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Heavy neutrinos in e^+e^- collisions

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The production and subsequent decay of a heavy neutrino, N, in e^+e^- collisions is investigated. Both the cases where the heavy neutrino is a Dirac or a Majorana particle have been studied and comparisons between V − A and V + A couplings of N have been made. For the pair production in the *s*-channel the total cross section for Majorana neutrinos is considerably smaller than that for Dirac neutrinos except when the collision energy is much larger than the mass of the neutrino. For the single neutrino production in the *t*-channel, which occurs if N mixes with the electron neutrino, the cross section of Majorana neutrinos is twice the cross section of Dirac neutrinos. In this reaction the cross section of a heavy neutrino with a right-handed coupling is by a typical factor of 1.5 larger than that of a neutrino with left-handed coupling. It is found that a correlated production of an equal-sign lepton pair in the decay of a $\bar{N}N$ system offers a promising signature to make a distinction between Dirac and Majorana neutrinos.

1. Introduction

Neutrinos with a non-zero mass fall into two categories, Dirac neutrinos and Majorana neutrinos. If a theory contains lepton number violating mass terms connecting neutrinos and antineutrinos, two-component Majorana neutrino or neutrinos necessarily result from the diagonalization of the mass lagrangian [1]. If the lepton number is conserved, however, massive neutrinos are, just like other fermions, four-component Dirac particles. In the limit of vanishing mass neutrinos are two-component chiral states and one can not make any difference between a Majorana neutrino and a Dirac neutrino provided the interactions are purely left-handed as they are in the standard model [2].

A comparative study of the properties of Dirac and Majorana neutrinos may yield important theoretical insights. In the standard electroweak model, as well as in the SU(5) grand unified theory, only left-handed chirality states of neutrinos (right-handed states of antineutrinos) exist and therefore neutrino mass terms, if any, are necessarily of the Majorana type.

In the $SU(2)_L \times SU(2)_R \times U(1)$ left–right symmetric models [3] both left-handed and right-handed neutrinos (and antineutrinos) exist so that both Dirac and Majorana masses are possible. If a neutrino is light, it is most naturally a Majorana particle as a result of a so-called see-saw mechanism [4]. According to this mechanism, when the left–right symmetry is broken at a scale $M \sim 10^2 - 10^{14}$GeV, a Majorana mass term of the order of M is induced for the right-handed component of the neutrino. Upon diagonalizing the mass lagrangian, two Majorana neutrinos are found, one very heavy with a mass $\sim M$ and one very light with a mass $\sim m_D^2/M$, where m_D is a typical mass of charged fermions. Nevertheless, also a light Dirac neutrino is possible in some versions of the left–right symmetric model [5]. In particular, a heavy neutrino, in which we are interested in this paper, could be either a Dirac or a Majorana particle in the left–right model without any clear preference.

Finally, in the simplest superstring-based E_6 models neutrinos are massless on tree level, but may obtain a Dirac mass through loop corrections [6].

For light neutrinos it is very difficult to experimentally distinguish the Dirac and Majorana cases from each other since differences of these two alternatives arise from tiny mass terms. The most decisive experiment for studying the character of the electron neutrino is neutrinoless double β-decay [7] which is possible only if the neutrino is a Majorana particle. From the measurements of this process we know by now that either ν_e is a Dirac particle or, if it is a Majorana particle, its mass is less than a few electronvolts [7]. It has been pointed out [8] that a distinction between Dirac and Majorana neutrinos could also be made by measuring the cross section of an elastic (anti)neutrino electron or (anti)neutrino nucleon scattering, but the cross sections for these processes are too small for this method to have any use in present experiments.

In this paper we will study methods for distinguishing *heavy* Dirac and Majorana neutrinos from each other in electron–positron collisions. The recent measurements of the width of the Z-boson at LEP have shown [9] that there exist at most three light neutrinos with standard electroweak couplings. Thus any additional neutrino species would have a mass in excess of about $M_Z/2 \simeq$ 45 GeV.

According to the standard model, a sequential heavy neutrino can be produced in e^+e^- collisions only in pairs, $e^+e^- \to \bar{N}N$. In the present more general approach we will also consider the production of a single heavy neutrino together with a light neutrino, $e^+e^- \to \bar{\nu}N(\bar{N}\nu)$ which is possible if the electron and the heavy neutrino couple to each other, for instance as a result of a νN mixing. We present the differential production cross sections and decay rates of heavy neutrinos and make comparisons between the Dirac and Majorana cases. We find that equal-sign lepton pair correlations in the decay of a $\bar{N}N$ system differ considerably in the Dirac and Majorana cases and offer a method to experimentally determine the character of N.

The paper is organized as follows. In sect. 2 we derive the differential cross sections for the processes $e^+e^- \to \bar{N}N, \bar{N}\nu, \bar{\nu}N$ in case of both a Dirac and a Majorana neutrino. In sect. 3 we present general formulas for the weak decays $N \to \ell^+\ell'^-\nu$. Our main interest is to apply the result to the case of a sequential neutrino with the standard V – A couplings. Nevertheless, all production and decay formulas are derived for arbitrary vector and axial vector couplings and comparisons are made between left- and right-handed heavy neutrinos. In sect. 4 we consider the equal-sign lepton pair correlations in the decay of $\bar{N}N$ pair and point out their usefulness for distinquishing Dirac and Majorana states. Conclusions are presented in sect. 5.

2. Production of heavy neutrinos in e^+e^- collisions

2.1. PAIR PRODUCTION

Let us start by studying the pair production of heavy neutrinos in the electron–positron annihilation. Define momenta and spins according to

$$e^-(k_1) + e^+(k_2) \to N(p_1, S_1) + \bar{N}(p_2, S_2). \tag{1}$$

We will assume that the beam particles are unpolarized. We will allow the neutral-current and charged-current couplings to have the most general vector axial-vector structure given by

$$\mathcal{L}_{NC} = -\frac{g}{4\cos\theta_W} Z^\mu [\bar{\ell}\gamma_\mu(V_{\ell\ell} - A_{\ell\ell}\gamma_5)\ell + \bar{N}\gamma_\mu(V_{NN} - A_{NN}\gamma_5)N], \tag{2}$$

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} W^\mu [\bar{\ell}\gamma_\mu(V_{\ell N} - A_{\ell N}\gamma_5)N + \bar{\ell}\gamma_\mu(V_{\ell i} - A_{\ell i}\gamma_5)\nu_i], \tag{3}$$

respectively, where ν_i is a light neutrino ($m_{\nu_i} \simeq 0$). Here $V_{\ell\ell} = 1 - 4\sin^2\theta_W$ and $A_{\ell\ell} = 1$. If N is a sequential Dirac neutrino, it has the standard neutral-current couplings $V_{NN} = A_{NN} = 1$. The charged-current couplings in eq. (3) include the possible neutrino mixing factors, e.g., if ν_i is a superposition $\nu_i = \sum U_{i\ell}\nu_\ell$ of the known light neutrinos ν_ℓ ($\ell = e, \mu, \tau$) one has $V_{\ell i} = A_{\ell i} = U_{\ell i}$. All

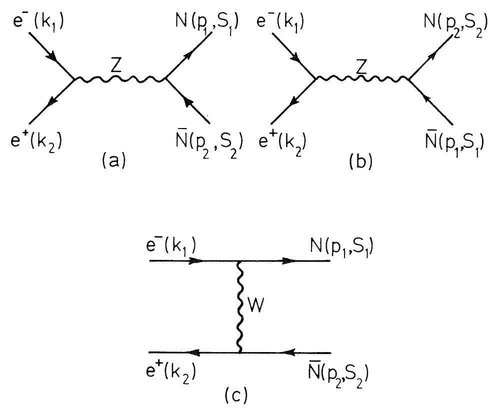

Fig. 1. Feynman diagrams for the pair production of a heavy neutrino N. In the Dirac case only the diagrams (a) and (c) contribute. If N is a Majorana particle (N = N̄) also diagram (b) should be taken into account.

fields in eqs. (2) and (3) are assumed to be presented in mass eigenstate basis. Note that we assume the GIM rule to be valid in $\mathcal{L}_{NC}$ by allowing only diagonal neutral currents (thereby excluding the possibility that the neutrino N is a mixed state of an ordinary left-handed neutrino and, for instance, a right-handed mirror neutrino [10]).

