects of mixing of ordinary leptons and mirror leptons can be much larger. For this reason we have undertaken to derive expressions for the cross sections of all the relevant charged current (CC) and neutral current (NC) reactions including the effects of mirror leptons and to make a statistical analysis to determine the range of mixing parameters allowed by the most recent data.

The present work extends earlier work [11-14] which concentrated exclusively on charged weak interactions. A more complete version of the present work has been published elsewhere [15-16].

The paper is organized as follows. In Section 2 we recapitulate the main features of the mirror mixing model. We discuss the constraints of charged current reactions in Section 3, of leptonic neutral current reactions in Section 4, and of semileptonic neutral current reactions in Section 5. Section 6 presents the results of the fits. In Section 7 we discuss the implications of non-universality and of the mirror mixing model for beam dump experiments. Section 8 contains our conclusions.

2. THE MIRROR MIXING MODEL

An example of a GUT based on a symmetry group containing mirror fermions is SO(11) [3b]. The fundamental spinor representation is real, 32-dimensional, and decomposes under SO(10) as

$$\underset{\sim}{32} = \underset{\sim}{16} + \underset{\sim}{16}^* ,$$

where the conventional lefthanded fermions of one family populate the $\underset{\sim}{16}$ spinor of SO(10). The $\underset{\sim}{16}^*$ contains a corresponding set of mirror fermions. Analogously to the ordinary fermions, the mirror electron L_e and its mirror neutrino N_e go into an $SU(2)_L$ doublet $\begin{pmatrix} N'^c_e \\ L'^c_e \end{pmatrix}_L$, the mirror quarks U, D form doublets $\begin{pmatrix} U'^c \\ D'^c \end{pmatrix}_L$ of three colors, and the remaining left-handed mirror fermions of $\underset{\sim}{16}^*$, D'_L, U'_L, L'_e, N'_{e_L} form 8 singlets (primes denote weak interaction eigenstates).

Consequently, the coupling of the ordinary fermions to the W-boson is left-handed, e.g.

$$\bar{\nu}'\gamma^\alpha(1-\gamma_5)e'W_\alpha ,$$

whereas the mirror fermions couple righthandedly to the same W-boson, e.g.

$$\bar{N}'_e\gamma^\alpha(1+\gamma_5)L'_e W_\alpha .$$

Let us denote the mass eigenstates [11]

$$\psi_\ell = \begin{pmatrix} \ell^- \\ L^-_\ell \end{pmatrix} , \quad \psi_{\nu_\ell} = \begin{pmatrix} \nu_\ell \\ N_\ell \end{pmatrix} , \tag{1}$$

where $l = e, \mu, \tau$. Allowing for the mixing of leptons with the corresponding mirror leptons through the mass matrix, the weak Lagrangian in the mass eigenstate basis takes the following form

$$\mathcal{L}^{CC} = -\frac{g}{2\sqrt{2}} W^\dagger_\alpha \sum_\ell \left\{ \bar{\psi}_{\nu_\ell}\gamma^\alpha \left[R(\varphi_\ell-\theta_\ell) - R(\varphi_\ell+\theta_\ell)\gamma_5\tau_3\right]\psi_\ell \right\} + h.c. ,$$

$$\mathcal{L}^{NC} = -\frac{g}{4\cos\theta_W} Z_\alpha \sum_\ell \Big\{ \bar{\psi}_{\nu_\ell} \gamma^\alpha \left[1 - R(2\varphi_\ell)\gamma_5\tau_3\right] \psi_{\nu_\ell} + $$
$$+ \bar{\psi}_\ell \gamma^\alpha \left[4\sin^2\theta_W - 1 + R(2\theta_\ell)\gamma_5\tau_3\right]\psi_\ell \Big\} \, , \qquad (2)$$

where the $R(\theta)$ are the 2x2 orthogonal mixing matrices diagonalizing the respective mass matrix. The particular V,A-structure of the Lagrangian (2) is a direct consequence of the rotation of chirality eigenstates to mass eigenstates.

If we restrict ourselves to the e and μ families, the Lagrangian (2) depends on the 1-L mixing angles θ_e, θ_μ and on the ν_1-N_1 mixing angles φ_e, φ_μ.

It is known from e^+e^- experiments that charged mirror fermions must be heavier than about 20 GeV. Mirror neutrinos, however, may be light ($< m_e$) or heavy ($> m_K$). Their couplings to the muon are known to be severely restricted for masses in the approximate range of 10 MeV and 200 MeV [17]. Therefore we will consider the following four extreme cases for the masses of mirror neutrinos of the first two generations:

Model a) $\quad m_{N_e}, m_{N_\mu} < m_e$

model b) $\quad m_{N_e} < m_e$, $m_{N_\mu} > m_K$

model c) $\quad m_{N_e}, m_{N_\mu} > m_K$

model d) $\quad m_{N_\mu} < m_e$, $m_{N_e} > m_K$.

Model c) with heavy mirror neutrinos decoupled from low energy phenomena also corresponds to the more general case that only ordinary fermions are present but with general V,A CC couplings and modified NC couplings.

The limit of the standard GWS model is obtained, as expected, when all the mixing angles vanish. The structure of the charged current constraints is, however, such that the V-A limit may also be obtained for some choices of non-vanishing mixing angles [12,13]. For instance in model c, this is ture if two of the angles vanish and the two other angles are equal but non-vanishing. This degeneracy is removed when both charged and neutral current constraints are used [15].

In models with light mirror neutrinos the neutrino beam produced in pseudoscalar or nuclear beta decay is a coherent superposition of the two mass eigenstates [12,14]. This leads to an oscillating probability for finding an interacting neutrino or mirror neutrino. The phenomenon is analogous to the well known case of flavor oscillations, but the additional novel feature is that now both chiralities interact. The rate of oscillations depends on the difference of the squared masses of neutrinos and mirror neutrinos, $\Delta m_\nu^2 = \left| m_N^2 - m_\nu^2 \right|$.

The main decay model of <u>light</u> mirror neutrinos as well as neutrinos is radiative decay $N_\ell \rightarrow \nu_\ell \gamma$ (or $\nu_\ell \rightarrow N_\ell \gamma$, whichever is kinematically allowed). For neutrinos lighter than, or of the order of 100 eV, however, there exist severe astrophysical constraints on the lifetime of unstable neutrinos [18]. These come from the observed cosmic photon spectrum and the 3° K background radiation, and do not allow for the existence of light neutrinos with lifetimes less than about 10^{20} s. For longlived neutrinos in the mass range 10-100 eV there are also limits coming from the observed galactic UV radiation [19].

