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CONSTRAINTS FOR MIXING BETWEEN LIGHT AND HEAVY NEUTRINOS

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We consider mixing of the known light neutrinos ν_e, ν_μ and ν_τ with hypothetical heavy neutrinos [with a mass $m > O(1)$ GeV]. Several phenomenological limits for such mixing are derived from the existing experimental information. We investigate the models in which the heavy neutrinos are assigned to left-handed doublet (sequential neutrinos), to right-handed doublets (mirror neutrinos) and to right-handed singlets. The most general and reliable upper bounds for the dominant mixing angles of ν_e, ν_μ and ν_τ are found to be 13.6°, 10.2° and 29.3°, respectively.

1. Introduction. If improved statistics will confirm the preliminary evidence [1] to the top quark production at the SPS collider, we will have a classification of all the basic fermions into three generations complete. This need not be the end of the story, however. In fact, there exists no convincing theoretical argument telling us that the appearance of fermions would end with the known three family replicants. New generations alike may follow in sequence, or families having some different kind of SU(2) × U(1) structure may emerge, at higher mass scales.

The first heralds of the possible new fermion generations may be neutrinos. Actually the theoretical scenarios built up in order to produce low [2] or strictly vanishing [3] masses of the known left-handed neutrinos seem all to require such new neutrino states to exist with a high mass. These heavy neutrinos would manifest themselves through mixing with the light ones, already in the present experiments.

In this work we consider the mixing between the known light neutrinos (ν_e, ν_μ, ν_τ) and the up to now unobserved heavy neutrinos, and its manifestations in the leptonic weak interactions. We present some new methods to exploit the most accurate experimental data to set strict and general limits to such mixing. Recently, an analogous study was made by Gronau, Leung and Rosner [4], mostly in the specific case of heavy right-handed singlet neutrinos. Our approach is more general in that we discuss also other group assignments for the additional heavy neutrinos. Furthermore, we consider only the most general and least model-dependent experimental constraints. Contrary to ref. [4] we do not consider here effects depending on the (unknown) masses of the heavy neutrinos. Our results apply for any neutral fermions with masses larger than O(1) GeV. Also, to keep our discussion general, we do not assume any a priori relations between the neutrino masses and/or the mixing angles.

2. Sequential heavy neutrinos. Let us start with the case where the heavy neutrinos belong to a left-handed doublet of a new sequential generation. We will assume that the light neutrinos (ν_ℓ, $\ell = e, \mu, \tau$) are dominantly mixed to only one corresponding heavy neutrino

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(N_ℓ), and, for simplicity, we will neglect the possible mixing among the light neutrinos. The dominant mixing is parametrized by an angle ϕ_ℓ according to the following equation

$$\begin{pmatrix} \nu_\ell \\ N_\ell \end{pmatrix} = \begin{pmatrix} \cos\phi_\ell & -\sin\phi_\ell \\ \sin\phi_\ell & \cos\phi_\ell \end{pmatrix} \begin{pmatrix} \nu'_\ell \\ N'_\ell \end{pmatrix}, \qquad (1)$$

where the primed (unprimed) fields stand for the current (mass) eigenstates. If only the left-handed fields ($\nu'_{\ell L}$, $N'_{\ell L}$) exist, the mass eigenstates are Majorana neutrinos, and eq. (1) (its charge conjugate) applies for the left-handed (right-handed) components of these eigenstates. If also right-handed fields ($\nu'_{\ell R}$, $N'_{\ell R}$) are present we assume that the mass eigenstates are Dirac neutrinos or, more generally, that the Dirac-type mixing dominates over the Majorana-type mixing. (The opposite situation will be dealt with later on for the singlet neutrino case.)

On the basis of the present knowledge of the masses of the observed neutrinos one is inclined to think that the neutrinos of the additional sequential generations are also very light. However, cosmological nucleosynthesis [‡1] limits the number of all light neutrinos to 3–4, thus allowing for only one additional generation with a light neutrino. The further generations should therefore include considerably heavier neutral leptons. Since cosmology also forbids [‡1] a too long lifetime (very small mixing) of the decaying neutrinos in the present interesting mass range (although the theoretically improbable absolute stability is also allowed for $m_N \geqslant 2$–3 GeV) we conclude that our assumptions about heavy mixing neutrinos of the further generations are also cosmologically reasonable.

