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LEPTONIC CHARGED WEAK INTERACTIONS IN THE LEFT-RIGHT SYMMETRIC MODEL

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An extensive analysis of the charged weak interactions of leptons within the left-right symmetric electroweak model is made. A general mass matrix of the neutrinos is allowed for, neglecting intergenerational mixing for simplicity, and the limits to the parameters of the model are obtained for various possible neutrino mass configurations. It is also pointed out that one of the muon decay parameters, δ, can uniquely distinguish between the different possibilities for the masses of the right-handed neutrinos.

1. Introduction

The exact V − A structure of charged weak interactions with the implied maximal parity (and charge conjugation) violation is a basic property of the present standard electroweak theory: the Glashow–Weinberg–Salam (GWS) model [1]. It is included in the model by judiciously choosing the chiral fermion representations under the $SU(2)\times U(1)$ group, the minimal gauge symmetry unifying weak and electromagnetic interactions. Thus the V − A structure is actually built in the model by hand and, consequently, no natural explanation is obtained for it.

Many extended models restoring the parity (and charge conjugation) invariance in the lagrangian have been proposed. After the most economical extension, the vector-like theory, has been excluded by the famous polarized electron–deuteron experiment [2], we are left with larger extensions of the GWS model. The most studied alternative is the left-right symmetric (LRS) model based on the $SU(2)_L\times SU(2)_R\times U(1)$ gauge group [3]. The obvious left-right symmetry of the lagrangian is violated by an asymmetric vacuum, giving different masses to the two sets of gauge bosons.

The question of the energy scale of parity restoration in the LRS model has been under much discussion recently. If the spontaneous symmetry breaking (SSB) of the model is done in a "minimal" way [4] using $\Delta_R=(0,1,2)$ Higgs representation to break the $SU(2)_R$ group, one naturally relates the parity restoration mass scale to the Majorana mass of right-handed neutrinos, and the light mass of the (almost purely) left-handed neutrino will be explained through the see-saw mechanism [5]. Then all the accurate purely leptonic charged current (CC) constraints are reduced to their V-A values, and information on the parity violation level is only obtained from the less accurate and theoretically less well-known semileptonic [6, 7] and purely hadronic processes.

The hadronic weak decays of baryons have been shown [7, 8] to limit only the possible mixing between the two charged weak bosons. Other estimates using the K_L-K_S mass difference or pionic decays of kaons can give a lower bound to the right-handed mass scale. However, the situation is still unsettled since some groups [9] report results consistent with low-mass parity restoration ($M_R \sim 200$ GeV) while others [10] are against it. There are several unknown (top-quark mass, Higgs masses) or inaccurately known factors which affect the result. Furthermore, whatever the final conclusion will be, it is dependent on the chosen Higgs sector and therefore not a general result.

Enlarging the minimal Higgs sector or using different Higgs multiplets to massify vector bosons (e.g. with $\delta_R = (0, 1, 0)$ and $\delta_L = (1, 0, 0)$) and neutrinos allows for a more general neutrino mass matrix. Then one can also use the purely leptonic high-accuracy data to obtain reliable information on the parameters of the left-right symmetric model. Up to now the leptonic constraints have been discussed only for the case of Dirac neutrinos [11, 12].

In this work we make a general study of the charged weak interactions of leptons, allowing for arbitrary mixing between the left- and right-handed neutrinos and treating the right-handed neutrino masses as *a priori* unknown parameters. We will, however, neglect intergenerational mixing to simplify our argument. Such an analysis is important in view of the on-going and future high-accuracy experiments planned to measure the muon [13] and beta [14] decay parameters by about one order of magnitude more precisely than today. We also note that if any deviation from V-A is seen in these experiments, it will not only imply an extended gauge structure, but also give information about the mixings and masses of the additional (mostly right-handed) neutrino states.

This work is divided as follows. In sect. 2 we review the leptonic CC interactions in the LRS model and discuss the neutrino mass matrix and its eigenstates. In sect. 3 the CC constraints are worked out and in sect. 4 several neutrino mass configurations are considered. In sect. 5 we present our conclusions.

2. Charged weak interactions of leptons in the LRS model

The assignment of fermions in the LRS model is symmetrical: the left (right)-handed states transform as doublets under $SU(2)_{L(R)}$. Hence the basic CC lagrangian (for one lepton generation) has the form:

$$\mathscr{L}^{CC} = -\sqrt{\tfrac{1}{2}}g[W_L^{\dagger\mu}\bar{l}_L\gamma_\mu\nu_{e_L} + W_R^{\dagger\mu}\bar{l}_R\gamma_\mu\nu_{e_R}] + \text{h.c.} \qquad (1)$$

In general, the left- and right-handed vector bosons W_L and W_R will get mixed through their mass matrix by an angle ω (dependent on the details of SSB):

$$\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} W_L \\ W_R \end{pmatrix}. \tag{2}$$