The first possibility to conform with these limits is to set θ_ℓ equal to zero. Since φ_ℓ can still be large and since there are no very strict bounds on the masses of neutrinos and mirror neutrinos, neutrino-mirror neutrino oscillation is possible. The scattering cross sections are then in general dependent on the oscillating term which includes the unknown neutrino mass factor Δm_ν^2. Assuming that this mass factor is such that, in all scattering processes to be considered, neutrino-mirror neutrino oscillations are already well developed before the beam hits the detector, it follows that the oscillating term is averaged and the mass dependence drops out. The beam is then effectively an incoherent mixture of neutrino and mirror neutrino mass eigenstates. Accordingly, we call this <u>incoherent</u> scattering.

This incoherent scattering is justified e.g. if m_N and m_ν are not accidentally degenerate and if the larger of them is at least of the order of 10 eV.

The second possibility which we consider here more thoroughly, is to set a strict upper bound on all light neutrino masses ($N = N_\ell, \nu_\ell$)

$$m_N \lesssim 2 \cdot 10^{-2} \text{ eV} ,$$

in which case oscillations turn on very slowly. Therefore the scattering cross sections are now independent of the arbitrary mass difference and also of the respective neutrino mixing angle φ_ℓ. We call this <u>coherent</u> scattering. In the following analysis we distinguish between coherent and incoherent scattering.

The third possibility, to which we shall return briefly in Sec. 7, is to have mirror neutrinos heavier than about 1 keV.

Because of the scarcity of data available, we cannot afford to keep the ν_1 and N_1 masses as free parameters in the fits. The models a-d are therefore extreme cases which permit us to neglect these masses.

3. CHARGED CURRENT REACTIONS

3.1 Muon decay

In contrast to the V-A theory, the muons produced in $\pi_{\mu 2}$ and $K_{\mu 2}$ decay no longer have the trivial longitudinal polarization $P_{\mu\pm} = \mp 1$. Moreover, the muons have four possible decay channels:

(i) $\mu \rightarrow e\nu_\mu\bar{\nu}_e$

(ii) $\mu \rightarrow e\nu_\mu\bar{N}_e$

(iii) $\mu \rightarrow eN_\mu\bar{\nu}_e$

(iv) $\mu \rightarrow eN_\mu\bar{N}_e$

Kinematically allowed processes in model a are all the channels (i) to (iv), in model b: (i) and (ii), in model c: (i), and in model d: (i) and (iii). The energy spectrum of the decay electron can be parametrized in the usual form [20]:

$$\frac{d\Gamma}{dx\, d\cos\theta} = F(x,\rho,\eta) + \xi F_\theta(x,\delta)\cos\theta + \\ + \xi' F_\phi(x,\delta')\cos\phi + \xi'' F_{\theta\phi}(x,\rho',\eta')\cos\theta\cos\phi + \\ + F_{T_1}(x,\alpha,\beta)\sin\theta\sin\phi\cos\psi + F_{T_2}(x,\alpha',\beta')\sin\theta\sin\phi\sin\psi. \tag{3}$$

For the definition of the decay geometry we refer to Scheck [20].

In pure V,A-models (with no S, P, T components in the charge changing form of the Lagrangian) some parameters vanish identically:

$$\eta = \alpha = \alpha' = \beta = \beta' = 0 .$$

The spectrum (3) determines the parameters ρ, δ, ξ and the Fermi coupling constant G_F. The parameter ξ'' has not been measured. The different subprocesses contribute additively to these parameters.

The longitudinal polarization of the final state electron is given by the parameter $\xi' = -P_{e^-}$.

We use the experimental values

$$\rho = 0.7517 \pm 0.0026 \quad [21] \quad , \quad \text{V-A value } \frac{3}{4}$$

$$\delta = 0.7551 \pm 0.0085 \quad [21] \quad , \quad \text{V-A value } \frac{3}{4}$$

$$-\xi P_{\mu^+} = 0.975 \pm 0.015 \quad [22] \quad , \quad \text{V-A value } 1 .$$

The value of ξ' is discussed in the next section.

The theoretical expressions for the above quantities in the different models are detailed elsewhere [12].

3.2 Nuclear beta decay

Measurement of the longitudinal polarization of the electron in the nuclear ß-decay provides a well-known test for the V-A hypothesis. Pure V-A predicts the value $P_{\beta^-} = - v/c$. The expression corresponding to the most general case is given e.g. by Marshak et al. [23]. In the models we are considering this reduces to (aside from the factor v/c) the same quantity as in the electron longitudinal polarization in the muon decay [12].

Thus we can average the measurement

$$- P_{e^+} = 1.008 \pm 0.054 \qquad [24]$$

from muon decay, and

$$- \frac{c}{v} P_{\beta}^{\text{Gamow-Teller}} = 1.001 \pm 0.008 \qquad [25]$$

from nuclear beta decay. The V-A value is 1.

3.3 Inverse muon decay

The process

$$\text{(i)} \qquad \nu_\mu + e^- \rightarrow \mu^- + \nu_e \qquad (4)$$

provides information about the V,A structure of the leptonic charged currents beyond what μ-decay without observation of the decay neutrinos gives.

In the fermion-mirror fermion mixing model there are three further processes that may contribute to the inverse muon decay:

(ii) $\nu_\mu + e^- \rightarrow \mu^- + N_e$

(iii) $N_\mu + e^- \rightarrow \mu^- + \nu_e$

(iv) $N_\mu + e^- \rightarrow \mu^- + N_e$

The differential cross sections for reactions (ii) to (iv) can be obtained from that of reaction (i) with obvious modifications. If the muonic mirror neutrino N_μ is light enough to be produced in the pseudoscalar decays, the neutrino beam scattering on electrons consists of two components ν_μ and N_μ, with well known relative probabilities.

The allowed processes in the different models are the following: model a (i)-(iv), model b (i) and (ii), model c (i), and model d (i) and (iii).

In the experimental studies the differential cross section is presented in the units of $G_F^2 s/2\pi$, the value of the cross section in the V-A limit. The expressions for this quantity which we dnote S, are in the different models given in Ref. 12.