As is well known, for the sequential neutrino mixing neutral weak currents remain diagonal (the GIM mechanism). The charged weak current instead transforms to the form

$$J^\alpha_{CC}(\ell) = -(g/2)\sqrt{2})\,\bar{\ell}\gamma^\alpha(1-\gamma_5)(\cos\phi_\ell\,\nu_\ell + \sin\phi_\ell N_\ell). \qquad (2)$$

This leads to the following redefined form of the relation between the Fermi constant G_F, measured in the muon decay, and the parameters of the standard model:

‡1 For a review see ref. [5].

$$G_F/\sqrt{2} \equiv (\widetilde{G}/\sqrt{2})\cos\phi_e\cos\phi_\mu$$
$$= (\pi\alpha/2M_W^2\sin^2\theta_W)\cos\phi_e\cos\phi_\mu. \qquad (3)$$

Owing to the recent measurement of M_W [6] we now know the values of all parameters (α, $\sin^2\theta_W$, M_W) on the right-hand side of eq. (3) and hence we can use this equation as a consistency check of the standard model to constrain the mixing angle factor $\cos\phi_e \times \cos\phi_\mu$. By using the standard values for G_F and α [7] and the best-fit values for $\sin^2\theta_W = 0.217 \pm 0.014$ [8] and $M_W = (82.2 \pm 2.2)$ GeV [6] we find $\cos\phi_e\cos\phi_\mu = 1.055 \pm 0.088$. (In the above value for $\sin^2\theta_W$ we have neglected the small dependence of ρ on the mixing angles; see below.)

The above estimate will considerably change when radiative corrections are taken into account. They will affect eq. (3) since the parameters α and θ_W are measured at low Q^2 while M_W is determined at the pole $Q^2 = M_W^2$. Once can evaluate this effect by replacing M_W in eq. (3) by $M_W(1-r)$. In the standard model Marciano and Sirlin have calculated the radiative correction factor to be $r = 0.0696 \pm 0.0020$ [8]. (This number does not, of course, include the effect of further generations.) Using their result we obtain

$$\cos\phi_e\cos\phi_\mu = 0.982 \pm 0.082. \qquad (4)$$

As we will see below, however, the above constraint with the present values of the parameters is not very competitive in limiting the angles ϕ_e and ϕ_μ. We therefore anticipate an improvement in the experimental accuracies of $\sin^2\theta_W$ and M_W which will soon be reached. Using the values $\Delta\sin^2\theta_W = 0.008$ and $\Delta M_W = 0.8$ GeV and assuming the central value to be one we find

$$\cos\phi_e\cos\phi_\mu \geqslant 0.96. \qquad (5)$$

We now turn to discuss other implications of the mixing. First let us consider the by now very accurate reactor neutrino experiments [9–11]. Both the production rate of electron-type antineutrinos in nuclear beta decays and the cross section of the positron production at the detection vertex are affected by the mixing, and according to eq. (2) are both proportional to $(\widetilde{G}\cos\phi_e)^2$, where $\widetilde{G}$ is defined in eq. (3). The experimental data is always given in a form where the measured rate is scaled by the theoretical prediction of the standard model. The latter is proportional to the fourth power of the Fermi constant, i.e. to $G_F^4 = (\widetilde{G} \cos \phi_e \cos \phi_\mu)^4$. Therefore, the ratio R_y of the experimental and theoretical yields, while insensitive to the electron neutrino mixing, depends on the muonic mixing angle ϕ_μ and is predicted to be larger than one for non-zero ϕ_μ. This is to be compared with the case of mixing among light neutrinos for which G_F remains unaffected and R_y is always smaller than one. In the most general case, of course, both types of mixing may be present simultaneously and cause a mutually compensating effect, so pretending no mixing. This is the case if only the average depletion of the flux is induced by the mixing between light neutrinos. The two effects can be resolved if actual oscillations of the beam are observed.

There is a further important subtlety to the above discussion. Namely, the modification of the leptonic vertex at the production does not show up in the experimental method used since the main part (approximately 90%) of the reactor neutrino flux is determined from the measured yield of electrons originating from the decaying U^{235} and Pu^{239} nuclei. In our case both the electron and the antineutrinos yields are modified by the same amount, and thus, fitting the former does not give any information on the latter. (This should again be compared with the mixing among light neutrinos, where no loss of information occurs during such fitting.) Accordingly, the only observable effect of the mixing comes from the modification of the detection vertex, and the experimentally measured quantity R_y hence depends on ϕ_μ through the relation

$$R_y = 1/\cos^2\phi_\mu . \tag{6}$$

Since the shape of the neutrino spectrum is unaltered one could use the most accurate flux measurement at any particular neutrino energy. Here we are, however, content with using the energy averaged flux measurements. Combining the results of the Bugey [9], Gösgen [10] and Grenoble [11] experiments one obtains $R_y = 1.000 \pm 0.032$, which converts to an upper bound $|\phi_\mu| \leqslant 10.2°$. As will later be discussed more explicitly, the above analysis remains unchanged for the other group assignments of the heavy neutrinos. Thus the reactor neutrino flux experiments provide us with a very general result, only subject to the above mentioned proviso about the possible interplay of the two different mixing effects.

