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13.

ANALYSIS OF LEPTONIC CHARGED WEAK INTERACTIONS

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Motivated by the desire to test the limits of the standard model via the Lorentz structure of charged weak interactions we perform an extended and systematic analysis of charge-changing leptonic vertices. Based on a helicity projection form of the effective hamiltonian, the analysis yields improved limits on noncanonical couplings from recent, high-precision measurements in muon decay. We also give limits on possible, charge-neutral exchanges with intrinsic electron and muon number.

1. Introduction

In a recent publication [1]* we have performed a comprehensive analysis of all available, charge-changing, leptonic weak interactions in terms of more general Lorentz structures than V − A. This analysis was guided by the general pattern of unified local gauge theories but was not restricted to any specific class of such models. It was found that the data on muon decay, inverse muon decay, $\pi\ell 2$ decays, and nuclear Gamow-Teller transitions did allow for sizeable deviations of leptonic charged current interactions from the canonical V − A form. Several possibilities were pointed out on how to improve on this unsatisfactory situation in testing the standard model at low energies.

In the meantime, several precision experiments on muon decay gave much improved information on the lifetime [2], the decay asymmetry near the upper end of the decay spectrum [3], and the polarization components of the positron from polarized μ^+ decay [4]. Furthermore, the asymmetry measurement [3] also allowed one to deduce a very precise value of the helicity $h(\nu_\mu)$ of the muon neutrino in the decay $\pi \to \mu\nu_\mu$ [5]. As we show below, this new set of data gives further constraints

* Henceforth referred to as I.

on some of the model cases considered in I, thus reducing the allowed range of deviations from the canonical couplings.

In this work we modify and extend the analysis given in I by incorporating the new data referred to above. Here we mostly make use of the helicity projection form (HPF) of the interaction that was proposed recently [6] and which is equivalent to the charge-changing form (CCF), used in I, and to the charge-retention form (CRF). This representation renders the relevance of the measurable quantities in terms of the underlying charged interaction even more transparent than in some of the model cases considered in I and allows one to make direct contact with specific model theories. In addition, we also consider the case of interactions containing charge-*retaining* vertices and give limits on the corresponding couplings.

The plan of the paper is the following: in sect. 2 we describe and discuss the helicity projection form of the interaction and give the formulae for the relevant observables. Sect. 3 summarizes the new data and our strategy in their analysis. In sect. 4 we study models with factorization and universality, whilst sect. 5 deals with V − A models with admixtures of other covariants. In sect. 6 we study exotic neutral interactions, i.e. models which contain additional couplings of the charge-retention type. Finally sect. 7 contains our conclusions.

2. Helicity projection form and formulae for observables

Following ref. [6] we write the effective charge-changing interaction for muon decay in the following form:

$$H=\sqrt{\tfrac{1}{2}}\,G_0\Big\{h_{11}(s+p)_{e\nu_e}(s+p)_{\nu_\mu\mu}+h_{12}(s+p)(s-p)+h_{21}(s-p)(s+p)$$
$$+h_{22}(s-p)(s-p)+g_{11}(v^\alpha+a^\alpha)_{e\nu_e}(v_\alpha+a_\alpha)_{\nu_\mu\mu}+g_{12}(v^\alpha+a^\alpha)(v_\alpha-a_\alpha)$$
$$+g_{21}(v^\alpha-a^\alpha)(v_\alpha+a_\alpha)+g_{22}(v^\alpha-a^\alpha)(v_\alpha-a_\alpha)$$
$$+f_{11}(t^{\alpha\beta}+t'^{\alpha\beta})_{e\nu_e}\big(t_{\alpha\beta}+t'_{\alpha\beta}\big)_{\nu_\mu\mu}+f_{22}(t^{\alpha\beta}-t'^{\alpha\beta})\big(t_{\alpha\beta}-t'_{\alpha\beta}\big)+\text{h.c.}\Big\}, \tag{1}$$

where the symbols s through t' stand for the covariants

$$s_{ik}=\overline{\Psi}_i\mathbb{1}\Psi_k,\qquad p_{ik}=\overline{\Psi}_i\gamma_5\Psi_k,\qquad v^\alpha_{ik}=\overline{\Psi}_i\gamma^\alpha\Psi_k,\qquad a^\alpha_{ik}=\overline{\Psi}_i\gamma^\alpha\gamma_5\Psi_k,$$
$$t^{\alpha\beta}_{ik}=\overline{\Psi}_i\sqrt{\tfrac{1}{2}}\,\sigma^{\alpha\beta}\Psi_k,\qquad t'^{\alpha\beta}_{ik}=\overline{\Psi}_i\sqrt{\tfrac{1}{2}}\,\sigma^{\alpha\beta}\gamma_5\Psi_k. \tag{2}$$

(The particle symbols are indicated only once for each class of covariants.) Barring the possibility of $\mu\to e$ decays with missing neutrals *other* than neutrinos [7, 8], this is the most general effective contact interaction. Clearly, it is equivalent to the CCF representation used in I, the relation to the complex coupling constants G_i, G_i' of I being given by

$$\left.\begin{matrix} G_S \\ G_P \end{matrix}\right\} = (h_{12} + h_{21}) \pm (h_{11} + h_{22}),$$

$$\left.\begin{matrix} G_S' \\ G_P' \end{matrix}\right\} = -(h_{12} - h_{21}) \pm (h_{11} - h_{22}),$$

$$\left.\begin{matrix} G_V \\ G_A \end{matrix}\right\} = (g_{11} + g_{22}) \pm (g_{12} + g_{21}),$$

$$\left.\begin{matrix} G_V' \\ G_A' \end{matrix}\right\} = (g_{11} - g_{22}) \mp (g_{12} - g_{21}),$$

$$\left.\begin{matrix} G_T \\ G_T' \end{matrix}\right\} = 2(f_{11} \pm f_{22}). \tag{3}$$

The representation (1), however, has a number of advantages over the standard charge-changing representation (CCF) in terms of G_i and G_i' (or, for that matter, the standard charge-retention representation (CRF)), viz.