Experimentally we have

$$S = 0.98 \pm 0.12 \quad [27].$$

3.4 Pseudoscalar meson leptonic decay

For the muon polarization in pion decay we have $-P_{\mu^+} = 0.99 \pm 0.16$ [27], V-A value 1.

The partial widths for 12-decays of the pseudoscalars π and K are given by

$$\Gamma(P \to \ell \bar{\nu}_\ell) = \frac{\tilde{G}^2}{8\pi} f_P^2 c_P^2 \frac{m_\ell^2}{m_P^3} (m_P^2 - m_\ell^2)^2 \cdot f(\Theta_\ell, \varphi_\ell) , \tag{5}$$

where f_p, $p = \pi, K$ is the decay constant and c_p stands for the Cabibbo factor. The coupling constant $\tilde{G}$ is defined by $\tilde{G}/\sqrt{2} = g^2/8M_W^2$. In models a, b and d also the nondiagonal decay $P \to l\bar{N}_1$ is allowed.

We use in our analysis the ratios of the total electronic and muonic 12-widths

$$R_P = \frac{\Gamma_{total}(P_{e2})}{\Gamma_{total}(P_{\mu 2})} = \frac{m_e^2 (m_P^2 - m_e^2)^2}{m_\mu^2 (m_P^2 - m_\mu^2)^2} \cdot (1 + \Delta_P) \cdot r , \tag{6}$$

where Δ_p is the $O(\alpha)$ radiative correction as calculated by Goldman and Wilson [28], $\Delta_\pi = -0.0347$ and $\Delta_K = -0.0372$. The factor r is a ratio of terms depending on the parameters of the lepton currents. The expressions for R_P in the different models are given in Ref. 12.

We use an average of R_π and R_K:

$$R_{\pi,K} = 1.016 \pm 0.017 \quad [21].$$

4. LEPTONIC NEUTRAL CURRENT REACTIONS

4.1 Elastic $\bar{\nu}_e e^-$-scattering

This reaction proceeds through both charge current W^- exchange and neutral current Z^o exchange. In models c and d the electron mirror neutrino N_e is heavy and thus decoupled from the low energy scattering phenomena. In models a and b the N_e mirror neutrino is light, so we have the two possibilities of coherent and incoherent scattering, as discussed in Sec. 2.

Let us first consider incoherent scattering. There the beam is an incoherent mixture of the two mass eigenstates $\bar{\nu}_e$ and $\bar{N}_e$. We then have the following four subprocesses which contribute to "elastic" $\bar{\nu}_e e^-$-scattering:

$$\begin{aligned} \bar{\nu}_e\, e^- &\rightarrow \bar{\nu}_e\, e^-\,, \\ \bar{\nu}_e\, e^- &\rightarrow \bar{N}_e\, e^-\,, \\ \bar{N}_e\, e^- &\rightarrow \bar{\nu}_e\, e^-\,, \\ \bar{N}_e\, e^- &\rightarrow \bar{N}_e\, e^-\,. \end{aligned} \tag{7}$$

The coupling constants for each of these subprocesses can be read off from the Lagrangian (2) with $\theta_e = 0$.

In the case of coherent scattering the bam can be regarded effectively as an incoherent mixture of weak eigenstates $\bar{\nu}'_e$ and $\bar{N}'_e$ whose interactions are easily seen in weak currents written in a mixed basis of neutrino weak eigenstates and charged lepton mass eigenstates:

$$J^{\mu}_{CC} = -\frac{g}{2\sqrt{2}}\Big\{\cos\theta_\ell\,\bar{\nu}'_\ell\,\gamma^\mu(1-\gamma_5)\ell - \sin\theta_\ell\,\bar{N}'_\ell\gamma^\mu(1+\gamma_5)\ell + \tag{8}$$

$$+\sin\theta_\ell\,\bar{\nu}'_\ell\,\gamma^\mu(1-\gamma_5)L_\ell - \cos\theta_\ell\,\bar{N}'_\ell\gamma^\mu(1+\gamma_5)L_\ell\Big\},$$

$$J^{\mu}_{NC} = -\frac{g}{4\cos\theta_W}\Big\{\cos\theta_\ell\cdot\bar{\nu}'_\ell\gamma^\mu(1-\gamma_5)\nu_\ell - \sin\theta_\ell\,\bar{N}'_\ell\gamma^\mu(1+\gamma_5)\nu_\ell \tag{9}$$

$$+\sin\varphi_\ell\,\bar{\nu}'_\ell\gamma^\mu(1-\gamma_5)N_\ell + \cos\varphi_\ell\,\bar{N}'_\ell\gamma^\mu(1+\gamma_5)N_\ell\Big\}.$$

As above, all the four subprocesses (7) with the incoming neutrinos being now the weak interaction eigenstates $\bar{\nu}'_e$ and $\bar{N}'_e$ add up incoherently to the total cross section.

The final result for both coherent and incoherent scattering cross sections has been given in Ref. 15. We note here that the cross section is proportional to the oscillation factor $P^e_{\nu\nu}$ given by

$$P^e_{\nu\nu} = 1 \qquad \text{(coherent case)},$$
$$P^e_{\nu\nu} = 1 - \tfrac{1}{2}\sin^2 2\varphi_e \qquad \text{(incoherent case)}. \tag{10}$$

Thus for models a and b in the incoherent case this reaction furnishes information on φ_e, whereas no other reaction does.

We note also that the cross section for this reaction in model b does depend also on φ_μ and θ_μ, which might seem surprising, since no muons or muon neutrinos participate in the reaction. The reason is that the scale of the cross section is given by a modified Fermi constant $\tilde{G}$, determined in muon decay and thus dependent (differently in the four models) on some of the four mixing angles [12]. This dependence is, however, rather weak.

We use the experimental cross section of the Savannah River reactor experiment [29] for low energy neutrinos,

$$\langle \sigma(\bar{\nu}_e e^-) \rangle_{low} = (7.6 \pm 2.2) \quad 10^{-46} \ \mathrm{cm}^2$$

together with the Davis antineutrino spectrum [30]. The higher energy neutrino cross section is more dependent on uncertainties in the anti-neutrino beam spectrum. As it is in serious conflict with other data we choose not to include it as a constraint.

The standard model with $\sin^2\theta_W = 0.239$ (from our best fit in the absence of mirror mixing) yields 5.58 10^{-46} cm^2 for this cross section.