The effective four-fermion CC lagrangian can then be written (in the low-$|q^2|$ limit) as follows

$$\mathscr{L}_{\text{eff}}^{\text{CC}} = -\tfrac{1}{8}g^2[AJ_{\ell_L}J^{\dagger}_{\ell'_L} + BJ_{\ell_R}J^{\dagger}_{\ell'_R} + C_1 J_{\ell_L}J^{\dagger}_{\ell'_R} + C_2 J_{\ell_R}J^{\dagger}_{\ell'_L}], \tag{3}$$

where the left- and right-handed lepton currents are

$$J^{\mu}_{\ell_{L(R)}} = \bar{\nu}_\ell \gamma^\mu (1 \underset{(+)}{-} \gamma_5)\ell, \tag{4}$$

and the following constants have been introduced:

$$A = \frac{\cos^2\omega}{M^2_{W_1}} + \frac{\sin^2\omega}{M^2_{W_2}},$$

$$B = \frac{\sin^2\omega}{M^2_{W_1}} + \frac{\cos^2\omega}{M^2_{W_2}},$$

$$C_1 = C_2 = C = \sin\omega\cos\omega\left(-\frac{1}{M^2_{W_1}} + \frac{1}{M^2_{W_2}}\right). \tag{5}$$

We also use the ratios x and y:

$$x = B/A = \frac{(\varepsilon+1)^2 r + (\varepsilon-1)^2}{(\varepsilon+1)^2 + (\varepsilon-1)^2 r},$$

$$y = C/A = \frac{(1-\varepsilon^2)(1-r)}{(\varepsilon+1)^2 + (\varepsilon-1)^2 r}, \tag{6}$$

where

$$\varepsilon = \frac{1+\tan\omega}{1-\tan\omega},$$

$$r = (M_{W_1}/M_{W_2})^2. \tag{7}$$

In the above formulae we are dealing with weak eigenstates of neutrinos. Next we will discuss the neutrino mass matrix and its eigenvalues, and then present the effective CC interactions in the neutrino mass eigenstate basis. Let us first simplify the most general mass matrix of (three generations of) neutrinos by neglecting the intergenerational mixing, and instead take the most general mass matrix for each generation separately. This choice makes our argument more clear and distinguishes it from the other, often studied, special case of sizeable flavour mixing and no $\nu_{\ell_L} - \nu^C_{\ell_L}$ mixing.

The most general mass matrix for one generation is [15]

$$\mathscr{L}_{\mathrm{m}}^{\nu} = -\tfrac{1}{2}(\bar{\nu}_{\ell_{\mathrm{L}}}, \bar{\nu}^{\mathrm{C}}_{\ell_{\mathrm{L}}})\begin{pmatrix} m_{\mathrm{L}}^{\ell} & m_{\mathrm{D}}^{\ell} \\ m_{\mathrm{D}}^{\ell} & m_{\mathrm{R}}^{\ell} \end{pmatrix}\begin{pmatrix} \nu^{\mathrm{C}}_{\ell_{\mathrm{R}}} \\ \nu_{\ell_{\mathrm{R}}} \end{pmatrix} + \mathrm{h.c.} \tag{8}$$

This symmetric mass matrix M_ℓ can be diagonalized by a unitary matrix U_ℓ:

$$D_\ell = U_\ell^{\mathrm{T}} M_\ell U_\ell = \begin{pmatrix} m_{\ell_1} & 0 \\ 0 & m_{\ell_2} \end{pmatrix}. \tag{9}$$

Defining a new neutrino basis

$$\begin{pmatrix} \nu_{\ell_1} \\ \nu_{\ell_2} \end{pmatrix}_{\mathrm{L}} = U_\ell^{\mathrm{T}} \begin{pmatrix} \nu_\ell \\ \nu_\ell^{\mathrm{C}} \end{pmatrix}_{\mathrm{L}},$$

$$\begin{pmatrix} \nu_{\ell_1} \\ \nu_{\ell_2} \end{pmatrix}_{\mathrm{R}} = U_\ell^{\dagger} \begin{pmatrix} \nu_\ell^{\mathrm{C}} \\ \nu_\ell \end{pmatrix}_{\mathrm{R}}, \tag{10}$$

one finds that the mass lagrangian takes the form:

$$\mathscr{L}_{\mathrm{m}} = -\tfrac{1}{2}(m_{\ell_1} \bar{\nu}^{\mathrm{C}}_{\ell_{1\mathrm{L}}} \nu_{\ell_{1\mathrm{R}}} + m_{\ell_2} \bar{\nu}^{\mathrm{C}}_{\ell_{2\mathrm{L}}} \nu_{\ell_{2\mathrm{R}}}) + \mathrm{h.c.} \tag{11}$$

If the mass eigenvalues m_{ℓ_i} are taken real and positive, the phases of the unitary matrix U_ℓ are fixed [16]. However, we will restrict ourselves here to the *CP*-conserving case of real matrices M_ℓ (in fact we already assumed *CP* invariance for the $\mathrm{W_L}$-$\mathrm{W_R}$ mass matrix in eq. (2)), whence the diagonalizing matrices U_ℓ are orthogonal:

$$U_\ell \equiv \begin{pmatrix} \cos\alpha_\ell & -\sin\alpha_\ell \\ \sin\alpha_\ell & \cos\alpha_\ell \end{pmatrix}. \tag{12}$$

Then the mass eigenvalues are real, but their signs η_i^ℓ are not known:

$$m_{\ell_i} \equiv \eta_i^\ell |m_{\ell_i}|, \qquad \eta_i^\ell = \pm 1. \tag{13}$$

Finally, the mass lagrangian (11) is diagonalized by Majorana states χ_i^ℓ:

$$\chi_i^\ell = \nu_{\ell_{i\mathrm{R}}} + \eta_i^\ell \nu^{\mathrm{C}}_{\ell_{i\mathrm{L}}}, \tag{14}$$

which have the following *CP* property:

$$\chi_i^{\ell\mathrm{C}} = \eta_i^\ell \chi_i^\ell. \tag{15}$$

As noted first by Wolfenstein [17] the relative *CP* factors are relevant for processes where two (or more) neutrino states contribute either as propagators (e.g. in neutrinoless double beta decay) or real particles (e.g. in $\nu_1 \to \nu_2 \gamma$ decay). The η_i^ℓ factors do not appear in the CC processes we are dealing with below. However, for completeness we will keep them in our general formulae.