(i) For massless particles, the combinations appearing in each individual term of eq. (1) project onto states of definite helicity. As a consequence, the number of interference terms in any decay rate or cross section is minimal (in fact, only scalar-pseudoscalar and tensor terms do interfere), and most observables will be functions of absolute squares like $|h_{ik}|^2, |g_{ik}|^2, \ldots,$ only.

(ii) The canonical V − A interaction is particularly simple because then $g_{22} = 1$ while all other coupling constants vanish. (Recall that in the standard form four constants are different from zero.)

(iii) The combinations of covariants as used in eq. (1) have an especially simple behaviour under Fierz transformations. Therefore, any nonvanishing coupling constant other than g_{22} can be identified directly in the hamiltonian either in its charge-changing or in its charge-retaining form, thus rendering the physical origin of such additional terms more transparent. In the sequel we shall make use of the helicity projection form (1) but we shall indicate the connection to the parameters discussed in I where this is useful.

The parameter A which determines the total rate of muon decay (cf. eq. (4) of I) is given by

$$A = 4\{4(|g_{22}|^2 + |g_{11}|^2 + |g_{12}|^2 + |g_{21}|^2) + |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 + 12(|f_{11}|^2 + |f_{22}|^2)\}. \tag{4}$$

For convenience, we write the decay parameters in terms of the difference to their V − A values. We find

$$\rho - \tfrac{3}{4} = -\frac{12}{A}\left\{|g_{12}|^2 + |g_{21}|^2 + 2|f_{11}|^2 + 2|f_{22}|^2 + \mathrm{Re}\left(h_{11}f_{11}^* + h_{22}f_{22}^*\right)\right\}, \tag{5}$$

$$\delta - \tfrac{3}{4} = \frac{36}{A\xi}\left\{|g_{12}|^2 - |g_{21}|^2 - 2|f_{11}|^2 + 2|f_{22}|^2 - \mathrm{Re}\left(h_{11}f_{11}^* - h_{22}f_{22}^*\right)\right\}, \tag{6}$$

$$1 - \xi = \frac{8}{A}\left\{4\left(|g_{11}|^2 + 2|g_{12}|^2 - |g_{21}|^2\right) + |h_{11}|^2 + |h_{21}|^2 - 4|f_{11}|^2 + 16|f_{22}|^2 - 8\,\mathrm{Re}\left(h_{11}f_{11}^* - h_{22}f_{22}^*\right)\right\}, \tag{7}$$

$$1 - \omega \equiv 1 - \xi\frac{\delta}{\rho} = \frac{8|g_{11}|^2 + 2|h_{21}|^2 + 2|h_{11} - 2f_{11}|^2}{4\left(|g_{11}|^2 + |g_{22}|^2\right) + |h_{12}|^2 + |h_{21}|^2 + |h_{11} - 2f_{11}|^2 + |h_{22} - 2f_{22}|^2}, \tag{8}$$

$$1 - \xi' = \frac{8}{A}\left\{4\left(|g_{11}|^2 + |g_{12}|^2\right) + |h_{22}|^2 + |h_{21}|^2 + 12|f_{22}|^2\right\}, \tag{9}$$

$$\left.\begin{matrix}\dfrac{\alpha}{A}\\[2ex] \dfrac{\alpha'}{A}\end{matrix}\right\} = \frac{8}{A}\left\{\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\}\left[g_{21}\left(h_{22}^* + 6f_{22}^*\right) \pm g_{12}\left(h_{11}^* + 6f_{11}^*\right)\right], \tag{10a}$$

$$\left.\begin{matrix}\dfrac{\beta}{A}\\[2ex] \dfrac{\beta'}{A}\end{matrix}\right\} = -\frac{4}{A}\left\{\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\}\left[g_{22}h_{21}^* \pm g_{11}h_{12}^*\right]. \tag{10b}$$

We note that the spectrum parameter η is given by $\eta = (\alpha - 2\beta)/A$, and the transverse polarization components P_{T1} and P_{T2} determine the quantities $\alpha/A, \ldots, \beta'/A$ [6]. The invariant, differential cross section for the reaction $\nu_\mu + e^- \to \mu^- + \nu_e$ is given in the appendix. We do not write down the expression for S, the ratio of the integrated cross section $\sigma(\nu_\mu + e^- \to \mu^- + \nu_e)$ and the formulae for $\pi\ell 2$ decay. These are obtained easily from eqs. (6) or (A.1)–(A.7) and (19) of I, respectively, upon insertion of eqs. (3). The electron polarization in Gamow-Teller interactions, in factorized and universal models, finally, is given by

$$P^{GT}/(v/c) = (g_{11} - g_{22})/(g_{11} + g_{22}). \tag{11}$$

3. Data and procedure of analysis

The new data on muon decay are summarized in table 1. All other data, not shown in this table, are the same as in I. As a first comment, we note that τ_μ is now known to an accuracy much higher than all other quantities and, therefore, it has a negligible effect on the error of the latter. We factor out the quantity $|g_{22}|^2$ in the differential decay distribution. Furthermore, we use the phase freedom in eq. (1) to choose g_{22} real positive. Up to a redefinition of the Fermi constant G_0 this is equivalent to setting $g_{22}=1$.