4.2 Elastic $\overset{(-)}{\nu}_\mu e^-$-scattering

These reactions proceed only through Z^o exchange. The cross sections follow easily from the previous case by changing φ_e to φ_μ and by neglecting the electron mass terms.

Now it is models a and d, containing a light muonic mirror neutrino, which allow to differentiate between coherent and incoherent scattering.

We take the experimental values from Mo [31], however, subtracting out the (then preliminary) cross sections of the CHARM experiment:

$$\sigma^{exp}(\bar{\nu}_\mu e^-) = (1.54 \pm 0.67)\cdot 10^{-42} E_{\bar{\nu}} \left[\frac{cm^2}{GeV}\right]$$ (standard model: 1.38),

$$\sigma^{exp}(\nu_\mu e^-) = (1.46 \pm 0.24)\cdot 10^{-42} E_{\nu} \left[\frac{cm^2}{GeV}\right]$$ (standard model: 1.50).

From the CHARM experiment [32] we take the cross section ratio

$$\frac{\sigma^{exp}(\nu_\mu e^-)}{\sigma^{exp}(\bar{\nu}_\mu e^-)} = 1.37 \begin{smallmatrix} +0.65 \\ -0.44 \end{smallmatrix}$$ (standard model: 1.09).

4.3 NC effects in $e^+e^- \to \mu^+\mu^-$

The formal expression for the differential cross section $\frac{d\sigma}{d\Omega}(e^+e^- \to \mu^+\mu^-)$ is the same for all the models considered in this paper. It is given by

$$\frac{d\sigma}{d\Omega}(e^+e^- \to \mu^+\mu^-) = \frac{\alpha^2}{4s}\left[F_1(s)(1+\cos^2\theta) + F_2(s)\cos\theta\right] , \tag{11}$$

where $F_1(S)$ and $F_2(S)$ are functions of the vector and axial vector NC coupling constants of the leptons, depending on the charged lepton mixing angles θ_e and θ_μ as well as on $\sin^2\theta_W$ [11].

The forward-backward asymmetry A^{FB} is then easily obtained from Eq. (11):

$$A^{FB} = \frac{3}{8}\,\frac{F_2(s)}{F_1(s)} . \tag{12}$$

For the experimental input we use the averaged values [33] given for three energy regions (mean $\sqrt{s}$ = 14, 22 and 34 GeV):

A^{FB}_{exp}(14 GeV) = (2.6 ± 3.0) % (standard model: -1.4),

A^{FB}_{exp}(22 GeV) = (-6.9 ± 3.4) % (standard model: -3.7),

A^{FB}_{exp}(34 GeV) = (-11.9 ± 1.5) % (standard model: -10.5).

There exists also separate information on the coefficient of the $(1 + \cos^2\theta)$-term in Eq. (11) coming from the quantity

$$\frac{\sigma - \sigma_{QED}}{\sigma_{QED}} = \frac{4s\,G_F}{\sqrt{2}\,e^2}\,h_{VV} \quad (13)$$

In our models, we have

$$h_{VV} = \frac{\tilde{G}}{G_F}\cdot\frac{1}{4}\left(1 - 4\sin^2\theta_W\right)^2. \quad (14)$$

We use the experimental average of PETRA measurements [34]

$$h_{VV}^{exp} = 0.009 \pm 0.040 \qquad \text{(standard model: 0.0005)}$$

as an additional constraint furnishing information on the weak mixing angle θ_W.

5. SEMILEPTONIC NEUTRAL CURRENT REACTIONS

5.1 Neutrino hadron scattering

As we have argued in Sec. 1 (and more fully in Ref. 11), the mixing angles between quarks and mirror quarks can be neglected in this study. This is fortunate because otherwise the proliferation of coupling parameters would make all semileptonic data useless. Instead we find that some of the cross sections of neutrino and antineutrino scattering on hadrons can be easily adapted to the mirror mixing model when the leptonic mixing angles are small. To second order in the mixing angles the cross sections for an arbitrary nuclear target N become:

$$\sigma_{CC}(\overset{(-)}{\nu}_\mu N) = \frac{\tilde{G}^2}{G_F^2} k_{CC}\, \sigma_{CC}^{V-A}(\overset{(-)}{\nu}_\mu N) ,$$

$$\sigma_{NC}(\nu_\mu N) = \frac{\tilde{G}^2}{G_F^2} \left\{ k_{NC}^1 \sigma_{NC}^{V-A}(\nu_\mu N) + k_{NC}^2 \sigma_{NC}^{V-A}(\bar{\nu}_\mu N) \right\}, \tag{15}$$

$$\sigma_{NC}(\bar{\nu}_\mu N) = \frac{\tilde{G}^2}{G_F^2} \left\{ k_{NC}^1 \sigma_{NC}^{V-A}(\bar{\nu}_\mu N) + k_{NC}^2 \sigma_{NC}^{V-A}(\nu_\mu N) \right\},$$

where the correction factors for models b and c are

$$k_{CC} \simeq 1 - \theta_\mu^2 - \varphi_\mu^2 ,$$

In models a and d the same equations hold, with changes only in Eqs. (16) and (20):

$$k_{NC}^1 \simeq 1 - 2\varphi_\mu^2 ,$$

$$k_{NC}^2 \simeq 0 . \tag{16}$$

We use the data on isoscalar targets $\bar{N}$ in the form of the very practical Paschos-Wolfenstein relation [35]

$$R^- = \frac{\sigma_{NC}(\nu_\mu \bar{N}) - \sigma_{NC}(\bar{\nu}_\mu \bar{N})}{\sigma_{CC}(\nu_\mu \bar{N}) - \sigma_{CC}(\bar{\nu}_\mu \bar{N})} = r^- R^-_{V-A} , \tag{17}$$

and the similar ratio 36 for the sums

$$R^+ = \frac{\sigma_{NC}(\nu_\mu \bar{N}) + \sigma_{NC}(\bar{\nu}_\mu \bar{N})}{\sigma_{CC}(\nu_\mu \bar{N}) + \sigma_{CC}(\bar{\nu}_\mu \bar{N})} = r^+ R^+_{V-A} , \tag{18}$$

where

$$R^+_{V-A} = \frac{1}{2} - \sin^2\theta_W + \frac{10}{9}\sin^4\theta_W ,$$

$$R^-_{V-A} = \frac{1}{2} - \sin^2\theta_W , \tag{19}$$

are the results in the V-A limit, and

$$r^+ \simeq r^- \simeq 1 - \theta_\mu^2 - \varphi_\mu^2 \ . \tag{20}$$

In models a and d the same equations hold, with changes only in Eqs. (16) and (20):