A basic difference between the Dirac and Majorana cases arises from the fact that if the neutrino N is a Majorana particle, the vector part of the neutral weak current vanishes identically, i.e. $V_{NN} = 0$, but on the other hand, the axial vector coupling becomes twice as large as that of a Dirac neutrino. These differences have not much effect in the case of the very light standard-model neutrinos since there the neutrinos remain almost exactly in the pure chiral state into which they are prepared in the left-handed charged-current production vertex. For a heavy neutrino the situation is different, as we will see.

The dominant contribution for the reaction (1) comes from the Z-exchange in the s-channel (see fig. 1a). If N is a Majorana state, the final state consists of two identical particles and one has to take into account another diagram, depicted in fig. 1b, where the neutrino momenta and spins of N and N̄ have been interchanged. It is the summing up of the amplitudes corresponding to these two diagrams that shows that the vector coupling disappears and the axial vector coupling is doubled in the case of a Majorana neutrino compared with the Dirac case.

The N̄N pair can be also produced via charged currents through a W-exchange in the t-channel (see fig. 1c). At the Z-peak energies the contribution of this diagram is, however, small as compared to the Z-exchange graphs. Moreover, the production via W-exchange will be proportional to the fourth power of the couplings V_{eN} and A_{eN}. Since it is plausible that if a heavy neutrino couples to the electron at all the coupling is small anyway, it is reasonable to neglect the contribution of the diagram of fig. 1c.

Averaging over the spins of the initial state particles, the matrix element corresponding to the graph in fig. 1a is given by (the electron mass is neglected)

$$\begin{aligned}
\frac{1}{4}\sum |M^{Z}|^2_{e^+e^-\to \bar{N}N} = \; & 4|D_Z(s)|^2 \left(\frac{g}{4\cos\theta_W}\right)^4 (V_{ee}^2 + A_{ee}^2)(V_{NN}^2 + A_{NN}^2) \\
& \times\Big\{ k_1\cdot p_2\, k_2\cdot p_1 (1+\lambda_e\lambda_N) + k_1\cdot p_1\, k_2\cdot p_2 (1-\lambda_e\lambda_N) \\
& + \alpha_N m_N^2\, k_2\cdot k_1 - h_1 h_2 (1+\lambda_e\lambda_N) m_N^2 k_1\cdot s_2 k_2\cdot s_1 \\
& - h_1 h_2 (1-\lambda_e\lambda_N) m_N^2 k_1\cdot s_1 k_2\cdot s_2 \\
& - \alpha_N h_1 h_2 \Big[p_2\cdot p_1 (k_2\cdot S_2 k_1\cdot S_1 + k_2\cdot S_1 k_1\cdot S_2) \\
& - p_2\cdot S_1 (k_2\cdot S_2 k_1\cdot p_1 + k_2\cdot p_1\cdot k_1\cdot S_2 - k_2\cdot k_1 p_1\cdot S_2) \\
& - p_1\cdot S_2 (k_2\cdot p_2 k_1\cdot S_1 + k_2\cdot S_1 k_1\cdot p_2) \\
& + S_2\cdot S_1 (k_2\cdot p_2 k_1\cdot p_1 + k_2\cdot p_1 k_1\cdot p_2 - k_2\cdot k_1 p_2\cdot p_1)\Big] \\
& + m_N(\lambda_e + \lambda_N)(h_2 k_2\cdot p_1 k_1\cdot S_2 - h_1 k_2\cdot S_1 k_1\cdot p_2) \\
& - m_N(\lambda_e - \lambda_N)(h_2 k_2\cdot S_2 k_1\cdot p_1 - h_1 k_2\cdot p_2 k_1\cdot S_1) \\
& - m_N\lambda_e\alpha_N \Big[h_2 (k_2\cdot S_2 k_1\cdot p_2 - k_2\cdot p_2 k_1\cdot S_2) \\
& + h_1 (k_2\cdot S_1 k_1\cdot p_1 - k_2\cdot p_1 k_1\cdot S_1)\Big]\Big\}, \qquad (4)
\end{aligned}$$

where h_1 and h_2 are the helicities of N and $\bar{N}$, respectively, the momenta and spins are defined in eq. (1), and the following notations have been introduced:

$$\lambda_k = \lambda\left(\frac{A_{kk}}{V_{kk}}\right), \quad \alpha_k = \alpha\left(\frac{A_{kk}}{V_{kk}}\right) \quad (k = e,N), \qquad (5)$$

where

$$\lambda(x) = \frac{2x}{1+x^2}, \quad \alpha(x) = \frac{1-x^2}{1+x^2}, \qquad (6)$$

$$D_i(s) = [(s - M_i^2) + i\Gamma_i M_i]^{-1} \quad (i = W,Z), \qquad (7)$$

where $s = (k_1 + k_2)^2$ is the c.m. energy squared. In the Dirac case, eq. (4) is the only amplitude to be taken into account. The differential cross section for the pair production of polarized heavy Dirac neutrinos is then

$$\left(\frac{\mathrm{d}\sigma_a(\theta, h_1, h_2)}{\mathrm{d}\cos\theta}\right)^{\text{Dirac}}_{e^+e^-\to N\bar{N}} = \frac{\beta s}{64\pi}|D_Z(s)|^2 \left(\frac{g}{4\cos\theta_W}\right)^4 (V_{ee}^2 + A_{ee}^2)(V_{NN}^2 + A_{NN}^2)$$
$$\times \Big[1 + \alpha_N(1-\beta^2) - h_1h_2\beta^2 + \lambda_N\beta(h_2 - h_1)$$
$$+ \cos\theta\,[2\lambda_e\lambda_N\beta(1-h_1h_2) + \lambda_e(h_2-h_1)(1+\beta^2)$$
$$+ \lambda_e\alpha_N(h_2-h_1)(1-\beta^2)]$$
$$+ \cos^2\theta\,[\beta^2 - h_1h_2 - \alpha_N h_1h_2(1-\beta^2) + \lambda_N(h_2-h_1)\beta]\Big], \tag{8}$$

where

$$\beta = \sqrt{1 - \frac{4m_N^2}{s}}\,, \tag{9}$$

and θ is the angle between the e^- beam and the momentum of the outgoing N. The corresponding cross section for an unpolarized final state is obtained by performing a summation over the helicities $h_{1,2} = \pm 1$.