There is one peculiarity about the eqs. (5) to (10): the coupling constant h_{12} does not appear in the electron observables with the exception of β and β'. Concerning these latter quantities h_{12} cannot be determined from them either because g_{11} is known to be very small from the other observables. In those models where this constant is not set equal to zero from the start we factor out the quantity $(g_{22}^2+\frac{1}{4}|h_{12}|^2)$. Again, we can set this factor equal to 1 through a redefinition of G_0.

Our strategy in analyzing the data is mostly the same as in I insofar as we distinguish between two general classes of models: in the first class we assume factorization and universality, in which case we can also use the available semi-leptonic data. In the second we study the case V − A, i.e. $g_{22}=1$, $g_{11}=0=g_{12}=g_{21}$, with admixture of specific covariants other than V − A. In this second class only the data from muon decay and inverse muon decay can be used.

Generally speaking we proceed as follows: complete fits to the data are often hampered by the fact that there are more parameters than data. Furthermore a controversial datum such as the measured value of $P_\mu\xi$ which contradicts the

TABLE 1
Data on muon decay

Quantity	Value	Ref.
$\tau_\mu\,[\mu\mathrm{s}]$	2.19703 ± 0.00004	[2]
$\delta-\frac{3}{4}$	0.0002 ± 0.0043	(a)
$1-P_\mu^{(\pi)}\xi\delta/\rho$	0.0011 ± 0.0023	[3]
$1-P_\ell$	0.002 ± 0.045	[4]
β/A	-0.002 ± 0.017	[4]
β'/A	-0.007 ± 0.016	[4]
$1-P_\mu^{(K)}$	0.030 ± 0.047	[12]

(a) Average of previous measurements as collected in particle data tables [13] and of preliminary value announced by the Berkeley-TRIUMF group (private information from M. Strovink).

The first six quantities belong to muon decay. The third observable contains $P_\mu^{(\pi)}$, the muon polarization from $\pi\mu2$ decay. P_ℓ is the longitudinal polarization of e^+ in μ^+ decay; β and β' are obtained from the transverse polarization as described in [4] (with α and α' being zero). The η-parameter is then found to be $\eta=0.004\pm0.034$. $P_\mu^{(K)}$ is the muon polarization as measured in $K\mu2$ decay [12].

information from $\rho, \delta, \omega \equiv \xi\delta/\rho$, and presumably is obscured by insufficient knowledge of the muon polarization in the experiment, tends to distort the fit. Therefore, we study the noncanonical covariants one (or at most two) at a time and give 1σ limits on the coupling constants. In particular, if a weighted average of experiments gives a value for the absolute square of a coupling constant, say $|g|^2 = a \pm \sigma$, we normalize the branch of the gaussian $\exp\{-(x-a)^2/2\sigma^2\}$ for $x \in [0, \infty]$ to unity and calculate x_1 so that the integral over the interval $[0, x_1]$ is 0.683. The value $\sqrt{x_1}$ is then the 1σ limit on $|g|$.

4. Models with factorization and universality

This class of models (called FSU or FSWU in I) assumes the weak charged interactions to be mediated by heavy charged bosons with spin 0, 1, or 2. For simplicity in models I to III we study only one such exchange per covariant at a time so as to make the sensitivity of a given observable to specific types of couplings as transparent as possible. Furthermore, as in ref. [1], we assume the couplings to be e-μ universal, but allowing also for the possibility of weak universality (i.e. proportionality of scalar-pseudoscalar coupling constants to the charged lepton masses) in the case of spin-zero exchange.

These assumptions impose the following constraints on the couplings in eq. (1):

$$h_{12}, h_{21} \text{ real, positive semi-definite}, \tag{12a}$$

$$h_{22} = h_{11}^* \quad \text{with } |h_{11}|^2 = h_{12}h_{21}, \tag{12b}$$

$$g_{11}, g_{22} \text{ real, positive semi-definite}, \tag{13a}$$

$$g_{21} = g_{12}^* \quad \text{with } |g_{12}|^2 = g_{11}g_{22}, \tag{13b}$$

$$f_{22} = f_{11}^* . \tag{14}$$

This class of models has a number of general properties which are evident from eqs. (5) to (10), viz.

$$\delta - \tfrac{3}{4} = 0, \qquad 1 - \xi \geqslant 0, \qquad \xi' = \xi, \qquad \beta' = 0. \tag{15}$$

Note that the first result (15) follows from the universality assumption only. Thus, any deviation of δ from $\frac{3}{4}$ would be an indication for violation of universality of electron and muon couplings to the exchanged charged bosons. The parameter ξ', being a polarization component, is smaller than or equal to one, by definition. The bounds on ξ, however, a priori are $-3 \leqslant \xi \leqslant 3$. The factorization and universality assumptions now constrain this parameter to $-1 \leqslant \xi \leqslant 1$.