$$\begin{aligned} k_{CC} &\simeq P_{\nu\nu}^\mu \left(1 - 2\theta_\mu^2\right) , \\ k_{NC}^1 &\simeq P_{\nu\nu}^\mu \left(1 - \theta_\mu^2\right) , \\ k_{NC}^2 &\simeq P_{\nu\nu}^\mu \ \theta_\mu^2 \end{aligned} \tag{21}$$

and

$$r^+ \simeq 1 + 2\theta_\mu^2 \quad , \qquad r^- \simeq 1 \ . \tag{22}$$

Moreover we use data on inclusive (anti)neutrino scattering on protons and neutrons. This data is expressed in terms of the ratios

$$\overset{(-)}{R}_N = \frac{\sigma_{NC}(\overset{(-)}{\nu}_\mu N)}{\sigma_{CC}(\overset{(-)}{\nu}_\mu N)} \ . \tag{23}$$

Using Eq. (15) one easily finds the relations

$$\begin{aligned} R_N &= \left(k_{NC}^1 / k_{CC}\right) R_N^{V-A} + \left(k_{NC}^2 / k_{CC}\right) \bar{R}_N^{V-A} \, S_N^{V-A} , \\ \bar{R}_N &= \left(k_{NC}^1 / k_{CC}\right) \bar{R}_N^{V-A} + \left(k_{NC}^2 / k_{CC}\right) R_N^{V-A} / S_N^{V-A} , \end{aligned} \tag{24}$$

where

$$S_N = S_N^{V-A} = \frac{\sigma_{CC}(\bar{\nu}_\mu N)}{\sigma_{CC}(\nu_\mu N)} \ . \tag{25}$$

The usual parametrization in the V-A limit, e.g., for the ratio R_N^{V-A} is [2,36]

$$R_N^{V-A} = f_1^N u_L^2 + f_2^N d_L^2 + f_3^N u_R^2 + f_4^N d_R^2 \,, \tag{26}$$

where the factors f_i contain the dependence on quark distributions and experimental conditions and the parameters $u_{L,R}$ and $d_{u,R}$ are the well known chiral NC coupling constants of quarks [36].

Although there are five measurements of R_p, two of R_n and one of $\bar{R}_p$ we can make use only of those experiments that report also the values of the constraints f_i (Eq. (26) including their errors. This restricts us to use only two measurements for one experiment [37]

$R_p = 0.47 \pm 0.064$ (standard model: 0.40),

$R_n = 0.22 \pm 0.031$ (standard model: 0.24).

Here the errors of the constants f_i have been propagated to the experimental errors of R_p and R_n.

For the Paschos-Wolfenstein ratio R^- the results of four experiments [38,39] can be combined to yield

$R^- = 0.264 \pm 0.008$ (standard model: 0.261).

The CFRR group [39] has also reported a value of R^+ which we can use:

$R^+ = 0.315 \pm 0.009$ (standard model: 0.325).

For elastic scattering there are four measurements of R_{el}^P, one of $\bar{R}_{el}^p$ and one of R_{el}^n. None of these experiments have evaluated the relevant constants f_i with errors. We therefore refrain from using them in the present work.

5.2 Charge asymmetries in polarized lepton hadron scattering

(i) Electron on deuterium

The asymmetry $A_D(x,y)$ is defined as

$$A_D(x,y) = \frac{d\sigma(e^-_R) - d\sigma(e^-_L)}{d\sigma(e^-_R) + d\sigma(e^-_L)} \quad . \tag{27}$$

Dividing out its proportionality on Q^2 we find for all the mixing models

$$\frac{A_D(x,y)}{Q^2} = \frac{3\tilde{G}}{5\sqrt{2}\,\pi\alpha}\left[\left(C_{1u} - \tfrac{1}{2}C_{1d}\right) + \left(C_{2u} - \tfrac{1}{2}C_{2d}\right)F(y)\right] , \tag{28}$$

where the y-dependent function is $F(y) = \dfrac{1 - (1-y)^2}{1 + (1-y)^2}$ and the coupling constants are

$$\begin{aligned} C_{1u} &= \left(\tfrac{4}{3}\sin^2\theta_W - \tfrac{1}{2}\right)\cos 2\theta_e \\ C_{2u} &= 2\sin^2\theta_W - \tfrac{1}{2} \\ C_{1d} &= \left(-\tfrac{2}{3}\sin^2\theta_W + \tfrac{1}{2}\right)\cos 2\theta_e \\ C_{2d} &= -\,C_{2u} \quad . \end{aligned} \tag{29}$$

Rewriting Eq. (28) in the form $A_D(x,y)/Q^2 = a_1(x) + a_2(x)F(y)$ we can then compare the theoretical formulas with the results of the SLAC experiment [40]:

$$a_1(x) = (-\,9.7 \pm 2.6) \times 10^{-5}\ \mathrm{GeV}^{-2} \qquad \text{(standard model: } -\,7.6\text{)},$$

$$a_2(x) = (4.9 \pm 8.1) \times 10^{-5}\ \mathrm{GeV}^{-2} \qquad \text{(standard model: } -\,0.7\text{)}.$$

(ii) Muon on carbon

There also exists a recent result for the asymmetry B from μC-scattering [41] where

$$B(P_1, P_2) = \frac{d\sigma(\mu^+, P_1) - d\sigma(\mu^-, P_2)}{d\sigma(\mu^+, P_1) + d\sigma(\mu^-, P_2)} . \tag{30}$$

The expression valid for B in all mixing models is

$$B(-P, P) = -\frac{3}{4} \cdot \frac{3\tilde{G}}{5\sqrt{2}\pi\alpha} \left[\cos 2\theta_\mu + P(4\sin^2\theta_W - 1) \right] F(y) Q^2 . \tag{31}$$

The experimental value is (with P = 0.81):

$$\frac{B(-P,P)}{F(y)Q^2} = -(1.40 \pm 0.35) \times 10^{-4} \ \mathrm{GeV}^{-2} \quad \text{(standard model: } -1.56\text{)}.$$

6. RESULTS OF THE FITS

The values of the mixing angles have been determined previously [12,13] using leptonic CC data alone. As noted in Ref. 12 there are two reasons for extending the analysis to neutral currents. One is that some NC processes already have the same level of experimental accuracy as the CC reactions and therefore they help to determine the mixing parameters, at the cost of introducing only one additional parameter, $\sin^2\theta_W$.