Let us now assume that N is a Majorana particle. As mentioned already, there are then two Z-exchange diagrams contributing to the process $e^+e^- \to NN$, those given in figs. 1a, b. Adding up the amplitudes corresponding to these diagrams has the same effect as making the replacements $V_{NN} \to 0$, $A_{NN} \to 2A_{NN}$ in the amplitude (4). The differential cross section for the pair production of polarized heavy Majorana-neutrinos is then given by

$$\left(\frac{\mathrm{d}\sigma_{(a,b)}(\theta, h_1, h_2)}{\mathrm{d}\cos\theta}\right)^{\text{Maj}}_{e^+e^-\to NN} = \frac{\beta^3 s}{32\pi}\left(\frac{g}{4\cos\theta_W}\right)^4 |D_Z(s)|^2 (V_{ee}^2 + A_{ee}^2)A_{NN}^2$$
$$\times [(1-h_1h_2)(1+\cos^2\theta) + 2(h_2-h_1)\lambda_e\cos\theta]\,. \tag{10}$$

If one sums over the helicities of the heavy neutrinos to obtain an unpolarized differential cross section, the asymmetric $\cos\theta$ term will disappear. On the

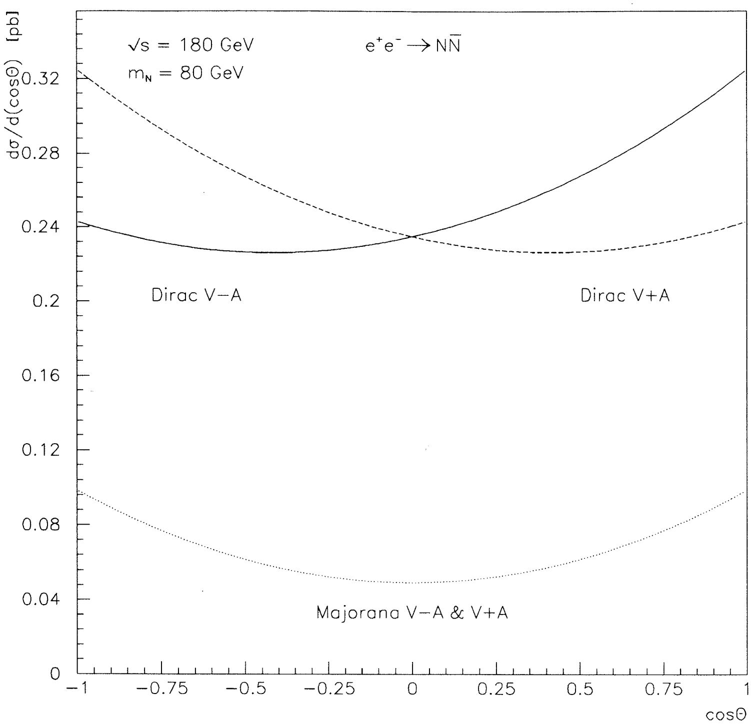

Fig. 2. The differential cross sections for the pair production of heavy neutrinos in the s-channel for V − A and V + A couplings ($\sqrt{s} = 180\,\text{GeV}, m_N = 80\,\text{GeV}$).

other hand, one can see from eq. (8) that in the Dirac case both the polarized and the unpolarized cross sections contain an asymmetric $\cos\theta$ term, in addition to a symmetric $1 + \cos^2\theta$ term. Hence one could in principle distinguish a Dirac and a Majorana neutrino from each other by measuring the forward–backward asymmetry. The effect is, however, not very large since the asymmetric term in the unpolarized Dirac cross section is proportional to the parameter λ_e which is very small due to smallness of $V_{ee} = 1-4\sin^2\theta_W \simeq 0.08$.

We have presented in fig. 2 differential cross sections for the pair production of Dirac neutrinos both with purely unpolarized left-handed and right-handed couplings. For Majorana neutrinos there is no difference between these two extreme cases. We have assumed the beam energy to be $\sqrt{s} = 180$ GeV (to be achieved in LEP200) and we have set the mass of the heavy neutrino to $m_N = 80$ GeV. Let us note that the cross section is considerably lower for a Majorana neutrino than for a Dirac neutrino due to the fact that in the Majorana case the pair production in the s-wave is suppressed by an additional velocity factor β^2, as pointed out in ref. [11].

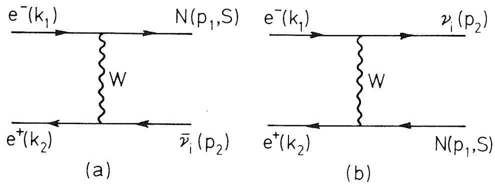

Fig. 3. Feynman diagrams for the single production of a heavy neutrino N. In the Dirac case only the diagram (a) contributes, in the Majorana case both diagrams (a) and (b) do.

2.2. SINGLE HEAVY NEUTRINO PRODUCTION

Let us now consider the production of a single heavy neutrino in the reactions

$$e^-(k_1) + e^+(k_2) \to N(p_1, S) + \bar{\nu}_i(p_2)\,, \tag{11a}$$

$$e^-(k_1) + e^+(k_2) \to \bar{N}(p_1, S) + \nu_i(p_2)\,. \tag{11b}$$

Since we assume that the neutral currents are diagonal, the reactions (11a) and (11b) can proceed only via a W-exchange as depicted in fig. 3a and 3b, respectively. These processes are favoured over the pair production due to a larger phase space, but on the other hand they are suppressed due to the presumably small non-diagonal coupling constants V_{eN} and A_{eN}. The matrix element for the production of a polarized heavy Dirac neutrino N via the diagram of fig. 3a is given by

$$\begin{aligned}
&\frac{1}{4}\sum |M_{(a)}|^2_{e^+e^-\to N\bar{\nu}_i} \\
&\quad = 4|D_W(t)|^2\left(\frac{g}{2\sqrt{2}}\right)^4 (|V_{eN}|^2 + |A_{eN}|^2)(|V_{ei}|^2 + |A_{ei}|^2) \\
&\quad \times [k_1\cdot p_2\; k_2\cdot p_1(1+\lambda_{eN}\lambda_{ei}) + k_1\cdot k_2\; p_1\cdot p_2(1-\lambda_{eN}\lambda_{ei}) \\
&\quad - m_N\, hk_1\cdot p_2\; k_2\cdot S(\lambda_{eN}+\lambda_{ei}) - m_N hk_1\cdot k_2\; p_2\cdot S(\lambda_{eN}-\lambda_{ei})]\,,
\end{aligned} \tag{12}$$

where h is the helicity of the heavy neutrino and

$$t = (k_1 - p_1)^2 = -\tfrac{1}{2}(s - m_N^2)(1-\cos\theta)\,, \tag{13}$$

and $(k = N, i)$

$$\lambda_{ek} = \frac{2\mathrm{Re}(V_{ek}A^*_{ek})}{|V_{ek}|^2 + |A_{ek}|^2}\,. \tag{14}$$