Let us now briefly comment on the mass matrix M in eq. (8) and its eigenvalues m_i and the mixing angle α (we drop the label ℓ for a moment). The eigenvalues are

$$m_\pm = \tfrac{1}{2}(m_{\mathrm{L}} + m_{\mathrm{R}} \pm \sqrt{(m_{\mathrm{L}} - m_{\mathrm{R}})^2 + 4m_{\mathrm{D}}^2}). \tag{16}$$

We wish to call the state with a smaller (absolute value of) mass the first neutrino state, and the other state with a larger one, the second neutrino. Thus for $m_L + m_R > 0$ we have $m_1 = m_-$ and $m_2 = m_+ > 0$ (implying $\eta_2 = +1$), while for $m_L + m_R < 0$ the other way around (with $\eta_2 = -1$). In the first case the mixing angle will be given by

$$\tan\alpha = \frac{m_D}{m_L - m_R} = \frac{1}{2m_D}\left(m_R - m_L - \sqrt{(m_L - m_R)^2 + 4m_D^2}\right), \quad (17)$$

while in the second case one has to replace the right-hand side of eq. (17) by its negative inverse. The mass eigenvalues have the same or opposite signs, depending on whether the quantity $m_L m_R - m_D^2$ is positive or negative.

Even in the case of large mass separation between the two neutrinos ($|m_2| \gg |m_1|$) it is possible in principle that the mixing angle α is large. For definiteness, eq. (17) is then reduced to $\tan\alpha = -m_L/m_R$ which implies that $|\alpha|$ can be of the order of one if $m_L \sim m_R \sim \pm m_D$. However, this requires that the parameters of the mass matrix be fine-tuned corresponding to the mass separation by as many orders of magnitude. For smaller $|\alpha|$, the need for fine tuning will be diminished, of course.

Let us now go back to study the effective lagrangian of the leptonic CC interactions and write it in the neutrino mass eigenstate basis. Using eqs. (10) and (14) to substitute the χ_i^ℓ for ν_ℓ's in the currents J_ℓ^μ of eq. (4), one finds that the effective lagrangian consists of four parts, all of which can be written in the form of eq. (3), however, with modified coefficients. E.g. for the part involving the two low-mass neutrinos χ_1^ℓ and $\chi_1^{\ell'}$ one has to replace A, B, C_1 and C_2 by $\tilde{A}$, $\tilde{B}$, $\tilde{C}_1$, and $\tilde{C}_2$, where

$$\begin{aligned}
\tilde{A} &= A\cos\alpha_\ell \cos\alpha_{\ell'}\eta_1^\ell\eta_1^{\ell'}, \\
\tilde{B} &= B\sin\alpha_\ell \sin\alpha_{\ell'}, \\
\tilde{C}_1 &= C\cos\alpha_\ell \sin\alpha_{\ell'}\eta_1^\ell, \\
\tilde{C}_2 &= C\sin\alpha_\ell \cos\alpha_{\ell'}\eta_1^{\ell'}.
\end{aligned} \quad (18)$$

The corresponding values of $\tilde{A}$, $\tilde{B}$, $\tilde{C}_1$ and $\tilde{C}_2$ for the three other channels $\chi_1^\ell\chi_2^{\ell'}$, $\chi_2^\ell\chi_1^{\ell'}$ and $\chi_2^\ell\chi_2^{\ell'}$ have been tabulated in table 1.

TABLE 1

The coefficients $\tilde{A}$, $\tilde{B}$, $\tilde{C}_1$ and $\tilde{C}_2$ of eq. (3) (replacing A, B, C_1 and C_2) for the three different neutrino channels with mass eigenstate neutrinos

	$\chi_1^\ell\chi_2^{\ell'}$	$\chi_2^\ell\chi_1^{\ell'}$	$\chi_2^\ell\chi_2^{\ell'}$
$\tilde{A}$	$-A\cos\alpha_\ell \sin\alpha_{\ell'}\eta_1^\ell\eta_2^{\ell'}$	$-A\sin\alpha_\ell\cos\alpha_{\ell'}\eta_2^\ell\eta_1^{\ell'}$	$A\sin\alpha_\ell\sin\alpha_{\ell'}\eta_2^\ell\eta_2^{\ell'}$
$\tilde{B}$	$B\sin\alpha_\ell\cos\alpha_{\ell'}$	$B\cos\alpha_\ell\sin\alpha_{\ell'}$	$B\cos\alpha_\ell\cos\alpha_{\ell'}$
$\tilde{C}_1$	$C\cos\alpha_\ell\cos\alpha_{\ell'}\eta_1^\ell$	$-C\sin\alpha_\ell\sin\alpha_{\ell'}\eta_2^\ell$	$-C\sin\alpha_\ell\cos\alpha_{\ell'}\eta_2^\ell$
$\tilde{C}_2$	$-C\sin\alpha_\ell\sin\alpha_{\ell'}\eta_2^{\ell'}$	$C\cos\alpha_\ell\cos\alpha_{\ell'}\eta_1^{\ell'}$	$-C\cos\alpha_\ell\sin\alpha_{\ell'}\eta_2^{\ell'}$

3. The charged current constraints

We will now derive the formulae for the most important quantities that restrict the structure of charged weak interactions of leptons. These constraints come from muon, pion and nuclear beta decay [11], inverse muon scattering [12] and high-energy neutrino scattering [6].