Our next topic is ρ, the parameter measuring the relative strength of neutral versus charged weak interactions. As noted above, the neutral currents remain diagonal and unmodified in the sequential neutrino case, while the charged weak cross sections will be suppressed by $\cos^2\phi_\ell$. Assuming the minimal Higgs pattern of the standard model one straightforwardly obtains, as only muonic neutrinos are used in the scattering experiments:

$$\rho = 1/\cos^2\phi_\mu . \tag{7}$$

Thus a non-vanishing mixing is, in this case, a way of making ρ larger than one. The present experimental value $\rho = 1.02 \pm 0.02$ [8] leads to the limit $|\phi_\mu| \leqslant 11.4°$. Since the neutral currents are sensitive to the group assignment of the heavy neutrino, this result only applies in the sequential case (for the other assignments, see below).

The most accurate but, on the other hand, theoretically the least reliable constraint on ϕ_μ can be derived from the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix for quarks. Because the Fermi constant and the leptonic couplings are affected by mixing, the analysis of CKM matrix elements is also modified. For example, the element $|U_{11}^{exp}|$ is determined by comparing the rate of the nuclear beta decay (proportional to $(\widetilde{G} \cos \phi_e)^2 \, |U_{11}^{th}|^2$) with the muon decay rate, and this gives the following relation between the experimental and theoretical matrix elements:

$$|U_{11}^{exp}|^2 = |U_{11}^{th}|^2/\cos^2\phi_\mu . \tag{8}$$

This same relation applies to all the known three elements of the first row ($U_{1i}, i = 1,2,3$) of the CKM matrix, since the most accurate experimental results for each of them come from the electronic decay modes. Then, unitarity of U^{th} allows us to use the equation

$$S^{exp}(N_f) \equiv \sum_{i=1}^{N_f} |U_{1i}^{exp}|^2 = 1/\cos^2\phi_\mu \tag{9}$$

as a constraint for ϕ_μ. This is, of course, academic as long as all the elements U_{1i} ($i = 1,\dots,N_f$) are not known. However, if one assumes the observed strong suppression of the off-diagonal elements to apply also for $i > 3$, one can saturate the above sum with the first two elements (the third one is already known to be negligible). Depending on whether one uses the averaged value for $|U_{12}^{exp}|$ or the value extracted from hyperon decay data only, one has [12] $S^{exp}(3) = 0.9965$

$\pm$ 0.0031 or 1.001 $\pm$ 0.0037, respectively. Even the latter, more conservative estimate would imply a very strict bound $|\phi_\mu| \leqslant 3.9^\circ$.

Most of the constraints discussed above concern the muonic mixing angle ϕ_μ only. In fact, for ϕ_e analogous accurate direct tests do not exist. (Such would be e.g. the either exclusive low-energy or inclusive high-energy neutral-over-charged cross-section ratio for an electron-type neutrino beam. The only such measurement known to us is that by the Irvine group [13] which, however, had unpracticable large errors.) The best present limit for ϕ_e is obtained indirectly from the universality test of electronic versus muonic decay rates of the pion:

$$R_\pi = \Gamma(\pi \to e\, \nu_e)/\Gamma(\pi \to \mu\, \nu_\mu)$$

$$= 1.239 \times 10^{-4} \cos^2\phi_e / \cos^2\phi_\mu , \qquad (10)$$

where the numerical factor includes the mass dependence and radiative effects [14] [‡2]. Using the averaged experimental value $R_\pi^{\exp} = (1.232 \pm 0.024) \times 10^{-4}$ [7] the following bounds are found for the two mixing angles:

$$0.9874 \leqslant \cos\phi_e / \cos\phi_\mu \leqslant 1.0069. \qquad (11)$$

Together with R_y (eq. (6)) this gives an upper bound $|\phi_e| \leqslant 13.6^\circ$.