The differential cross section corresponding to diagram 3a is then given by

$$\left(\frac{d\sigma(\theta,h)}{d\cos\theta}\right)^{\text{Dirac}}_{e^+e^-\to N\bar{\nu}_i}$$
$$= \frac{G_F^2 M_W^4 s}{256\pi}|D_W(t)|^2\left(1-\frac{m_N^2}{s}\right)^2\left(1+\frac{m_N^2}{s}\right)$$
$$\times\left(|V_{eN}|^2+|A_{eN}|^2\right)\left(|V_{ei}|^2+|A_{ei}|^2\right)$$
$$\times\left\{[1+(1+\tilde{\beta})\cos\theta+\tilde{\beta}\cos^2\theta](1+\lambda_{eN}\lambda_{ei})+\frac{4s}{s+m_N^2}(1-\lambda_{eN}\lambda_{ei})\right.$$
$$\left.-h[\tilde{\beta}+(1+\tilde{\beta})\cos\theta+\cos^2\theta](\lambda_{eN}+\lambda_{ei})-h\frac{4s}{s+m_N^2}(\lambda_{eN}-\lambda_{ei})\right\}, \tag{15}$$

where

$$\tilde{\beta}=\frac{s-m_N^2}{s+m_N^2}. \tag{16}$$

Since the charged weak couplings of the light neutrinos are known to be left-handed to a good accuracy, we will set $\lambda_{ei}=1$ in what follows.

The matrix element $\frac{1}{4}\sum|M_{(b)}|^2_{e^+e^-\to\nu_i\bar{N}}$ for the process (11b) (cf. fig. 3b) can be obtained from eq. (12) by interchanging k_1 and k_2 and by reversing the sign of the helicity h. Accordingly the corresponding differential cross section is obtained from eq. (15) via the replacements $h\to -h$ and $\cos\theta\to-\cos\theta$, whence t is replaced by $u=-\frac{1}{2}(s-m_N^2)(1+\cos\theta)$.

In the case that N has the ordinary V – A couplings its production is peaked in the forward direction, whereas the backward direction is forbidden due to conservation of angular momentum. If N has, however, V + A couplings there is a certain probability for a production in the backward direction, too, while the forward direction is still preferred. Let us note that in the V + A case the angular distribution is determined by the W-propagator factor alone, as one can see from eq. (15).

Using the same numerical values as above, we have presented in fig. 4a the differential cross sections for the production of a single Dirac neutrino via W-exchange for both the V – A and V + A case.

If the heavy neutrino N and the light neutrino ν_i are both Majorana particles, the two processes depicted in figs. 3a,b add up coherently. Let their *CP* phases be given as follows:

$$N^c=e^{i\varphi_N}N,\quad \nu_i^c=e^{i\varphi_i}\nu_i, \tag{17}$$

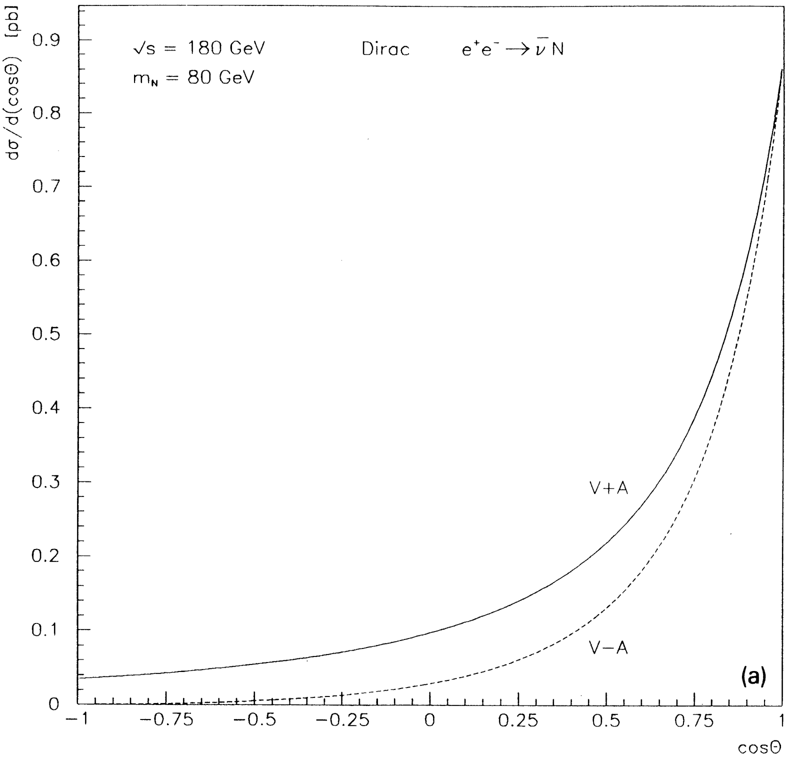

Fig. 4. The differential cross sections for the single neutrino production in the t-channel for V − A and V + A couplings: (a) Dirac neutrinos, (b) Majorana neutrinos ($\sqrt{s} = 180\,\mathrm{GeV}, m_{\mathrm{N}} = 80\,\mathrm{GeV}, |V_{\mathrm{eN}}| = |A_{\mathrm{eN}}| = 0.1$).

whence the charged-current lagrangian (3) can be rewritten in the form

$$\mathcal{L}^{\mathrm{cc}} = \frac{g}{2\sqrt{2}} W^{\mu} [\mathrm{e}^{-i\varphi_i} \bar{\nu}_i \; \gamma_\mu (V_{\mathrm{e}i} + A_{\mathrm{e}i}\gamma_5) e^{\mathrm{c}} + \mathrm{e}^{-i\varphi_{\mathrm{N}}} \bar{N} \; \gamma_\mu (V_{\mathrm{eN}} + A_{\mathrm{eN}}\gamma_5) e^{\mathrm{c}}] + \text{h.c.}, \qquad (18)$$

where e^{c} is the positron field. Then the interference term of the diagrams 3a and 3b is given by

$$\frac{1}{4}\sum (M_{(\mathrm{a})} M^*_{(\mathrm{b})} + \text{h.c.})_{\mathrm{e}^+\mathrm{e}^- \to \mathrm{N}\nu_i} = -16\left(\frac{g}{2\sqrt{2}}\right)^4 \mathrm{Re}(D^*_{\mathrm{W}}(t) D_{\mathrm{W}}(u)) \mathrm{Re}(\mathrm{e}^{i(\varphi_{\mathrm{N}}-\varphi_i)}((V^*_{\mathrm{eN}})^2 - (A^*_{\mathrm{eN}})^2) \times (V_{\mathrm{e}i}^2 - A_{\mathrm{e}i}^2))\; k_1 \cdot k_2 \; p_1 \cdot p_2 \; . \qquad (19)$$