The muon decay parameters ρ, ξ, δ and ξ' for each possible decay channel can be straightforwardly derived using the above equations (with $\ell=\mu$, $\ell'=\mathrm{e}$) and general formulae [18]. The corresponding decay rate is proportional to the factor $K=\tilde{A}^2+\tilde{B}^2+\tilde{C}_1^2+\tilde{C}_2^2$. We list in table 2 our results for these parameters, in a general notation for all channels. (One can explicitly see that the η_i^ℓ factors do not appear since the constants $\tilde{A}$, $\tilde{B}$, $\tilde{C}_1$ and $\tilde{C}_2$ are squared).

The total cross section for the inverse muon decay process can also be derived from the above lagrangian (now with $\ell=\mathrm{e}$, $\ell'=\mu$) and the general result [19] with a non-trivial longitudinal polarization h of the neutrino ($h=P^P_{\chi_i^\mu}$, see later). Our result is (again, for any particular channel)

$$\sigma \sim \tilde{A}^2(1-h)+\tilde{B}^2(1+h)+0.375[\tilde{C}_1^2(1+h)+\tilde{C}_2^2(1-h)]\,. \qquad (19)$$

In what follows, we will divide the cross section with its V-A value and call the ratio S.

The semileptonic effective CC interactions can, of course, be represented by formulae similar to eq. (3). One finds easily that the lognitudinal polarization of the neutrino χ_i^ℓ from pseudoscalar ($P^{\mathrm{P}}_{\chi_i^\ell}$, $\ell=\mathrm{e}, \mu, \ldots$, $\mathrm{P}=\pi, \mathrm{K}, \ldots$) and nuclear beta decays ($P^{\mathrm{N}}_{\chi_i^\ell}$, for pure Fermi transitions $\mathrm{N}=\mathrm{F}$ and pure Gamow–Teller transitions $\mathrm{N}=\mathrm{GT}$) [20] can be expressed compactly as follows:

$$P^{\mathrm{P}}_{\chi_i^\ell}=-\frac{c}{v}P^{\mathrm{N}}_{\chi_i^\ell}=\frac{2V_\ell A_\ell}{V_\ell^2+A_\ell^2}\,. \qquad (20)$$

TABLE 2

Formulae of the muon decay parameters for each separate neutrino channel using the notation of eq. (3) (with A, B, C_1 and C_2 replaced by $\tilde{A}$, $\tilde{B}$, $\tilde{C}_1$ and $\tilde{C}_2$)

$$\rho=\frac{3}{4}\frac{\tilde{A}^2+\tilde{B}^2}{\tilde{A}^2+\tilde{B}^2+\tilde{C}_1^2+\tilde{C}_2^2}$$

$$\xi=\frac{\tilde{A}^2-\tilde{B}^2-3\tilde{C}_1^2+3\tilde{C}_2^2}{\tilde{A}^2+\tilde{B}^2+\tilde{C}_1^2+\tilde{C}_2^2}$$

$$\delta=\frac{3}{4}\frac{\tilde{A}^2-\tilde{B}^2}{\tilde{A}^2-\tilde{B}^2-3\tilde{C}_1^2+3\tilde{C}_2^2}$$

$$\xi'=\frac{\tilde{A}^2-\tilde{B}^2-\tilde{C}_1^2+\tilde{C}_2^2}{\tilde{A}^2+\tilde{B}^2+\tilde{C}_1^2+\tilde{C}_2^2}$$

TABLE 3

Coefficients V_ℓ and A_ℓ for the longitudinal polarization of the two neutrino states χ_1^ℓ and χ_2^ℓ (if kinematically allowed) in three processes (pseudoscalar decays and nuclear Gamow–Teller and Fermi transitions)

	V_ℓ	A_ℓ
$P^{\mathrm{P}}_{\chi_1^\ell}$ and $P^{\mathrm{GT}}_{\chi_1^\ell}$	$-(A-C)\cos\alpha_\ell\eta_1^\ell+(B-C)\sin\alpha_\ell$	$-(A-C)\cos\alpha_\ell\eta_1^\ell-(B-C)\sin\alpha_\ell$
$P^{\mathrm{P}}_{\chi_2^\ell}$ and $P^{\mathrm{GT}}_{\chi_2^\ell}$	$(A-C)\sin\alpha_\ell\eta_2^\ell+(B-C)\cos\alpha_\ell$	$(A-C)\sin\alpha_\ell\eta_2^\ell-(B-C)\cos\alpha_\ell$
$P^{\mathrm{F}}_{\chi_1^\ell}$	$(A+C)\cos\alpha_\ell\eta_1^\ell+(B+C)\sin\alpha_\ell$	$(A+C)\cos\alpha_\ell\eta_1^\ell-(B+C)\sin\alpha_\ell$
$P^{\mathrm{F}}_{\chi_2^\ell}$	$-(A+C)\sin\alpha_\ell\eta_2^\ell+(B+C)\cos\alpha_\ell$	$-(A+C)\sin\alpha_\ell\eta_2^\ell-(B+C)\cos\alpha_\ell$