The other reason is that some NC processes make restrictions on the mixing angles beyond what the CC data do. As found in Refs. 12, and 13 the CC data leave the V-A limit in model c arbitrary in the sense that many non-trivial values of mixing angles give the V-A predictions. Now, the inclusion of NC data resolves this problem. For example e^+e^--asymmetry measures essentially the product $\cos 2\theta_\mu \cdot \cos 2\theta_e$, and the $\overset{(-)}{\nu}_\mu e$-scattering depends on $\cos 2\varphi_\mu$. Thus the predictions of the standard GWS theory (including the V-A structure in the CC sector) are obtained in model c only when all the mixing angles vanish.

Furthermore, as noted in Ref. 12, since the CC data do not involve any scattering experiment with $\overset{(-)}{\nu}_e$ beams, the corresponding neutrino mixing angle φ_e is left arbitrary in models a and b. Including now the elastic $\bar{\nu}_e e$ -scattering in the analysis we get also φ_e constrained in models a and b with incoherent scattering.

We group the models we have tested into four classes:

(i) Models a, b and d with coherent neutrino scattering (call them a_{coh}, b_{coh}, d_{coh}). The respective neutrino mixing angle(s) φ_ℓ remain(s) undetermined.

(ii) The same models with incoherent neutrino scattering (a_{inc}, b_{inc}, d_{inc}). In this case, as explained in Sec. 2, the corresponding charged lepton mixing angle(s) θ_ℓ must be vanishingly small (for the case of light mirror neutrinos, $m_N \lesssim 100$ eV, that we are now interested in). This implies a non-universal V-A structure for charged currents and the standard NC couplings for charged leptons. However, neutrino mixing angles φ_ℓ can now be restricted.

(iii) The general 5-parametric model c and the two 3-parametric special cases: model c_{V-A} where the charged lepton mixing angles are set to zero and model c_{univ} where the universality constraints $\theta_e = \theta_\mu$ and $\varphi_e = \varphi_\mu$ are imposed.

(iv) The standard (universal) GWS model.

In all models we find that the mirror mixing angles are consistent with zero, although their best values may differ from zero. We tabulate in Table 1 for each angle its best fit value and, in brackets, the 68.3 % upper confidence limit. This confidence limit corresponds to the requirement that all the non-vanishing mixing angles are simultaneously within their confidence limits, however θ_W is allowed to assume any value. In the mixing models we therefore do not give any errors for $\sin^2\theta_W$. For the standard GWS model we use another procedure: since θ_W is then the only free parameter we give it with its usual 1σ error.

To give an idea of the variation of $\sin^2\theta_W$ allowed in the mixing models, we plot in Fig. 1 the contour of the simultaneous 63.8 % confidence region of all the 5 parameters in model c projected onto the $(\sin^2\theta_W, \varphi_\mu)$-plane.

We review now briefly the results presented in Table 1 for the mixing angles for model c.

Firstly, the effect of mixing on $\sin^2\theta_W$ is to decrease it. This can particularly be seen in models b and c where the largest best fit values for the mixing angles are obtained. The value of $\sin^2\theta_W$ in the standard model is

$$\sin^2\theta_W \quad) \quad 0.239 \pm 0.007$$

The reason for this value being about 1σ higher than in other global fits of NC data [2,42] is that we have used the data on inclusive (anti)neutrino scattering on isoscalar target (which determines $\sin^2\theta_W$ most accurately) in the Paschos-Wolfenstein form.

Secondly, the charged lepton mixing angles are much more constrained (in coherent models) than the neutrino mixing angles (in incoherent models). This leaves the possibility of large neutrino-mirror neutrino oscillations valid. Note that in coherent models one or two φ_ℓ's remain totally unrestricted.

Thirdly, the inclusion of NC data in addition to CC (see Ref. 12) has only slightly modified the values and limits of the mixing angles of models a, b and d. However, model c is now much better determined. This is due to the resolution of the above mentioned multiple V-A limit problem of CC data.

7. BEAM DUMP EXPERIMENTS

The observation in the beam dump experiments [43,44] is that the prompt neutrino flux from the beam dump produces electrons and muons in the ratio

$$R_1 = \frac{e^+ + e^-}{\mu^+ + \mu^-} \simeq 0.52 \pm 0.15$$

This in contrast to the expectation, $R_1 = 1$, if universality holds and if the main source of prompt neutrinos is the decay of charmed hadrons.

Various suggestions have been made to explain this e-μ asymmetry. If the electron neutrino oscillates to a neutrino of another flavour, say ν_τ [45], the amount of produced electrons would decrease correspondingly. Also speculations on charged Higgses with non-universal couplings [46] or an enhanced purely leptonic branching ratio of the charmed particles [47] have been proposed.

None of these alternatives, however, has reached the observed level of asymmetry without introducing other undesirable features. The flavor oscillation required surpasses by far the oscillations allowed by present dedicated experiments. Enhancing the leptonic branching ratio of D and F leads at most to $R_1 = 0.9$ 47 . One the other hand, the possible mixing of mirror and ordinary fermions naturally leads to a modified non-universal V,A structure of weak currents, which might be the cause of the leptonic asymmetry [16].

Let us first consider the status of e-μ universality in charged weak currents (CC), independently of the mirror mixing model. Modifying the conventional V-A structure to a general mixture of (real) V and A couplings [11,12,14]

$$J_e^\alpha = \bar{e}\,\gamma^\alpha \left(V_e - A_e \gamma_5\right) \nu_e \,,$$
$$J_q^\alpha = \bar{q}\,\gamma^\alpha \left(V_q - A_q \gamma_5\right) q' \,. \tag{32}$$

We can obtain limits on the ratios of lepton couplings,

$$\lambda_i \equiv \frac{A_i}{V_i} \quad , i = e, \mu \quad ; \qquad \varkappa \equiv \frac{V_\mu}{V_e} \,.$$

We find [11,12] the following best fit values (and 1σ-limits) for them:

$$\lambda_e = 1.085 \quad (< 1.15) \,,$$
$$\lambda_\mu = 1.00 \quad (< 1.115)$$
$$(0.905 <)\ \varkappa = 1.047 \quad (< 1.115)^{*)} \,. \tag{33}$$

The constraints used (except R_p) depend only on λ_e and λ_μ and are symmetrical with respect to the replacements $\lambda_i \quad 1/\lambda_i$. In Eq. (33) we have given only the upper bounds and best fit values for $\lambda_i \geqslant 1$. The parameter $\varkappa$, which directly measures e-μ universality is constrained only by the remarkably accurate pseudoscalar rates R_p, cf. Section 3.4.