It is easy to work out the relation to the fit parameters used in I. We have

$$
\begin{aligned}
G'_{\mathrm{S}} &= G'^{*}_{\mathrm{P}} = (h_{21} - h_{12}) + 2i\,\mathrm{Im}\,h_{11}\,,\\
G_{\mathrm{S}}G_{\mathrm{P}} &= (h_{12} + h_{21})^2 - (h_{11} + h_{22})^2\,,\\
\mathrm{tg}\,\alpha_{\mathrm{S}} &= 2\,\mathrm{Im}(h_{11})/(h_{21} - h_{12})\,,\\
G'_{\mathrm{V}} &= G'^{*}_{\mathrm{A}} = (g_{11} - g_{22}) - 2i\,\mathrm{Im}\,g_{12}\,,\\
G_{\mathrm{V}}G_{\mathrm{A}} &= (g_{11} + g_{22})^2 - (g_{12} + g_{21})^2\,,\\
\mathrm{tg}\,\alpha_{\mathrm{V}} &= 2\,\mathrm{Im}(g_{12})/(g_{22} - g_{11})\,. \qquad (16)
\end{aligned}
$$

We now proceed to a number of representative case studies based on the factorization and universality constraints (12) to (14).

Model I contains only V, A and T couplings and is equivalent to model FSU 1 of I, up to the following modifications.

As α and α' are now found to be zero within a small error band [4] (of about 0.005) the product $g_{12}f^{*}_{11}$ is very small, cf. eqs. (10a), and it is no more possible to determine the real and imaginary part of f_{11} (parameters G_{T} and $\mathrm{Im}\,G'_{\mathrm{T}}$ in I) separately. We assume $g_{22} = 1$, and leave g_{11} and f_{11} free, g_{12} being constrained by the universality condition (13b). Limits on g_{11} and $|f_{11}|$ are primarily obtained from ρ and from ω combined. The expressions for α and α', with the new experimental limits

$$
\left.\begin{matrix}\alpha/A\\ \alpha'/A\end{matrix}\right\} \simeq 6\left.\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\}(g^{*}_{12}f_{11}) = 0 \pm 0.005 \qquad (17)
$$

yield a rather tight bound on $|g_{12}| \cdot |f_{11}|$. As f_{11} is compatible with zero, this cannot be used to improve the limit on $|g_{12}|$ which, by the universality relation, is essentially the root of the limit on g_{11}.

Model II assumes that the interaction is given by the exchange of one heavy boson with spin 1 which couples universally. It is easy to see that in this case the only HPF parameter to appear in the formulae is g_{11}/g_{22} and, in particular, that the phase of g_{12} remains undetermined. The muon polarization in the $\pi \to \mu\nu_{\mu}$ and $\mathrm{K} \to \mu\nu_{\mu}$ decays, as well as the electron polarization in Gamow-Teller transitions, is given by

$$
P^{(\pi)}_{\mu} = \frac{g_{22} - g_{11}}{g_{22} + g_{11}} = P^{(\mathrm{K})}_{\mu} = -P^{\mathrm{GT}}\frac{c}{v}\,.
$$

Likewise, the other relevant parameters ρ, ω, ξ', depend only on the ratio g_{11}/g_{22}.

TABLE 2
Models assuming factorization and universality

Model	Coupling	Constraining parameters[a]	1σ limit on couplings	Comments
I	g_{11}, g_{12} f_{11}	ρ; ω; ξ'; α; α'	$\|g_{12}\| < 0.056$ $\|f_{11}\| < 0.037$ $g_{11} < 0.003$	$f_{22} = f_{11}^*$; $\arg g_{12}^* f_{11}$ remains undetermined
II	g_{11}/g_{22}	ρ; ξ'; $P_\mu^{(\pi,K)}$ p^{GT}	$g_{11}/g_{22} < 9.6 \times 10^{-4}$	$\|g_{12}\| = \sqrt{g_{11} g_{22}}$; $\arg g_{12}$ undetermined
III	h_{11} h_{21}	ω; ξ' β	$\|h_{11}\| < 0.076$ $h_{21} < 0.076$	$h_{22} = h_{11}^*$; $\|h_{11}\| = \sqrt{h_{12} h_{21}}$; $\arg h_{11}$ undetermined
IV	g'_{11}	ω	$g'_{11} < 0.038$	2 spin-one exchanges with real couplings
	g'_{12}	ρ; ξ'	$g'_{12} < 0.036$	$g'_{12} = g'_{21} = \sqrt{g'_{11} g'_{22}}$

[a] $\omega \equiv \xi\delta/\rho$.

The second column shows which coupling constants are being probed in the models indicated in the first column and described in the text. The third column shows those observables which yield the strongest limits. The asymmetry parameter ξ is equal to ξ' in models I to III but is not used because no direct measurement is available. Other observables not shown assume their V − A values.

As the deviation from the V − A values is quadratic in this ratio for the case of ω, but linear for the case of ρ and ξ', useful limits stem from these latter two, combined with the information on P_μ. Note that g_{11} can at most equal 0.1% of g_{22} (fourth line of table 2).