The corresponding constants V_ℓ and A_ℓ for each process and for the two decay modes (if kinematically allowed) are given in table 3. The rate of each decay mode is proportional to the denominator of eq. (20). The electron-to-muon ratio R_{P} of the pseudoscalar meson decay that we are going to use as one constraint, can be calculated from these rates. The longitudinal polarizations of the charged leptons ℓ^- are easily obtained from eq. (20) and table 3. If the corresponding χ_2^ℓ state is heavy, they are just

$$P_\ell \equiv P^{\mathrm{P}}_{\ell^-} = -\frac{c}{v}P^{\mathrm{GT}}_{\ell^-} = \frac{1-[(x-y)/(1-y)]^2\tan^2\alpha_\ell}{1+[(x-y)/(1-y)]^2\tan^2\alpha_\ell}$$

$$-\frac{c}{v}P^{\mathrm{F}}_{\ell^-} = \frac{1-[(x+y)/(1+y)]^2\tan^2\alpha_\ell}{1+[(x+y)/(1+y)]^2\tan^2\alpha_\ell}\,. \tag{21}$$

If χ_2^ℓ is light enough to contribute to the decay, the formulae [11] of the Dirac case are recovered.

Another accurate piece of information on the CC structure is obtained from high-energy muon-neutrino scattering, using its Y dependence ($Y = 1 - E_\mu/E_\nu$) at high x-values. One can find an upper bound on right-handed interactions from the ratio of Y-dependent and constant terms in the differential cross section. We call this ratio R_Y and find the following formula for it: if χ_2^μ is heavy

$$R_Y = \frac{y^2(1-h\cos^2 2\alpha_\mu)}{\cos^2\alpha_\mu(1-h)+x^2\sin^2\alpha_\mu(1+h)}\,, \tag{22}$$

and if χ_2^μ is light

$$R_y = \frac{y^2[(1-y)^2+(x-y)^2]}{(1-y)^2+x^2(x-y)^2}\,. \tag{23}$$

All the used CC constraints with their most recent experimental values (and 1σ errors) are listed in table 4. Note that we have included the recent and very accurate result [21] for the quantity called N, which is the following combination of the muon decay constants

$$N = \xi\frac{\delta}{\rho}P_\mu\,. \tag{24}$$

TABLE 4

All the used leptonic and semileptonic CC constraints with their experimental best fit values and 68% CL errors (and references)

ρ	0.7517 ± 0.0026	[19]
$P_\mu \xi$	0.975 ± 0.015	[25]
δ	0.7551 ± 0.0085	[19]
ξ'	1.008 ± 0.054	[19]
N	0.9989 ± 0.0023	[22]
P_μ	1.002 ± 0.045	[24]
R_π	0.9953 ± 0.0097	[19]
$-(c/v)P_{\mathrm{e}^-}^{\mathrm{GT}}$	1.001 ± 0.008	[19]
$-(c/v)P_{\mathrm{e}^-}^{\mathrm{F}}$	0.97 ± 0.19	[23]
S	0.98 ± 0.12	[19]
R_Y	0.000 ± 0.005	[26]

Considerable progress has also been made recently in the measurement of P_μ at SIN [24].

4. Different neutrino mass configurations

4.1. HEAVY RIGHT-HANDED NEUTRINOS

Having now derived the expression for the CC constraints, we can use these formulae to study several special cases, following from different assumptions on the (unknown) masses of the right-handed neutrinos. One possibility is that both χ_2^{e} and χ_2^{μ} are too heavy to be produced in the weak decays. Then, as discussed in sect. 2, large neutrino mixing angles α_ℓ would require some fine tuning of the neutrino mass parameters.

In this mass configuration the formulae for the muon decay parameters are just those given in table 2, with $\tilde{A}$, $\tilde{B}$, $\tilde{C}_1$ and $\tilde{C}_2$ listed in eq. (18). One can see that all the constraints reduce to their trivial values, if either the neutrino mixing angles α_ℓ or the LRS parameters r and ω vanish. Therefore, in the general case, only related bounds for the α_ℓ angles and the LRS parameters can be obtained. In principle, it is possible to have a fairly large r with small α_ℓ's or vice versa. Anyway, one can deduce from the R_Y parameter (22) that y and thus ω have to be small. Similarly, the R_P ratio requires the two neutrino mixing angles to be almost equal. We still wish to notice, in view of later discussion, that the parameters δ and $P_\mu \xi$ can be either smaller or larger than their V − A values. However, this deviation is suppressed by two (instead of one) small numbers squared, thus making the observation of any deviation impossible even in the forthcoming experiments.

4.2. LIGHT RIGHT-HANDED NEUTRINOS

Another, and more interesting, possibility is that both of the (mostly) right-handed neutrinos χ_2^{e} and χ_2^{μ} are light enough to contribute to the weak decays we are considering. (For simplicity, we will neglect here all effects of finite neutrino masses). Then one finds after adding up all the different contributions to each decay process that the neutrino mixing angles α_ℓ drop out and the corresponding formulae reduce to those for Dirac neutrinos [11, 12] (although χ_1^ℓ and χ_2^ℓ do not have to combine to form a Dirac neutrino). Therefore we call here a neutrino with a light χ_2^ℓ a Dirac-like neutrino.