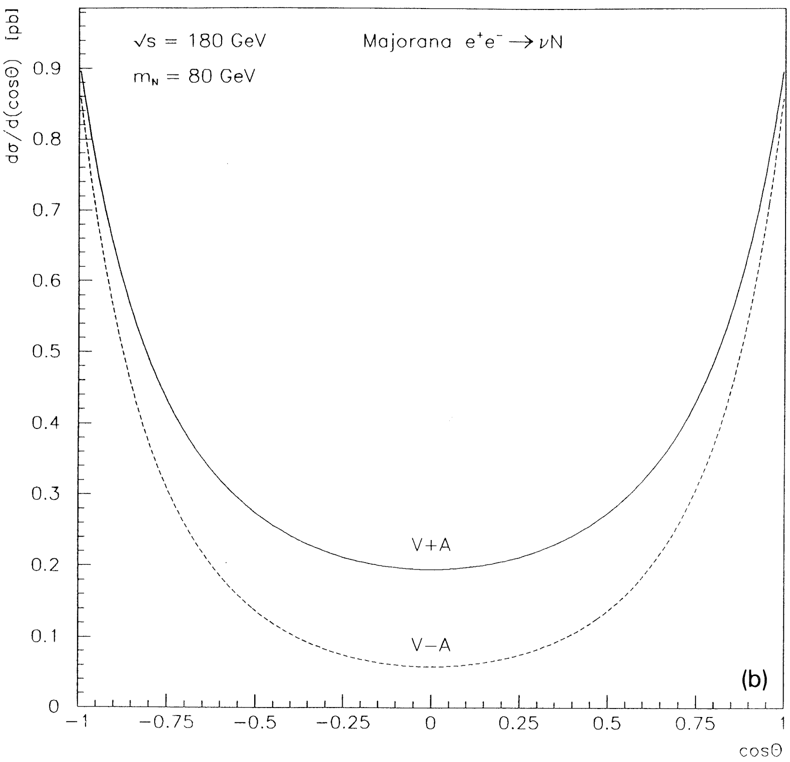

Fig. 4. Continued

This amplitude vanishes if either of the electron-neutrino vertices is purely chiral whence a massless ν_i is in that case produced in the opposite helicity states in the two diagrams. This is also now the case as we have assumed that ν_e has a purely left-handed coupling.

The polarized differential cross section of the process $e^+e^- \to N\nu_i$, where N and ν_i are Majorana particles, is then

$$\left(\frac{d\sigma(\theta,h)}{d\cos\theta}\right)^{\mathrm{Maj}}_{e^+e^-\to N\nu_i} = \left(\frac{d\sigma(\theta,h)}{d\cos\theta}\right)^{\mathrm{Dirac}} + \left(\frac{d\sigma(\pi-\theta,-h)}{d\cos\theta}\right)^{\mathrm{Dirac}}, \quad (20)$$

where $(d\sigma/d\cos\theta)^{\mathrm{Dirac}}$ is given in eq. (15).

In fig. 4b this cross section is presented for $\sqrt{s} = 180\,\mathrm{GeV}$ and $m_N = 80\,\mathrm{GeV}$ in both the V − A and V + A cases. The cross section is naturally symmetric with respect to the scattering angle θ for all types of couplings of the heavy neutrino. Just like for the Dirac neutrino, the cross section is larger in the case where the Majorana neutrino N has V + A couplings. For example, in the

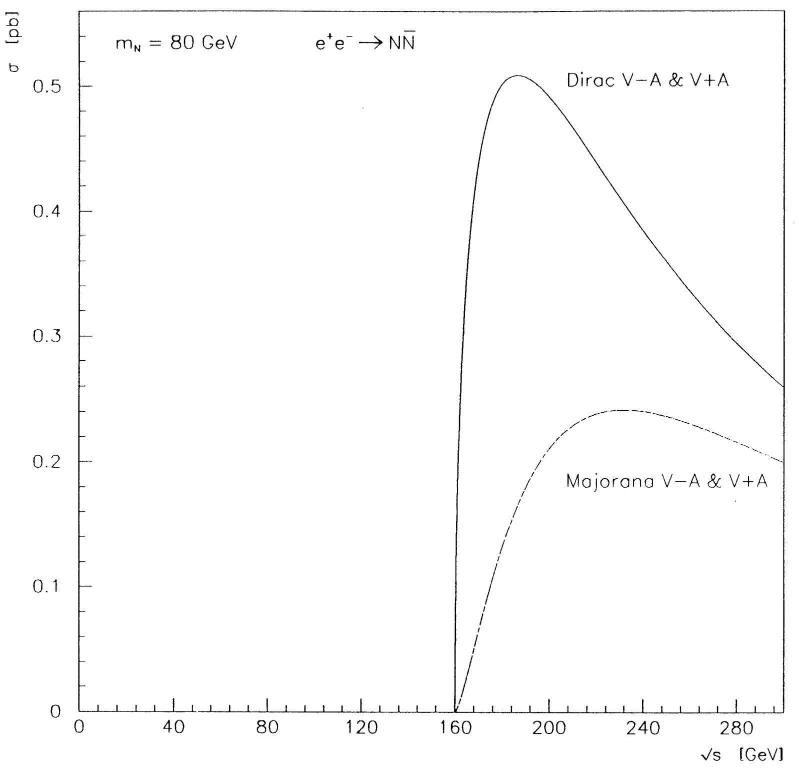

Fig. 5. The total cross sections of the pair production of heavy neutrinos in the s-channel as a function of the c.m. collision energy for Dirac and Majorana neutrinos and for V − A and V + A couplings (m_N = 80 GeV).

transverse direction it is about three times more effective to produce a V + A Majorana neutrino than a V − A neutrino. In the beam directions, where the angular distributions are peaked, there is, however, no large difference between the V − A and V + A cases.

2.3. TOTAL CROSS SECTIONS

The total production cross sections of the heavy neutrinos through the processes (1) and (11) are obtained from the corresponding differential cross sections (8) and (15) in the Dirac case, and (11) and (20) in the Majorana case by summing over the polarizations and integrating over the scattering angle θ. We will not write down here the ensuing lengthy expressions but are content with giving just some numerical examples.

Fig. 5 depicts the total cross section as a function of the collision energy $\sqrt{s}$

for the pair production of N($m_N=80$ GeV) in the process (1). As one can deduce from eqs. (8) and (10) the unpolarized total cross sections are the same for both left- and right-handed couplings. At very large collision energies, $\sqrt{s} \gg M_N$, the cross sections for production of Dirac and Majorana neutrinos are equal. Near the threshold they behave, however, quite differently: in the Majorana case the cross section is proportional to β^3 while in the Dirac case it grows–and descends– more steeply, as proportional to $\frac{1}{4}\beta(3+\beta^2)$. In fig. 6 we present the cross sections as a function of the neutrino mass for the collision energy $\sqrt{s}$ = 180 GeV. It can be seen from this figure that if the mass of the neutrino m_N is close to the collision energy $\sqrt{s}$, the cross sections for Dirac and Majorana neutrinos can differ by a large factor in favour of the Dirac neutrinos, making it possible with even a quite modest amount of data to distinguish between the two alternatives provided, of course, that m_N is determined accurately enough.

In fig. 7 the total cross sections for a single N production due to the process (11) are given for Dirac and Majorana neutrinos for both left- and right-handed couplings. The mixing between ν_e and N is assumed to be 10%, that is, $|V_{eN}| = |A_{eN}| = 0.1$. The cross section in the Majorana case is now by a factor of two larger than in the Dirac case because there are two contributing diagrams. Furthermore, the cross sections are substantially larger for a right-handed heavy neutrino than for a neutrino with the standard V – A couplings. This is due to the fact that for the left-handed heavy neutrinos the collision proceeds through a spin-1 state which fully forbids the back scattering, while for right-handed heavy neutrinos the system is in a spin-0 state and the back scattering is also possible. If the cross sections can be determined accurately enough in experiment one can hence use the single N channel to make difference between the Dirac and Majorana alternatives and also to make conclusion about the Lorentz structure of the weak couplings of N. Whether this is possible depends, apart from the unknown value of the eN coupling, also on the mass m_N of the neutrino. It should be noted that in the νN channel it is easier to observe an exotic heavy neutrino than an ordinary left-handed neutrino.