We can thus conclude that both the V,A structure and the universality of the charged currents of the first two generations are known to within an error of 10-15%.

The beam dump ratio can now be expressed [16] by

$$R_1 \simeq \frac{1}{\varkappa^4} \cdot \frac{1 + 6\lambda_e^2 + \lambda_e^4}{1 + 6\lambda_\mu^2 + \lambda_\mu^4} \,. \tag{34}$$

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*) This limit is further tightened by the new measurement of Bryman et al. [48] of R_π.

Inserting the limits (33) one finds that the maximum non-universality allowed yields

$$R_1 \gtrsim 0.96 \ . \tag{35}$$

Thus non-universality of the charged currents alone cannot explain the beam dump result in this general formalism.

Consider next the mirror lepton models c and d where N_e is heavy enough not to be produced in weak decays of either light or heavy particles ($m_{Ne} > m_D$). Then the situation is similar to the previous general case, and the limit (35) holds.

Models a and b, however, with a light mirror neutrino N_e, offer a new situation: there appears an oscillating pattern in the cross sections due to neutrino-mirror neutrino oscillations. In particular, the cross sections of the $\overset{(-)}{\nu}_e$ beam are modified (see Refs. 12, 14-16 for details) with an oscillation factor at the distance x

$$P^e_{\nu\nu}(x) = 1 - \frac{1}{2}\sin^2 2\varphi_e \left(1 - \cos\frac{2\pi x}{L}\right) . \tag{36}$$

The oscillation length is, as usual, $L = 4\pi E_\nu/\Delta m_e^2$, where $\Delta m_e^2 = \left| m_{N_e}^2 - m_{\nu_e}^2 \right|$. For Δm^2_e sufficiently large $P^e_{\nu\nu}(x)$ is averaged to $\bar{P}^e_{\nu\nu} = 1 - \frac{1}{2}\sin^2 2\varphi_e$, and an effectively incoherent scattering follows.

In an experiment with reactor antineutrinos it is enough to take $\Delta m_e^2 \gtrsim 0.1 - 0.2\ \mathrm{eV}^2$ in order to have the beam flux incoherent and thus depleted by the factor $\bar{P}^e_{\nu\nu}$, while in all beam dump experiments a somewhat heavier mass scale is needed (in most of them [43,44a,44b] $\Delta m_e^2 \gtrsim 20\ \mathrm{eV}^2$). This means that if the observed electron deficiency in beam dump experiments is due to neutrino-mirror neutrino oscillations it would also lead to a diminished flux in reactor experiments. However, the recent reactor experiments [49] tell us that the reactor antineutrino flux is known to an inaccuracy at 5-10 %. Accordingly, the ratio R_1 can only be allowed to decrease to the value 0.9 in models a and b with light ($< m_e$) mirror neutrinos.

One can thus conclude that neither very light ($< m_e$) nor very heavy ($> m_D$) mirror neutrinos can provide an explanation for the observed smallness of the ratio R_1. What then about mirror neutrinos with masses between these two limits ? There is an experimental limit [50] from K^+_{e3}-decay which tells that no sizeable depletion of the ν_e-flux is allowed. Since the experimental arrangement is roughly the same here as in the beam dump experiments, we see that mirror neutrinos with a mass ($<$ 200 MeV) small enough to be produced in K^+_{e3}-decay cannot have a large enough mixing with ν_e's to make the R_1-ratio small. However, there still remains the possibility that the electron-like mirror neutrino is heavier than 200 MeV and smaller than about 1 GeV. This is discussed in more detail in Ref. 16.

8. CONCLUSIONS

Mirror fermions appear in many theories based on large gauge symmetries, as discussed in Sec. 1. Their effects through mixing with ordinary fermions on low energy CC and NC processes have been analyzed in this work.

We note that in order to suppress the flavor changing neutral currents, the mixing in the quark sector must be vanishingly small. However, no such constraint exists for the leptons, not even from the highly accurate data on the anomalous magnetic moments of the electron and muon [15]. Therefore we have concentrated on the mixing effects in the leptonic sector.

We parametrize the mixing in terms of charged and neutral lepton mixing angles θ_i and φ_i . We also consider various possibilities for the masses of the mirror neutrinos. In particular, we make a nearly complete classification of the masses of the mirror neutrinos for the case of two generations. These correspond to the possibilities of having either a light ($< m_e$) or massive ($> m_K$) electron or muon mirror neutrino. We call these models a, b, c and d. Of these models the model c can be interpreted in more general terms than within the mirror mixing model. It corresponds to any model where the charged currents have a general V,A structure (see also [11]) and the neutral currents have a modified axial vector part.

We argue in that various astronomical limits from the cosmic photon spectrum and the 3 ^{o}K background radiation put severe constraints on the models with light ($\lesssim 100$ eV) mirror neutrinos. These constraints can be evaded in two ways. Either the (mirror) neutrinos are very light (10^{-2} eV) or the charged lepton mixing angle θ_ℓ is negligible ($< 10^{-5}$). In the first case neutrino-mirror neutrino oscillation, which is possible if mirror neutrinos are light, can not be seen in present experiments since it has not yet started. The beam is completely coherent and we call this the coherent case.

In the other case θ_ℓ is zero and this leads to a (non-universal) V-A structure for the charged currents and to standard NC couplings for charged leptons. However, now neutrino-mirror neutrino oscillation is possible. In principle. the oscillation phenomena are dependent on the unknown neutrino mass term, but for simplicity, we discuss here only the possibility that the beam is oscillating rapidly and is therefore effectively incoherent. We call this case incoherent scattering.

We have then proceeded to analyze the various models described above in order to determine the mixing angles and $\sin^2\theta_W$. We use altogether

7 CC and 15 NC constraints in the fit. Table 1 gives the best fit values and 1σ errors of the mixing angles in all the models we consider.