It is instructive to study this model also in terms of CCF parameters. In this case the correlated limits on the two parameters G_A/G_V and α_V are obtained from the relation

$$\frac{g_{11}}{g_{22}} = \frac{G_V + G_A - 2\sqrt{G_V G_A}\cos\alpha_V}{G_V + G_A + 2\sqrt{G_V G_A}\cos\alpha_V} \tag{18a}$$

[with $\sqrt{G_A/G_V}$ being the ratio of the absolute magnitudes of the axial vector and vector couplings g_A and g_V, respectively, and $g_A = -g_V e^{-i\alpha_V}$, as defined in I]. The limit $g_{11}/g_{22} < 9.6 \times 10^{-4}$ yields a limiting contour in the plane $\left(\sqrt{G_A/G_V}, \alpha_V\right)$ which is approximately a circle around the point (1,0) with radius $2\sqrt{g_{11}/g_{22}}$. The absolute boundaries are

$$0.94 \leqslant \sqrt{\frac{G_A}{G_V}} \leqslant 1.06, \qquad |\alpha_V| \leqslant 2.5°, \tag{18b}$$

and provide the best-known measure of the equality of the complex coupling constants g_V and $(-g_A)$.

Model III contains V − A plus factorized S, P structures, i.e.

$$g_{12} = 0 = g_{21}, \qquad g_{11} = 0, \qquad f_{11} = 0 = f_{22},$$

$$h_{12} \geqslant 0, \qquad h_{21} \geqslant 0, \qquad h_{22} = h_{11}^{*}, \qquad |h_{11}|^2 = h_{12} h_{21}, \tag{19}$$

and is analogous to our former FSU 3 and FSWU 2 models in I. As h_{12} is unknown, we factor out the combination $g_{22}^2 + \frac{1}{4}h_{12}^2$. The phase of h_{11} cannot be determined. The limits on $|h_{11}|$ and h_{12} stem primarily from ω and ξ' (fifth line of table 2). β gives a similar bound on h_{21} only if h_{12} is assumed small. Again it is instructive to express this model in terms of CCF parameters, viz.

$$h_{\substack{12\\21}} = \tfrac{1}{4}(G_S + G_P) \pm \tfrac{1}{2}\sqrt{G_S G_P}\cos\alpha_S .$$

In CCF we have *three* parameters $(G_S/G_V, G_P/G_V, \alpha_S)$ instead of *two* in HPF and, accordingly, since h_{12} remains undetermined, we find correlated boundary contours for the CCF parameters. This is another manifestation of the simplicity of the HPF parametrization. In particular, the (still allowed) case of large h_{12} corresponds to $\alpha_S = 0$ and $G_S \simeq G_P$, in which case G_S/G_V remains unbounded. (This was found in the global analysis on computer in I.) G_S, G_P and α_S are the squares of the absolute magnitudes of the primordial coupling constants and their relative phase, respectively, as defined in I.

Model IV assumes the standard W_L exchange (i.e. $g_{22} = 1$, $g_{11} = g_{12} = g_{21} = 0$) plus the exchange of one more spin-one object (not degenerate with W_L) with real and universal couplings

$$g'_{11} \geqslant 0, \qquad g'_{22} \geqslant 0, \qquad g'_{12} = g'_{21} = \sqrt{g'_{11} g'_{22}} .$$

The best limit on g'_{11} stems from ω, the best limit on g'_{12} from ρ and ξ' combined, as shown in the last two lines of table 2. This model is interesting in the context of models with two nondegenerate W-bosons W_1 and W_2. For instance, if the couplings of (ℓ_f, ν_f) to W_1 and W_2 are equal then the information on g'_{11} yields a limit on the ratio of boson masses, viz. $M_2 > 5M_1$ (at 1σ), up to small mixing effects. In the manifestly left-right symmetric model based on $SU(2)_L \times SU(2)_R \times U(1)$ [9], finally, g_{12} is a measure of the mixing angle Φ of W_1 and W_2 [sect. 6.2.3 in [6]] (mass eigenstates versus interaction states), viz. $|\Phi| < 0.036$.

5. V − A models with admixtures of other covariants

As in I we have studied the case of dominant V − A interaction to which small, noncanonical, in general complex structures are added. In contrast to our earlier analysis, which gave only a selected set of examples, we now study in a systematic way the noncanonical couplings one by one in order to exhibit more clearly the sensitivity of individual observables to the different couplings. Thus, we set $g_{22} = 1$ and add to it one by one the couplings indicated in the second column of table 3.

As noted earlier, the coupling h_{12} cannot be determined from direct or inverse muon decay, unless g_{11} is different from zero and not small. By a redefinition of G_0 in eq. (1) we can always take out a factor $(g_{22}^2 + \frac{1}{4}|h_{12}|^2)$ from the denominators of expressions (5)–(10) and obtain limits on the other coupling constants normalized to this constant. As g_{11} is always found to be small, this procedure is indistinguishable from setting $g_{22} = 1$, $h_{12} = 0$ – the choice that we have adopted.