3. Weak decays of a heavy neutrino

The dominant decay mode of a heavy neutrino N, either Dirac or Majorana, is a weak decay into a charged lepton and a pair of quarks. Accordingly, the production of a N$\bar{\text{N}}$ pair in e^+e^- collision experiments will be easily detected by two energetic leptons, either same or oppositely charged depending on the neutrino type, and from a few (4–5) hard jets of hadrons. Once the energy threshold is open for N$\bar{\text{N}}$-pair to be produced it will be one of the dominant

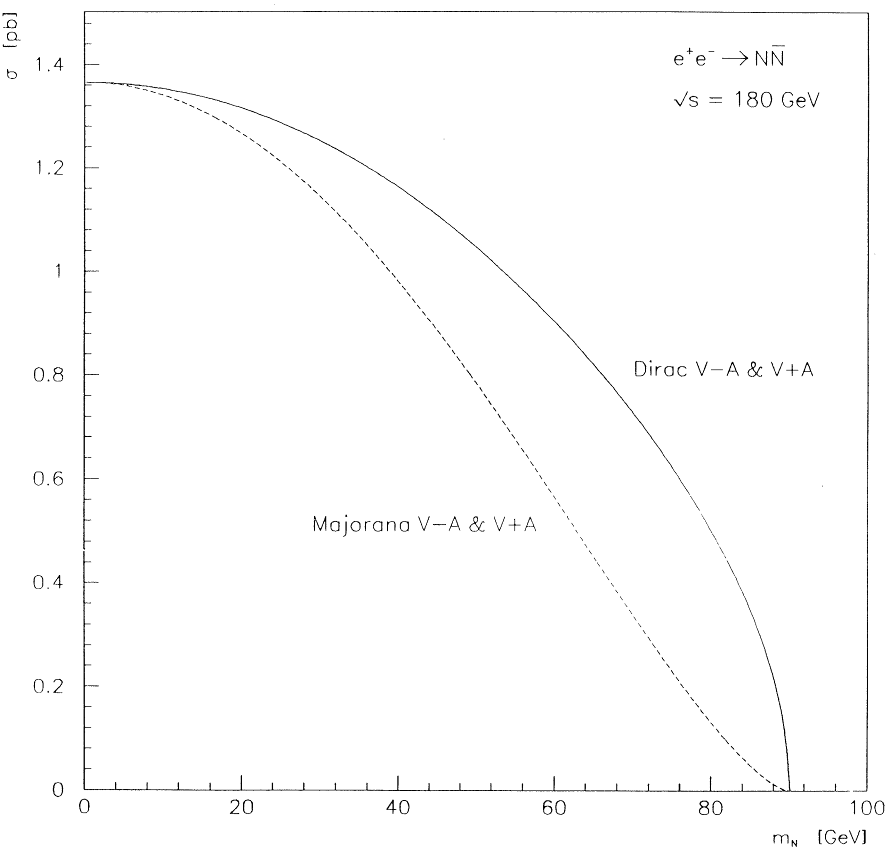

Fig. 6. The total cross sections of the pair production of Dirac and Majorana heavy neutrinos in the s-channel as a function of the neutrino mass M_N ($\sqrt{s}$ = 180 GeV).

reactions to occur and its signal is very clear. There is hardly any competitive background from for instance the pair production of heavy quarks or possible heavy charged leptons, the decay products of which always contain either only jets or a sizeble amount of invisible energy carried by light neutrinos.

The experimental signal of the $N\bar{\nu}$-production is also quite unique. There is no other reaction giving one hard lepton and two jets accompanied with a sizeable amount of invisible energy (once m_N is not too close to M_W) associated with the light neutrino. The cross section depends, however, on the unknown mixing angle which may be small and therefore the signal may not be as prominent as the signal from the $N\bar{N}$ production.

In the following we will study the leptonic decay channels of the heavy neutrino and antineutrino and show that these channels can be used to make

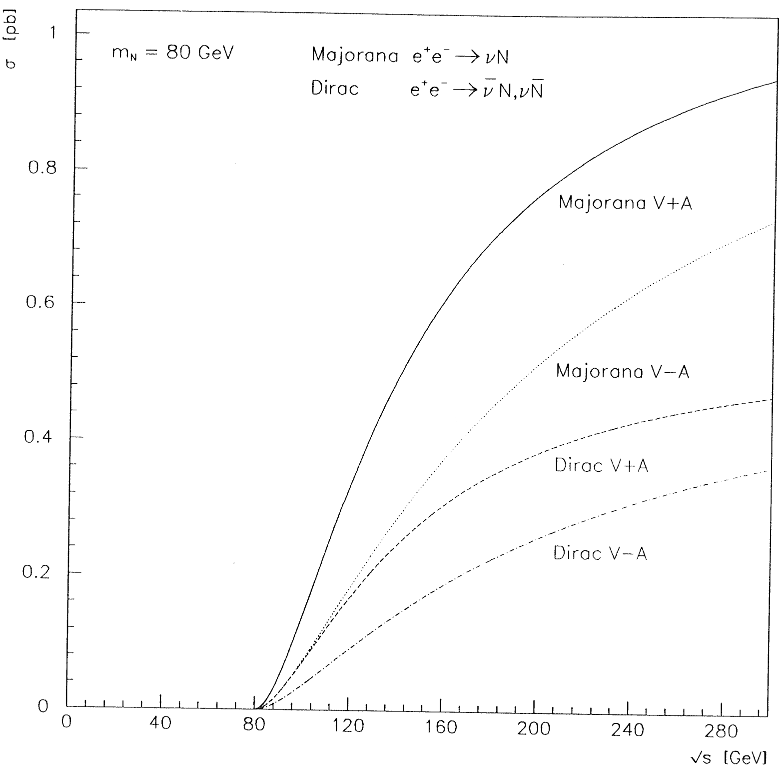

Fig. 7. The total cross sections of a single neutrino production in the t-channel as a function of the c.m. collision energy for Dirac and Majorana neutrinos and for V − A and V + A couplings ($m_N = 80\,\mathrm{GeV}, |V_{eN}| = |A_{eN}| = 0.1$).

distinction between the Dirac and Majorana cases.