However, for the scattering processes a more detailed knowledge of the mass of a Dirac-like neutrino would be necessary, since the χ_1^ℓ and χ_2^ℓ states are produced coherently and an oscillation pattern is induced. This is very much similar to what was found [6, 12, 27] for light mirror neutrinos and was called neutrino-mirror neutrino oscillation. Also here all scattering cross sections of an incoming ℓ-type neutrino are of the generic form

$$\sigma(\alpha_\ell) = P^\ell_{\nu\nu}(s)\sigma(0)\,, \tag{25}$$

where

$$P^\ell_{\nu\nu}(s) = 1 - \tfrac{1}{2}\sin^2 2\alpha_\ell(1 - \cos(2\pi s/L)) \tag{26}$$

is the oscillation factor at a distance s with an oscillation length

$$L = \frac{4\pi E_\nu}{\Delta m^2}\,, \tag{27}$$

$$\Delta m^2 = m_2^2 - m_1^2\,. \tag{28}$$

If $s \ll L$ there is no information on α_ℓ (coherent case) and if $s \gg L$ (incoherent approximation $P^\ell_{\nu\nu}(s)$ is averaged to

$$\bar{P}^\ell_{\nu\nu} = 1 - \tfrac{1}{2}\sin^2 2\alpha_\ell \tag{29}$$

in which case a bound on α_ℓ can be obtained. $\sigma(0)$ is a (modified) cross section which may depend on all other parameters (r, ω, α_k) except on α_ℓ. An example of $\sigma(0)$ is given by the inverse muon scattering constraint S, which was calculated for two Dirac-like neutrinos in ref. [12]. (See also later in table 6.)

Thus in this case there is no unambiguous information on the mixing angle α_μ and, since no ν_e-scattering experiment is now included, no information at all on α_e. For further enlightment about the neutrino mixing angles one would have to use other constraints, e.g. neutrino oscillation experiments. We do not go more deeply into this question here. However, we wish to point out that also the neutrinoless double beta process will put a combined limit on the mixing angle α_e, neutrino mass eigenvalues m_{e_1} and m_{e_2} (including their sign) and the LRS parameters.

For completeness we have presented the 1σ boundaries of the most strict CC constraints in this Dirac-like case in fig. 1. One can see that the parameters ω and r are limited essentially by ρ and N; and the latter constraint is in slight disagreement with an earlier result for $P_\mu\xi$ [25]. A global test gives the following bounds:

$$\sqrt{r} = 0.15\ (<0.22)\,,$$

$$(-2.7°<)\omega = -0.2°\ (<1.3°)\,. \tag{30}$$

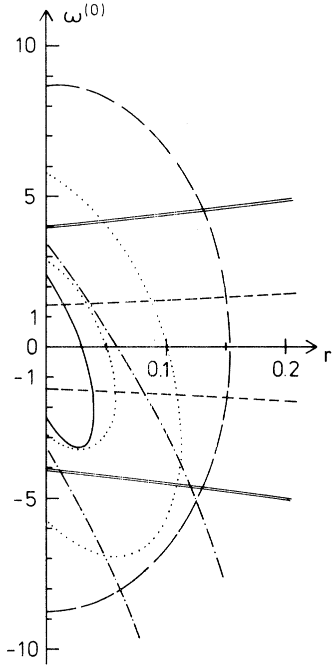

Fig. 1. The 68% CL boundary curves set by the most relevant CC constraints on the parameters r and ω in case both χ_2^e and χ_2^μ are light. The straight line is the new parameter N, the dashed, dotted, dash-dotted, long-dashed and double lines come from ρ, $P_\mu \xi$, P_e^{GT}, ξ' and R_Y, respectively. All curves, except the strip formed by $P_\mu \xi$ enclose the origin. N and $P_\mu \xi$ are in slight disagreement.

Thus the heavier vector boson would in this case have to be at least 4.5 times more massive than the lighter boson. In the incoherent case for the muonic neutrino masses, the angle $|\alpha_\mu|$ is restricted to less than 16°.

We wish to point out here a few remarks on the muon decay parameters in this Dirac-like case (for formulae, see refs. [11, 12]). Namely, the parameter $P_\mu \xi$ cannot be larger than one for any values of r and ω. Furthermore, the parameter δ is trivially $\frac{3}{4}$. As we shall see later on, different assumptions on the masses of χ_2^ℓ states will lead to different predictions for these parameters, thus opening the possibility of obtaining information on the neutrino masses from these parameters.

4.3. ASYMMETRIC NEUTRINO MASS CONFIGURATIONS

There are still other possible configurations for the right-handed neutrino masses. It may well be (as for the "minimal" Higgs choice) that all the χ_2^ℓ's are too heavy to be produced in weak decays. It is also possible that the two χ_2^ℓ states have somewhat different masses, e.g. such that one of the states could contribute to muon and pion decays but the other could not. (It has been pointed out [28] that even with the minimal Higgs model the χ_2^e state can be as light as tens of MeV. This shows how indeterminate all the mass predictions are, and that all possible mass configurations should therefore be studied.) This possibility, though somewhat accidental, is worth studying because of its phenomenological implications.

As noted in ref. [6], some of the muon decay parameters, asymmetric with respect to the exchange of the two neutrinos, will attain different formulae for the two cases of light χ_2^e or χ_2^μ, respectively. We will now study these generation-asymmetric models more carefully. For simplicity and clarity of our argument, we will again neglect all effects of finite masses. A complete analysis can be made using our coupling constants and adding the trivial phase-space factors with finite χ_2^ℓ masses.