Table 3 shows our results for this class of model. As before, cf. sect. 4, the third column indicates the observables which constrain the coupling at stake (all couplings not shown in the second column are now zero with the exception, of course, of g_{22} which is 1). The table shows that tensor as well as noncanonical vector/axial vector couplings are excluded at the level of about 0.03 and 0.04 (1σ), respectively. The

TABLE 3
Models assuming V − A plus other covariants

Model	Coupling	Constraining parameters[(a)]	1σ limit on coupling	Comments
I	h_{11}	ω	$\lvert h_{11}\rvert < 0.076$	$\xi = \omega$
II[(b)]	h_{22}	ξ'	$\lvert h_{22}\rvert < 0.30$	
III[(b)]	h_{21}	ω	$\lvert h_{21}\rvert < 0.076$	$\xi = \omega$
		β	$-0.060 < \mathrm{Re}\, h_{21} < 0.076$	
		β'	$-0.092 < \mathrm{Im}\, h_{21} < 0.036$	
IV	g_{11}	ξ'; ω	$\lvert g_{11}\rvert < 0.038$	$\xi = \omega$
V[(c)]	$g_{12} = g_{21}$	ρ, ξ'	$\lvert g_{12}\rvert < 0.036$	$\xi = \xi'$
VIa[(c)]	g_{12}	ρ; δ; ξ'	$\lvert g_{12}\rvert < 0.038$	$1 - \xi = 2(1 - \xi')$
VIb	g_{21}	ρ; δ	$\lvert g_{21}\rvert < 0.044$	$1 - \xi = -2\lvert g_{21}\rvert^2$
VII[(d)]	f_{11}	ρ; δ; ω	$\lvert f_{11}\rvert < 0.026$	$1 - \xi = -(1 - \omega)$
VIII[(d)]	f_{22}	ρ; δ; ξ'	$\lvert f_{22}\rvert < 0.027$	$1 - \xi = \frac{4}{3}(1 - \xi')$

[(a)] $\omega \equiv \xi\delta/\rho$.
[(b)] Model with S, P structures.
[(c)] Model with V, A structures.
[(d)] Model with T structures.

In all of these models g_{22} was set equal to 1 whilst all couplings not indicated in the second column were set equal to zero. Those observables that do not appear in column 3 or 5 assume their V − A values. Column 5 indicates the additional information that will be obtained from a direct measurement of ξ.

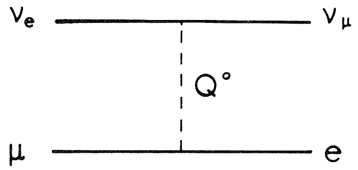

Fig. 1. Exchange of a neutral particle Q^0 with intrinsic family numbers ($L_e = \pm 1$, $L_\mu = \mp 1$) and spin $J = 0, 1$ or 2.

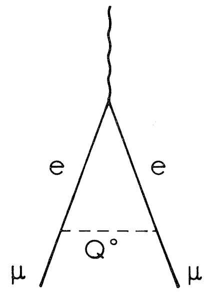

Fig. 2. Contribution of Q^0 exchange to muonic g-factor anomaly.

limits on scalar/pseudoscalar couplings are not as good with h_{22} being the least well determined (model II). In the case of h_{21} (model III), on the other hand, we obtain independent limits on its real and imaginary parts.

6. Neutral models

We have also studied the more speculative possibility of exchanges of the kind shown in fig. 1 being added to the standard $W^\pm$ diagram. A particle of this kind would be electrically neutral, would have total lepton number zero, $L = 0$, but would carry equal and opposite muonic and electronic family numbers ($L_e = \pm 1$, $L_\mu = \mp 1$)⋆. Particles of this kind would contribute to muon decay, to the g-factor anomalies of muon (fig. 2) and electron (analogous diagram with e and μ interchanged), to ν_μ and $\bar{\nu}_\mu$ electron scattering, the hyperfine structure of muonium and to the asymmetry in $e^+e^- \to \mu^+\mu^-$ ($e^+e^- \to \tau^+\tau^-$) (fig. 3). The particle Q^0 should not couple to quarks, however, because this would give rise to neutrinoless muon capture ($\mu^- \to e^-$) on nuclei and to kaon decays into μe pairs⋆⋆. Although this is speculative we find it interesting to give limits on the couplings of Q^0 to leptons as they are obtained from muon decay.

⋆ There would then probably also be analogous objects mediating between e and τ, μ and τ.

⋆⋆ If Q^0 carries spin 1, its mixing with Z^0 must be very small so as to avoid conflict with the experimental limits on $\mu \to e\gamma$ and $\mu \to e\bar{e}e$. For some more detailed estimates see ref. [10].

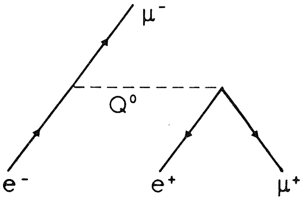

Fig. 3. Contribution of Q^0 exchange to $e^+e^- \to \mu^+\mu^-$.