Let us consider the decay of a polarized heavy neutrino N into three light leptons:

$$\mathrm{N}(p_\mathrm{N}, S) \to \ell(p_-) + \bar{\ell}'(p_+) + \nu_i(p_\nu). \tag{21}$$

This process is only possible via charged currents as we have assumed the neutral currents to be flavour diagonal (see fig. 8a). Neglecting the charged lepton and light neutrino masses the matrix element for the decay (21) is given by

$$\begin{aligned}\sum |M_{(a)}|^2_{\mathrm{N}\to\ell\bar{\ell}'\nu_i} = \frac{g^4}{4} |D_\mathrm{W}(t)|^2 \, [\, (A+B)\, p_\mathrm{N}\cdot p_+ p_\nu \cdot p_- \\ + (A-B) p_\mathrm{N}\cdot p_\nu p_+ \cdot p_- - 2m_\mathrm{N}(C+D) S\cdot p_+ p_\nu \cdot p_- \\ - 2m_\mathrm{N}(C-D) S\cdot p_\nu p_+ \cdot p_- \,],\end{aligned} \tag{22}$$

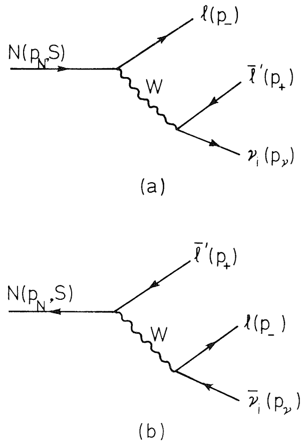

Fig. 8. Feynman diagrams for the leptonic decay of a heavy neutrino N. In the Dirac case only diagram (a) contributes, in the Majorana case both (a) and (b) ($\ell, \ell' = e^-, \mu^-, \tau^-$).

where we have defined

$$\begin{aligned} A &= (|V_{\ell N}|^2 + |A_{\ell N}|^2)\,(|V_{\ell' i}|^2 + |A_{\ell' i}|^2)\,, \\ B &= 4\mathrm{Re}(V_{\ell N} A^*_{\ell N})\,\mathrm{Re}(V_{\ell' i} A^*_{\ell' i})\,, \\ C &= (|V_{\ell' i}|^2 + |A_{\ell' i}|^2)\mathrm{Re}(V_{\ell N} A^*_{\ell N})\,, \\ D &= (|V_{\ell N}|^2 + |A_{\ell N}|^2)\mathrm{Re}(V_{\ell' i} A^*_{\ell' i})\,, \end{aligned} \tag{23}$$

and $t = (p_N - p_-)^2$. In the special case where N is purely left-handed we have

$$A = B = 2C = 2D = 4|V_{\ell N}|^2 |V_{\ell' i}|^2\,, \tag{24}$$

where V is now interpreted as a leptonic mixing matrix.

If N is a Majorana particle it can also decay into the following channel:

$$N(p_N, S) \to \ell(p_-) + \bar{\ell}'(p_+) + \bar{\nu}_i(p_\nu), \tag{25}$$

obtained by a charge conjugation from (21) (see fig. 8b). In this process the total lepton number is violated by two units, but this violation can be directly observed only if the outgoing light antineutrino is detected. Since this is not feasible, the only way to distinguish a decaying Majorana neutrino from a Dirac neutrino is to examine the detailed decay distributions of the charged leptons ℓ and ℓ' in (21) and (25), or to determine the total decay rate.

The matrix element corresponding to the diagram in fig. 8b can be obtained from eqs. (22) and (23) by interchanging the subscripts $\ell \leftrightarrow \ell'$ and the momenta $p_+ \leftrightarrow p_-$, and by reversing the sign of the terms proportional to m_N. If also ν_i is a Majorana particle the two diagrams in fig. 8 add coherently. The interference term is given by

$$\sum (M_{(a)} M^*_{(b)} + \text{h.c.}) = g^4 \text{Re}(D_W(t) D^*_W(u)) \times [m_N \text{Re}(Q e^{i(\varphi_i - \varphi_N)}) S \cdot p_\nu - \text{Re}(P e^{i(\varphi_i - \varphi_N)}) p_N \cdot p_\nu], \quad (26)$$

where $u = (p_N - p_+)$, and

$$\begin{aligned} P &= (V_{\ell N} V_{\ell' N} - A_{\ell N} A_{\ell' N})(V^*_{\ell i} V^*_{\ell' i} - A^*_{\ell i} A^*_{\ell' i}) \\ &\quad -(V_{\ell N} A_{\ell' N} - A_{\ell N} V_{\ell' N})(V^*_{\ell' i} A^*_{\ell i} - V^*_{\ell i} A^*_{\ell' i}), \\ Q &= (V_{\ell N} V_{\ell' N} - A_{\ell N} A_{\ell' N})(V^*_{\ell' i} A^*_{\ell i} - V^*_{\ell i} A^*_{\ell' i}) \\ &\quad -(V_{\ell N} A_{\ell' N} - V_{\ell' N} A_{\ell N})(V^*_{\ell i} V^*_{\ell' i} - A^*_{\ell i} A^*_{\ell' i}). \end{aligned} \quad (27)$$

One can see from eq. (27) that if one of the vertices is purely chiral, the interference term (26) vanishes. As already mentioned above, the charged-current couplings of the light neutrinos can deviate from the purely left-handed form only by a very small amount. Therefore, taking $\lambda_{ei} = 1$, the interference term will disappear. Consequently, we cannot obtain any information about the relative CP-phase of ν_i and N. The smallness of the interference term also means that the decay rate of a Majorana neutrino is always larger than the rate of a Dirac neutrino.

Let us now restrict ourselves to the case where the heavy neutrino N belongs to a possible fourth sequential family with the standard V – A interactions. By intergrating the matrix elements over the momenta of the final-state neutrino and antilepton we obtain the following differential decay rates:

$$\begin{aligned} \frac{d\Gamma_{(a)}}{dx\, d\cos\theta} &= \frac{G_F^2 m_N^5}{384\pi^3} |V_{\ell N}|^2 |V_{\ell' i}|^2 [f_{(a)}(x,r) + \cos\vartheta\, g_{(a)}(x,r)], \\ \frac{d\Gamma_{(b)}}{dx\, d\cos\theta} &= \frac{G_F^2 m_N^5}{384\pi^3} |V_{\ell' N}|^2 |V_{\ell i}|^2\ f_{(b)}(x,r)(1 - \cos\vartheta), \end{aligned} \quad (28)$$

corresponding to the diagrams 8a and 8b, respectively. Here ϑ is the angle between the spin of N and the momentum $\boldsymbol{p}_-$ of the final-state lepton ℓ, and we have defined

$$x = \frac{2 m_N E_\ell}{M_W^2}, \quad r = \left(\frac{m_N}{M_W}\right)^2, \quad (29)$$

$$\begin{aligned} f_{(a)}(x,r) &= \frac{2x^2(3r - 2x)}{r^4(1 + x - r)^2}, \quad g_{(a)}(x,r) = \frac{2x^2(r - 2x)}{r^4(1 + x - r)^2}, \\ f_{(b)}(x,r) &= \frac{12x^2(r - x)}{r^4(1 - x)}. \end{aligned} \quad (30)$$

Apart from the mixing factors the total decay rates into the above two channels are the same. The mass m_N dependence of the total rate is determined by the function

$$F(r) = \int_0^r f_{(a)}(x,r)\mathrm{d}x = \int_0^r f_{(b)}(x,r)\mathrm{d}x$$
$$= \frac{2}{r^4}[6r - 3r^2 - r^3 + 6(1-r)\ln(1-r)]. \tag{31}$$

Thus, if N is a Majorana neutrino, its total decay rate into all channels is twice the rate of the Dirac neutrino independently of its mass. The angular distributions, on the other hand, appear to be very different for the diagrams 8a and 8b. The lepton spectrum in the latter case is strongly peaked to the backward direction with an average of $\langle\cos\vartheta\rangle_{(b)} = -\frac{1}{3}$ independently of the lepton energy E. In the former case one has

$$\langle\cos\vartheta\rangle_{(a)} = \frac{G(r)}{3F(r)}, \tag{32}$$

where

$$G(r) = \int_0^r g_{(a)}(x,r)\mathrm{d}x = \frac{2}{r^4}[6r - 7r^2 + r^3 + 2(1-r)(3-2r)\ln(1-r)]. \tag{33}$$