We discussed the question of non-universality and its implications for the beam-dump experiment. We showed in a general model that universality of charged currents is well enough known not to be able to explain the whole observed e/μ-asymmetry. We also discussed the beam dump experiment within the mirror mixing model for many choices of the mirror neutrino mass. Various constraints seem to limit the mixing small enough to give an e/μ-ratio of only 0.9. However, the question is still open for the mass range 200 MeV $\lesssim m_{N_e} \lesssim$ 1 GeV [16].

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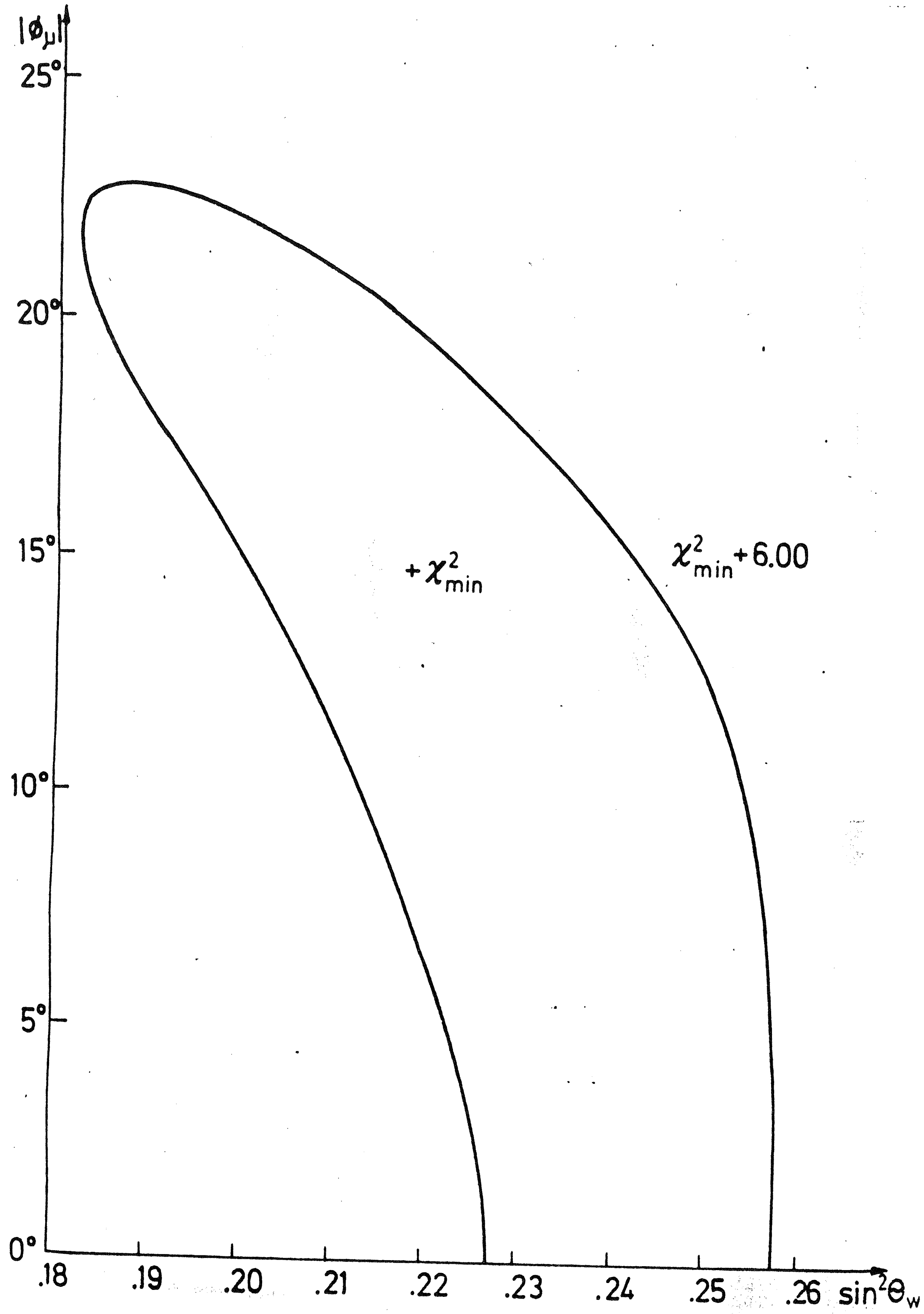
Model	$\sin^2\theta_W$	$\lvert\theta_e\rvert$	$\lvert\phi_e\rvert$	$\lvert\theta_\mu\rvert$	$\lvert\phi_\mu\rvert$	$\chi^2/\langle\chi^2\rangle$
a_{coh}	0.239	$2.3^\circ(<3.8^\circ)$	-	$0^\circ(<2.5^\circ)$	-	13.10/18
b_{coh}	0.232	$2.3^\circ(<4.1^\circ)$	-	$0^\circ(<9.0^\circ)$	$8.4^\circ(<12.8^\circ)$	12.08/18
d_{coh}	0.239	$0^\circ(<7.4^\circ)$	$0^\circ(<8.2^\circ)$	$0^\circ(<3.0^\circ)$	-	13.74/18
a_{inc}	0.239	0	$0^\circ(<27.5^\circ)$	0	$5.5^\circ(<16.3^\circ)$	13.80/18
b_{inc}	0.232	0	$0^\circ(<36.3^\circ)$	$0^\circ(<9.3^\circ)$	$8.4^\circ(<12.9^\circ)$	12.84/18
d_{inc}	0.239	$0^\circ(<7.4^\circ)$	$0^\circ(<8.0^\circ)$	0	$5.5^\circ(<18.2^\circ)$	14.69/18
c	0.218	$6.1^\circ(<16.4^\circ)$	$11.5^\circ(<20.5^\circ)$	$0.1^\circ(<13.2^\circ)$	$14.5^\circ(<21.9^\circ)$	11.88/17
c_{V-A}	0.223	0	$10.1^\circ(<17.8^\circ)$	0	$12.5^\circ(<19.0^\circ)$	12.34/19
c_{univ}	0.223	$3.2^\circ(<10.0^\circ)$	$0^\circ(<8.0^\circ)$	$=\theta_e$	$=\phi_e$	13.00/19
GWS	$0.239\pm.007$	0	0	0	0	14.74/21

Table 1



$|\phi_\mu|$
25°
20°
15°
10°
5°
0°
.18
.19
.20
.21
.22
.23
.24
.25
.26
$\sin^2\theta_W$
$+\chi^2_{min}$
$\chi^2_{min}+6.00$

Fig 1