Let us denote the corresponding coupling constants (in helicity projection form) by

$$\eta_{ik}\,(\text{scalar-pseudoscalar}), \qquad \gamma_{ik}\,(\text{vector-axial vector}), \qquad \varphi_{ii}\,(\text{tensor-pseudotensor}). \tag{20}$$

These effective couplings fulfil factorization but there is no reason why they should be universal. Therefore, unlike relations (12)–(14), we only have the relations

$$\eta_{12}\eta_{21} = \eta_{11}\eta_{22}, \qquad \gamma_{12}\gamma_{21} = \gamma_{11}\gamma_{22}. \tag{21}$$

The contribution to $(g-2)$ is smaller than the known standard weak contributions and hence undetectable at present. The same statement probably also applies to the hyperfine structure in muonium. As to the asymmetry in $e^+e^- \to \mu^+\mu^-$, the limits on the coupling constants do not upset the agreement of the data with the standard model. For instance, if Q^0 carries spin 1 and is heavy, the asymmetry is found to be

$$A \simeq 2K(s) - \frac{2G_0}{\sqrt{2}\,e^2}(\gamma_{11}+\gamma_{22}) \times s. \tag{22}$$

Here s is the squared c.m. energy, $K(s)$ denotes the asymmetry due to $\gamma - Z^0$ interference [6].

We find the following expressions for observables in muon decay (knowing that the new couplings are small compared to $g_{22}=1$):

$$\rho - \tfrac{3}{4} \simeq -\tfrac{3}{16}\left[|\eta_{11}|^2 + |\eta_{22}|^2 + |\eta_{12}|^2 + |\eta_{21}|^2 - 4\left(|\varphi_{11}|^2 + |\varphi_{22}|^2\right)\right], \tag{23}$$

$$\delta - \tfrac{3}{4} \simeq \tfrac{9}{16}\left[-|\eta_{11}|^2 + |\eta_{22}|^2 - |\eta_{12}|^2 + |\eta_{21}|^2 + 4\left(|\varphi_{11}|^2 - |\varphi_{22}|^2\right)\right], \tag{24}$$

$$1 - \omega \equiv 1 - \xi\frac{\delta}{\rho} \simeq 2\left[|\gamma_{11}|^2 + |\gamma_{12}|^2 + 4|\varphi_{11}|^2\right], \tag{25}$$

$$1 - \xi \simeq 1 - \xi\frac{\delta}{\rho} + \tfrac{1}{2}\left[-|\eta_{11}|^2 + 2|\eta_{22}|^2 - |\eta_{12}|^2 + 2|\eta_{21}|^2 + 4|\varphi_{11}|^2 - 8|\varphi_{22}|^2\right], \tag{26}$$

$$1-\xi' \simeq \tfrac{1}{2}\left[|\eta_{21}|^2+|\eta_{22}|^2+4|\gamma_{11}|^2+4|\gamma_{12}|^2+12|\varphi_{22}|^2\right], \tag{27}$$

$$\left.\begin{array}{c}\dfrac{\alpha}{A}\\[2ex] \dfrac{\alpha'}{A}\end{array}\right\} \simeq \frac{1}{2}\left\{\begin{array}{c}\mathrm{Re}\\ \mathrm{Im}\end{array}\right\}\left[\eta_{12}\eta_{22}^{*} \pm \eta_{21}\eta_{11}^{*}\right], \tag{28}$$

$$\left.\begin{array}{c}\dfrac{\beta}{A}\\[2ex] \dfrac{\beta'}{A}\end{array}\right\} \simeq \frac{1}{2}\left\{\begin{array}{c}\mathrm{Re}\\ \mathrm{Im}\end{array}\right\}\left[(1+\gamma_{22})\gamma_{12}^{*} \pm \gamma_{11}\gamma_{21}^{*}\right]. \tag{29}$$

We note, in particular, that α and α' are quadratic in the η_{ik} whilst β and β' are linear in γ_{12}. In this class of models α and α' are bound to be very small whereas β and β' may be different from zero.

We have studied the three possible spins of Q^0 separately: $J=0$ is the case of model I, $J=1$ is the case of model II, and $J=2$ the case of model III. Our results are summarized in table 4. In model I we obtain 1σ constraints of the type

$$|\eta_{11}|^2+|\eta_{12}|^2<0.017, \qquad |\eta_{22}|^2+|\eta_{21}|^2<0.017,$$

from which we conclude limits on $|\eta_{ik}|$ of the order of 0.09 (if they have equal magnitudes) to 0.13 (if η_{12}, η_{21} or η_{11}, η_{22} vanish). The constraints from the data are much better, where applicable, in the other two cases, cf. third and fourth, as well as

TABLE 4
Models assuming V − A plus neutral exchanges

Model	Neutral couplings	Constraining parameters[a]	1σ limit on couplings	Comments
I	η_{ik}	ρ; δ; ξ'; α; α'	$\|\eta_{ik}\| < 0.09$–0.13	$1-\xi \simeq \|\eta_{22}\|^2+\|\eta_{21}\|^2-\frac{1}{2}(\|\eta_{11}\|^2+\|\eta_{12}\|^2)$
II	γ_{11}	ω; ξ'	$\|\gamma_{11}\| < 0.038$	$\xi=\omega$;
	γ_{12}	β; β'	$-0.038 < \mathrm{Re}\,\gamma_{12} < 0.030$	γ_{21}, γ_{22} constrained
			$-0.018 < \mathrm{Im}\,\gamma_{12} < 0.046$	through eq. (21)
III	φ_{11}; φ_{22}	ρ; δ; ω; ξ'	$\|\varphi_{11}\| < 0.019$	$1-\xi=10\|\varphi_{11}\|^2-4\|\varphi_{22}\|^2$;
			$\|\varphi_{22}\| < 0.056$	phases of φ_{ii}
				undetermined

[a] $\omega \equiv \xi\delta/\rho$.