Before closing the discussion on sequential neutrinos we make a remark on the mixing in the τ sector. With ν_τ not being even directly observed, there is not too much experimental data available. However, the $\tau - \mu$ universality can be tested by comparing the electronic decay width of the τ, $\Gamma(\tau \to e\, \nu\nu)$, with the total width of the muon, $\Gamma(\mu \to e\, \nu\nu)$. Scaling this ratio with masses one obtains

$$\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{\Gamma(\tau \to e\nu\nu)}{\Gamma(\mu \to e\nu\nu)} \equiv \left(\frac{G_\tau}{G_F}\right)^2 = \left(\frac{\cos\phi_\tau}{\cos\phi_\mu}\right)^2 . \qquad (12)$$

By combining the data on the total lifetime of τ and its electronic branching ratio [‡3] gives us the value of the "Fermi constant" in τ decay of $G_\tau^{\exp} = (1.120 \pm 0.086) \times 10^{-5}$ GeV^{-2} which, when combined with R_y leads to the present best upper bound for $|\phi_\tau|$ of 29.3°.

[‡2] For a review see ref. [15].

[‡3] For a recent review and references see ref. [16].

3. Heavy right-handed singlet neutrinos. Let us now consider the case of singlet neutrinos. We assume that in addition to the field $\nu'_{\ell L}$ there exists a right-handed SU(2) × U(1)-singlet state $N'_{\ell R}$ in every family, and that the two states mix with each other through a general Dirac–Majorana mass matrix. While the charged currents maintain the same form as above, eq. (2), the neutral currents are changed to the following form involving also non-diagonal interactions

$$J^\alpha_{NC}(\nu_\ell, N_\ell) = -(g/4\cos\theta_W)$$
$$\times \{\cos^2\phi_\ell\, \bar\nu_\ell \gamma^\alpha(1-\gamma_5)\nu_\ell + \sin^2\phi_\ell\, \bar N_\ell \gamma^\alpha(1-\gamma_5) N_\ell$$
$$+ \sin\phi_\ell \cos\phi_\ell\, [\bar\nu_\ell \gamma^\alpha (1-\gamma_5) N_\ell + \bar N_\ell \gamma^\alpha(1-\gamma_5)\nu_\ell]\}, \qquad (13)$$

where the mass eigenstates (Majorana neutrinos) ν_ℓ and N_ℓ are given by

$$\begin{pmatrix} \nu_\ell \\ N_\ell \end{pmatrix} = \begin{pmatrix} \cos\phi_\ell & -\sin\phi_\ell \\ \sin\phi_\ell & \cos\phi_\ell \end{pmatrix} \begin{pmatrix} \nu'_{\ell L} + \nu'^{C}_{\ell R} \\ N'^{C}_{\ell L} + N'_{\ell R} \end{pmatrix}. \qquad (14)$$

Accordingly, all the constraints discussed above for the sequential neutrino mixing remain valid except for the one coming from ρ, which, due to the suppression of the diagonal ν_ℓ-interaction in eq. (13), will now obtain the form

$$\rho = \cos^2\phi_\mu . \qquad (15)$$

Since the central experimental value of ρ is greater than one, we obtain now a somewhat better bound than in the sequential neutrino case, $|\phi_\mu| \leqslant 8.2^\circ$ (2σ-limit). Rigorously speaking, this estimate is exact only if N_μ is too massive to be produced in the neutrino scattering experiments that determine ρ. Indeed, if no mass suppression would exist, ρ would be trivially equal to one. However, in the N mass range we are dealing with, the kinematical suppression is already sizeable. Furthermore, the charged decay modes of N_μ will by far dominate the neutral ones, and thus, if the mixing angle is not totally negligible, N_μ events would be counted as charged current events.

The above estimate is reliable also in a model with an enlarged Higgs sector. Namely, using a triplet Higgs to induce a Majorana mass for the doublet neutrino $\nu'_{\ell L}$ would, just like the mixing, decrease ρ. Thus a compensation between the two effects is not possible.

4. Heavy mirror neutrinos. Finally, let us briefly discuss an assignment where $N'_{\ell R}$ is a member of a right-handed mirror family [17]. If there are no singlet fields $(\nu'_{\ell R}, N'_{\ell L})$, the relation between the current and mass (Majorana) eigenstates is as in the singlet neutrino case, eq. (14). In case the singlet fields do exist, we assume, in analogy to the sequential neutrino case, that the Dirac-type νN-mixing dominates over possible Majorana mixings, thus justifying the parametrization of eq. (1). Allowing also for the possible mixing between the charged lepton ℓ and its mirror partner (in analogy to eq. (1) parametrized by the angle θ_ℓ) one finds the following form for the weak currents:

$$J^\alpha_{CC}(\ell) = -(g/2\sqrt{2}) \times \bar{\ell}\gamma^\alpha\{[\cos(\phi_\ell - \theta_\ell) - \cos(\phi_\ell + \theta_\ell)\gamma_5]\,\nu_\ell + [\sin(\phi_\ell - \theta_\ell) - \sin(\phi_\ell + \theta_\ell)\gamma_5]\,N_\ell\}, \qquad (16)$$