If χ_2^e is light, both χ_1^e and χ_2^e (forming a Dirac-like electron neutrino) contribute to each decay process with their respective couplings given in table 1. We have presented the ensuing formulae for the constraints explicitly in table 5. One finds that $P\mu\xi$ is always smaller than or equal to one. (Equality is obtained if both α_μ and ω vanish.) However, contrary to the above, the parameter δ is now restricted to be always greater than or equal to $\frac{3}{4}$ ($\delta=\frac{3}{4}$ for $\omega=0$, independently of α_μ). The accurately constrained N is now seen to take the following form:

$$N=\frac{1-[(x-y)/(1-y)]^2\tan^2\alpha_\mu}{1+[(x-y)/(1-y)]^2\tan^2\alpha_\mu}\,\frac{1-x^2\tan^2\alpha_\mu}{1+x^2\tan^2\alpha_\mu}, \tag{31}$$

TABLE 5

The muon and pseudoscalar decay parameters and the inverse muon decay scattering constraint S for such a neutrino mass configuration where χ_2^e is light enough and χ_2^μ too heavy to contribute to the low-energy processes

$$\rho=\frac{3}{4}\frac{\cos^2\alpha_\mu+x^2\sin^2\alpha_\mu}{\cos^2\alpha_\mu+x^2\sin^2\alpha_\mu+y^2}$$

$$\xi=\frac{\cos^2\alpha_\mu-x^2\sin^2\alpha_\mu-3y^2\cos^2 2\alpha_\mu}{\cos^2\alpha_\mu+x^2\sin^2\alpha_\mu+y^2}$$

$$\delta=\frac{3}{4}\frac{\cos^2\alpha_\mu-x^2\sin^2\alpha_\mu}{\cos^2\alpha_\mu-x^2\sin^2\alpha_\mu-3y^2\cos^2 2\alpha_\mu}$$

$$\xi'=\frac{\cos^2\alpha_\mu-x^2\sin^2\alpha_\mu-y^2\cos^2 2\alpha_\mu}{\cos^2\alpha_\mu+x^2\sin^2\alpha_\mu+y^2}$$

$$N=P_\mu\frac{1-x^2\tan^2\alpha_\mu}{1+x^2\tan^2\alpha_\mu}$$

$$P_\mu=\frac{1-[(x-y)/(1-y)]^2\tan^2\alpha_\mu}{1+[(x-y)/(1-y)]^2\tan^2\alpha_\mu}$$

$$R_P=\frac{1+[(x-y)/(1-y)]^2}{\cos^2\alpha_\mu+[(x-y)/(1-y)]^2\sin^2\alpha_\mu}$$

$$S=\frac{\cos^2\alpha_\mu(1+P_\mu)+x^2\sin^2\alpha_\mu(1-P_\mu)+0.375y^2(1+P_\mu\cos^2 2\alpha_\mu)}{2(\cos^2\alpha_\mu+x^2\sin^2\alpha_\mu+y^2)}$$

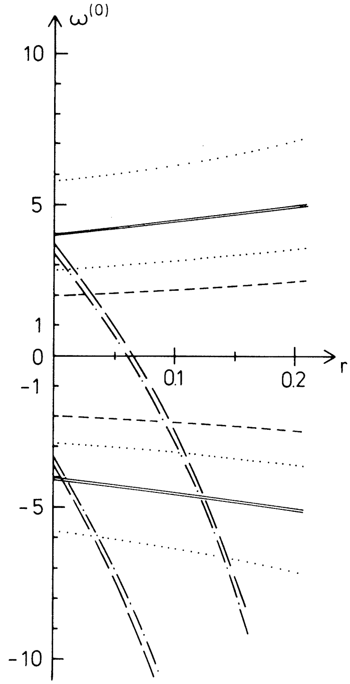

Fig. 2. Similar boundaries as in fig. 1, but now for χ_2^e light and χ_2^μ heavy (with $\alpha_\mu = 0$). The meaning of the lines is as in fig. 1, except that the long-dashed line comes now from the electron-to-muon decay ratio R_π. Note that N does not set any limit and that ρ and $P_\mu \xi$ contradict each other.

which is always less than or equal to one. It essentially measures α_μ, and for $\alpha_\mu = 0$ it reduces to one. Therefore, contrary to the case of two Dirac-like neutrinos, N does not impose now a strict constraint for the LRS parameters.

We have plotted in fig. 2 the most relevant constraints restricting r and ω, taking α_μ equal to zero, for simplicity. As ρ, δ, $P_\mu \xi$ and ξ' all limit the same function, y, only the most accurate restriction coming from ρ is given, together with the slightly contradicting result for $P_\mu \xi$. A global fit to the data gives the following values for the parameters:

$$\sqrt{r} = 0.20\,(<0.35)\,,$$
$$(-4.4°<)\omega = -2.3°\,(<2.4°)\,,$$
$$\alpha_\mu = 0° \pm 11°\,. \tag{32}$$

In this case W_2 must only be less than three times heavier than W_1. Since $\delta - \frac{3}{4} \approx \frac{9}{4}\omega^2$ to leading order, one sees that deviations of δ from $\frac{3}{4}$ up to 1% are still allowed, making the future tests [13] with increased accuracy extremely interesting.