The average value (32) is smaller in absolute magnitude, varying from $-\frac{1}{3}$ to 0 for neutrino mass increasing from 0 to M_W. For the averaged energies of the charged lepton ℓ one obtains in the limit $m_N \ll M_W$

$$\langle E_\ell\rangle_{(a)} = \tfrac{7}{20}m_N, \quad \langle E_\ell\rangle_{(b)} = \tfrac{3}{10}m_N. \tag{34}$$

4. Lepton pair correlations

The fact that the lifetime of a Dirac neutrino is twice the lifetime of a Majorana neutrino (which is true also when the decay modes to quarks are taken into account) may not be easy to observe in experiment. The problem is the very shortness of the lifetime (unless the mixings are extremely small, of course). It would also require a large sample of heavy neutrino decays in order to decide whether N is a Dirac or a Majorana particle just by looking at the single lepton spectrum because of the smallness of the differences between Dirac and Majorana cases as shown above.

We now show that a correlated production of lepton pairs may offer a useful method to experimentally distinguish between Dirac and Majorana neutrinos. We will not go here into details of the angular or energy distributions as we did above, but just consider the total decay rates. A more detailed analysis of lepton correlations which takes into account the differences in the energy and angular distributions in an appropriate way would give even larger differences for between Dirac and Majorana neutrinos than will be obtained below. Furthermore, we will restrict ourselves here to the case of a left-handed heavy neutrino, and assume, for simplicity, that the possible new charged lepton is heavier than the heavy neutrino. Analogous results to be derived below under these simplified assumptions can be found more general cases as well.

Let us now assume that a $N\bar{N}$ pair has been produced in reaction (1), and consider its subsequent decay producing a correlated $\ell\ell'$ pair, where $\ell, \ell' = e^-, \mu^-$ or τ^-. Since the two same-sign leptons cannot originate from the same neutrino, the branching ratio for the production is just the product of the individual branching ratios:

$$B(\ell\ell') = B(N \to \ell)B(\bar{N} \to \ell') + (\ell \leftrightarrow \ell', \text{ if } \ell \neq \ell') \,. \tag{35}$$

If N is a Dirac neutrino, one has

$$\begin{aligned} B_D(N \to \ell) &= \frac{\Gamma_D(N \to \ell)}{\Gamma_D(N \to \text{all})} = \frac{|V_{\ell N}|^2}{\sum_{\ell''} |V_{\ell'' N}|^2}, \\ B_D(\bar{N} \to \ell) &= \frac{\Gamma_D(\bar{N} \to \ell)}{\Gamma_D(\bar{N} \to \text{all})} = \frac{\sum_i |V_{\ell i}|^2}{3n_q + \sum_{i,\ell''} |V_{\ell'' i}|^2}, \end{aligned} \tag{36}$$

where n_q is the effective number of quark pairs that can be produced in N-decay (quark and lepton masses are neglected for simplicity) and the sums run over $\ell'' = e, \mu, \tau$ and $i = 1, 2, 3$. It follows that

$$B_D(\ell\ell') = \frac{|V_{\ell N}|^2 \sum_i |V_{\ell' i}|^2 + (\ell \leftrightarrow \ell', \text{ if } \ell \neq \ell')}{(3n_q + \sum_{i,\ell''} |V_{\ell'' i}|^2) \sum_{\ell''} |V_{\ell'' N}|^2} \,. \tag{37}$$

In the Majorana case one has simply

$$\begin{aligned} B_M(N \to \ell) &= \frac{\Gamma_M(N \to \ell)}{\Gamma_M(N \to \text{all})} \\ &= \frac{\Gamma_D(N \to \ell) + \Gamma_D(\bar{N} \to \ell)}{2\Gamma_D(N \to \text{all})} \\ &= \tfrac{1}{2}(B_D(N \to \ell) + B_D(\bar{N} \to \ell)) \,. \end{aligned} \tag{38}$$

Accordingly,

$$B_M(\ell\ell') = \begin{cases} 2B_M(N \to \ell)B_M(N \to \ell') & \text{for } \ell \neq \ell' \\ B_M^2(N \to \ell) & \text{for } \ell = \ell' \end{cases} . \quad (39)$$

Let us now call $\hat{\ell}$ that lepton whose non-diagonal coupling to the heavy neutrino is largest (if the analogy with quark sector works $\hat{\ell} = \tau$) and neglect the other non-diagonal couplings to N. Using the above eqs. (36) and (38) we find in the case of $n_q = 2\ (3)$ and $\ell = \hat{\ell} \neq \ell'$

$$\begin{aligned} B_D(\hat{\ell}\ell') &\simeq \frac{1}{3n_q + 3} = \tfrac{1}{9}\ (\tfrac{1}{12}) , \\ B_M(\hat{\ell}\ell') &\simeq \frac{3n_q + 4}{2(3n_q + 3)^2} = \tfrac{5}{81}\ (\tfrac{13}{288}) \end{aligned} \quad (40)$$

and for $\ell = \ell' = \hat{\ell}$

$$\begin{aligned} B_D(\hat{\ell}\hat{\ell}) &\simeq \frac{1}{3n_q + 3} = \tfrac{1}{9}\ (\tfrac{1}{12}) , \\ B_M(\hat{\ell}\hat{\ell}) &\simeq \frac{1}{4}\left(\frac{3n_q + 4}{3n_q + 3}\right)^2 = \tfrac{25}{81}\ (\tfrac{169}{576}) . \end{aligned} \quad (41)$$

We thus see that the correlated production of same-sign leptons of different flavours is larger for a heavy Dirac neutrino by a factor of $\frac{9}{5}\ (\frac{24}{13})$ compared with the heavy Majorana neutrino. The situation is, however, reversed for the pair production of same-sign leptons of the same flavour, where the Majorana case dominates over the Dirac case by a factor of $\frac{25}{9}\ (\frac{169}{48})$. Note that these ratios between the Majorana and Dirac cases are almost independent on the number of quark pairs n_q. Furthermore, since the pair same-sign leptons of the dominant flavour $\hat{\ell}$ is present in half of the all decays of a heavy Majorana neutrino pair, the method of studying same-sign leptons of the same flavour provides an effective test of the existence and nature of heavy neutrinos to be made in the future e^+e^- collider experiments.

5. Summary

We have investigated production of a heavy neutrino N in e^+e^- collisions considering both the pair production $e^+e^- \to N\bar{N}$ and the single production $e^+e^- \to N\bar{\nu}_i$, $\bar{N}\nu_i$, where ν_i is a light neutrino. We have considered the case where the neutrinos are Dirac particles as well as the case where they are Majorana particles. Differences between the two cases have been pointed out.

We also considered the subsequent decay of the neutrino N into leptonic and semileptonic channels, again making a distinction between the Dirac and Majorana cases. The correlated production of the equal-sign lepton pair in the decay of the $N\bar{N}$ system was found to be a possible method in experiment to distinguish heavy Dirac and Majorana neutrinos from each other.

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