$$J^\alpha_{NC}(\nu_\ell, N_\ell) = -(g/4\cos\theta_W) \times \{\bar{\nu}_\ell\gamma^\alpha(1-\cos 2\phi_\ell\gamma_5)\,\nu_\ell + \bar{N}_\ell\gamma^\alpha(1+\cos 2\phi_\ell\gamma_5)N_\ell - \sin 2\phi_\ell\,[\bar{\nu}_\ell\gamma^\alpha\gamma_5 N_\ell + \bar{N}_\ell\gamma^\alpha\gamma_5\nu_\ell]\}, \qquad (17)$$

where the mass eigenstates ν_ℓ and N_ℓ are given by eq. (14).

The essential difference to the former two cases is that the Lorentz structure of both charged and neutral weak currents is modified, and therefore none of the above considered constraints remains valid straightforwardly. While we refer the reader to ref. [18] for a detailed phenomenological analysis of the model we only wish to make here the following remarks. If the mixing between charged particles is suppressed with respect to that between the neutrinos (i.e. $\theta_\ell \ll \phi_\ell$) one recovers for eq. (16) exactly the same (left-handed) form as above in eq. (2) and, accordingly, the constraints based on R_y, R_π, S and G_τ remain as they were obtained above. However, the neutral currents reveal their non-trivial Lorentz structure which complicates the determination of $\sin^2\theta_W$ from neutrino scattering data, and thus the information from the consistency relation (eq. (3)) and ρ parameter is not straightforwardly available. For such models a more direct way to obtain $\sin^2\theta_W$ is to compare the masses of the vector bosons W and Z [‡4]. In the other, perhaps less probable case $\phi_\ell \ll \theta_\ell$, all constraints are applicable without modifications, but they limit in this case the charged lepton mixing angles θ_ℓ.

[‡4] A possible difference (after radiative corrections) in the values of $\sin^2\theta_W$ obtained by these two methods could also reflect an effect of the mixing.

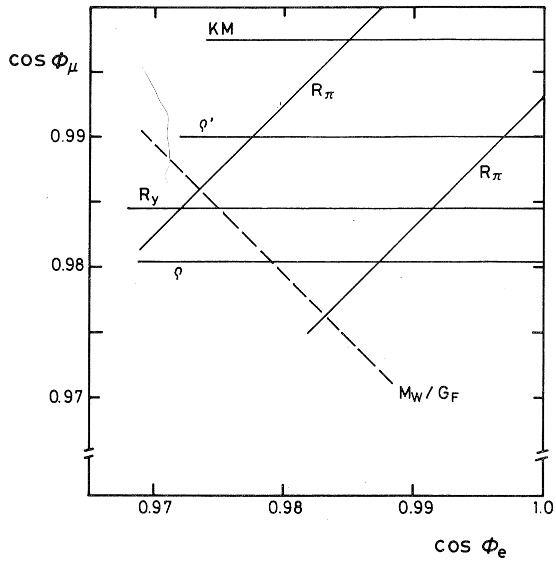

Fig. 1. Bounds to the electronic and muonic mixing angles ϕ_e and ϕ_μ in the $\cos\phi_e$–$\cos\phi_\mu$ plane. The horizontal lines are lower limits to $\cos\phi_\mu$ coming from the CKM matrix (eq. (9)), the reactor neutrino flux measurement (R_y, eq. (6)) and the ρ-parameter (ρ refers to the sequential neutrino model, eq. (7), and ρ' to the singlet neutrino case, eq. (15)). The pion decay ratio (eq. (11)) gives the band between the two R_π lines, and the dashed line (M_W/G_F) perpendicular to it is a future estimate for the consistency test of the standard model, eq. (5) (the present limit is out of the scope of this figure). All values are 1σ limits, except the 2σ limit for ρ'.

5. Summary. Most of our results are compactly summarized in fig. 1. There the phenomenological constraints for the mixing angles of electronic and muonic neutrinos are presented. We recall that the results apply to the case where the light neutrinos (ν_e, ν_μ, ν_τ) mix with so far unobserved heavy neutrinos with a mass larger than O(1) GeV. Most of the constraints are valid for different SU(2) × U(1) group assignments of the heavy neutrinos, i.e., for a left-handed doublet (new sequential generations), a right-handed singlet or a right-handed doublet (mirror families). The most general and reliable upper bounds for the electronic, muonic and tauonic mixing angles are found to be 13.6°, 10.2° and 29.3°, respectively.

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