A similar analysis has been made also for the case of light χ_2^μ and heavier χ_2^e. The CC constraints for this case have been listed in table 6. While ρ, being symmetric

TABLE 6

The same as table 5, but now χ_2^{μ} is light and χ_2^{e} heavy

$$\rho=\frac{3}{4}\frac{\cos^2\alpha_e+x^2\sin^2\alpha_e}{\cos^2\alpha_e+x^2\sin^2\alpha_e+y^2}$$

$$\xi=\frac{\cos^2\alpha_e-x^2\sin^2\alpha_e+3y^2\cos^2 2\alpha_e}{\cos^2\alpha_e+x^2\sin^2\alpha_e+y^2}$$

$$\delta=\frac{3}{4}\frac{\cos^2\alpha_e-x^2\sin^2\alpha_e}{\cos^2\alpha_e-x^2\sin^2\alpha_e+3y^2\cos^2 2\alpha_e}$$

$$\xi'=\frac{\cos^2\alpha_e-x^2\sin^2\alpha_e+y^2\cos^2 2\alpha_e}{\cos^2\alpha_e+x^2\sin^2\alpha_e+y^2}$$

$$N=P_\mu\frac{1-x^2\tan^2\alpha_e}{1+x^2\tan^2\alpha_e}$$

$$P_\mu=\frac{1-2y+2xy-x^2}{1-2y-2xy+x^2+2y^2}$$

$$R_P=\frac{\cos^2\alpha_e+[(x-y)/(1-y)]^2\sin^2\alpha_e}{1+[(x-y)/(1-y)]^2}$$

$$S=P^{\mu}_{\nu\nu}(s)\frac{\cos^2\alpha_e+x^2[(x-y)/(1-y)]^2\sin^2\alpha_e+0.375y^2[\sin^2\alpha_e+[(x-y)/(1-y)]^2\cos^2\alpha_e]}{[1+[(x-y)/(1-y)]^2][\cos^2\alpha_e+x^2\sin^2\alpha_e+y^2]}$$

with the interchange of the two neutrinos, is obtained just by changing α_μ to α_e, the other parameters obtained attain different expressions. The parameter $P_\mu\xi$ can obtain a value greater or smaller than one. Therefore, if $P_\mu\xi$ is found to be larger than one, it suggests this neutrino mass configuration. Even more clear is the information from δ, since it is now always smaller than or equal to $\frac{3}{4}$. Thus a possible observation of a deviation from this value would signal for one of the asymmetric neutrino mass configurations (as well as an extended gauge structure) in a definite way.

We have plotted again the most relevant constraints on r and ω (taking $\alpha_e=0$) in fig. 3. One can see that N and ρ form now the most strict bounds and that the early measurements on $P_\mu\xi$ are in an even more clear conflict with N than in the case of two Dirac-like neutrinos. A global fit gives the following limits:

$$\sqrt{r}=0.17\ (<0.28)\,,$$
$$\omega=0°\pm1.9°\,,$$
$$(-11°<)\alpha_e=-3.6°\ (<4.6°)\,. \tag{33}$$

Since $\delta-\frac{3}{4}\approx-\frac{9}{4}\omega^2$, a deviation of up to 0.3% from $\frac{3}{4}$ is possible.

We still want to point out that for many reasonable types of charged weak interactions [19] the δ parameter was found to be trivially $\frac{3}{4}$. This makes the study of δ even more worthwhile and the present suggestion even more probable, in case some deviation is observed.

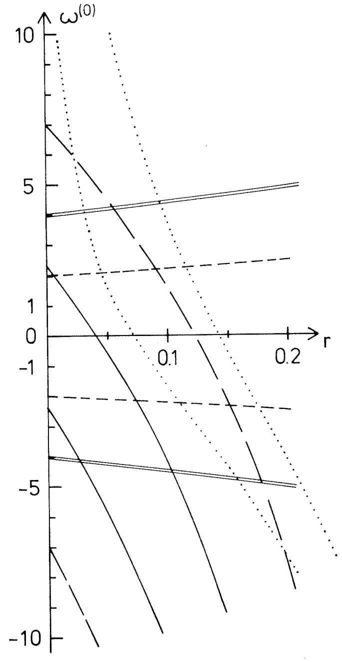

Fig. 3. Similar bounds as in fig. 1, but now for χ_2^{μ} light and χ_2^{e} heavy (with $\alpha_e = 0$). The meaning of the lines is as in fig. 1, except that the long-dashed line comes from R_π. N and $P_\mu \xi$ are now in a more clear conflict than in fig. 1.

5. Conclusions

We have studied the charged weak interactions of leptons in the left-right symmetric $SU(2)_R \times SU(2)_L \times U(1)$ theory, going beyond the restrictions set on the model by choosing the "minimal" Higgs representation [4]. We have considered a general *CP* conserving mass matrix for neutrinos (neglecting, for clarity, the interfamily mixings), and calculated the form of the most accurate leptonic and semileptonic CC constraints. The bounds on the parameters of the LRS model (the mass ratio and mixing angle of the vector bosons) and the neutrino mixing angles were derived for several neutrino mass configurations.

For heavy right-handed neutrinos, the neutrino mixing angles should naturally be small, and thus little information on the LRS parameters is obtained. On the other hand, if the right-handed neutrinos are light (forming "Dirac-like" neutrinos), the data imply strict limits to the LRS parameters ($M_{W_2} \geq 4.5 M_{W_1}$; $-2.7° < \omega < 1.3°$). For any Dirac-like neutrino, the coherence of the two states affects the neutrino scattering experiments. A general treatment of the corresponding cross section was indicated.

The possible generation asymmetric neutrino mass configurations were also discussed. If the electron-type right-handed neutrino is light and the muon-type one is heavy, the recent high-accuracy result [21, 22] for muon decay was shown to be trivial, thus allowing for an even lighter mass for the second vector boson. Furthermore, a possible deviation of δ from $\frac{3}{4}$ was shown to uniquely select one of the asymmetric mass configurations. This deviation (depending in the first order only on ω and not on the naturally small neutrino mixing angle) may be so large that it could be observed in the next generation of experiments.

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