The neutral couplings η_{ik}, γ_{ik} and φ_{ii} are defined by an expression analogous to eq. (1), written with the ordering $(\bar{e}\mu)(\bar{\nu}_\mu \nu_e)$. To this the standard W-exchange is added ($g_{22}=1$). The observables not shown assume their V − A values. The fifth column indicates when and how a direct measurement of ξ will add to the existing constraints.

fifth lines of table 4. In the spin-1 case γ_{11} and γ_{12} are constrained to zero within a few percent of $g_{22}=1$, whilst γ_{22} and γ_{21} remain undetermined. Most interesting is model III which yields very tight bounds on both types of tensor couplings.

7. Summary and conclusions

In this work we update and extend our previous analysis of leptonic vertices occurring in charged leptonic and semileptonic processes, on the basis of the new high-precision data in muon decay that became available recently. We apply for the first time the helicity projection form (HPF) of the effective hamiltonian whose primary merits are the following: it exhibits in a more transparent manner the sensitivity of a given observable to specific types of couplings. It minimizes the number of interference terms so that an improvement in accuracy for one of the observables does not require a new fit of all the couplings. Because it makes use of left- and right-handed fields, contact to specific unified models is made in a simple and transparent manner. Finally, its simple behaviour under Fierz reordering allows for physical interpretation of any noncanonical Lorentz structure in a straightforward way.

The analysis is based on the assumption that no other muon decay modes [7, 8] are present and that neutrinos are essentially massless. Other decay modes $\mu^- \to e^- +$ neutrals, if present, could produce compensating effects in the decay parameters, as long as these are deduced from observation of the electron only. In the case of massive neutrinos [11], neutrino state mixing and the possible occurrence of Majorana states render the analysis considerably more involved.

Generally speaking, we find that the HPF couplings are now well constrained, relative to the dominant V − A structure g_{22}, the progress being brought about in some cases by the new measurement of $\omega \equiv \xi\delta/\rho$ [3], in others by the measurements of the complete polarization of the positron in μ^+ decay [4]. Vector, axial vector and tensor couplings are generally found to be zero within about 0.04 (in units of g_{22}) or better. The limits on scalar-pseudo-scalar couplings are not so good: the constant h_{12} remains undetermined, h_{22} is badly known and could be as large as 0.3 (cf. model II of table 3). The limits on h_{11} and h_{21} are typically of the order of 0.08. Improvements on ξ', and an independent measurement of ξ would help to narrow down these limits, (cf. the comments in tables 3 and 4). In the framework of models with factorization and universality (sect. 4) we note that the deviation of δ from its V − A value $\frac{3}{4}$ is a test of the assumed eμ universality.

We also study the more speculative possibility of exchanging a neutral particle Q^0 with quantum numbers ($L_e = \pm 1, L_\mu = \mp 1$) (composite?) and spin $J = 0, 1$ or 2. For $J=1$ and $J=2$ we obtain rather good limits on some of or all effective couplings. In the case $J=0$ there is room for such a contribution at the level of up to 0.13.

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Appendix

INVARIANT DIFFERENTIAL CROSS SECTION FOR THE REACTION $\nu_\mu e^- \to \mu^- \nu_e$

Let $s=(p^{(\nu_\mu)}+p^{(e)})^2$, $t=(p^{(\nu_\mu)}-p^{(\mu)})^2$ and $u=m_\mu^2+m_e^2-s-t$ be the standard kinematic variables and let h be the helicity of the incoming muon neutrino. As in I we define the kinematic functions

$$g_1(x):=\left(x-m_\mu^2\right)\left(x-m_e^2\right), \qquad x=s \quad \text{or} \quad t,$$

$$g_2(s,t):=(s+t)\left(s+t-m_\mu^2-m_e^2\right),$$

and write the cross section as

$$\frac{d\sigma}{dt}=\left(\frac{d\sigma}{dt}\right)_{VA}+\left(\frac{d\sigma}{dt}\right)_{SP}+\left(\frac{d\sigma}{dt}\right)_{T}+\left(\frac{d\sigma}{dt}\right)_{SP-T}+\left(\frac{d\sigma}{dt}\right)_{SP-VA}+\left(\frac{d\sigma}{dt}\right)_{VA-T}. \tag{A.1}$$

We then have

$$\left(\frac{d\sigma}{dt}\right)_{VA}=\frac{G_0^2}{2\pi s^2}\left\{\left[|g_{22}|^2 g_1(s)+|g_{12}|^2 g_2(s,t)\right](1-h)+(1\leftrightarrow 2;\, h\to -h)\right\}, \tag{A.2}$$

$$\left(\frac{d\sigma}{dt}\right)_{SP}=\frac{G_0^2}{8\pi s^2}g_1(t)\left\{\left(|h_{11}|^2+|h_{21}|^2\right)(1-h)+(1\leftrightarrow 2;\, h\to -h)\right\}, \tag{A.3}$$

$$\left(\frac{d\sigma}{dt}\right)_{T}=\frac{G_0^2}{2\pi s^2}\left[2g_1(s)+2g_2(s,t)-g_1(t)\right]\left\{|f_{11}|^2(1-h)+(1\leftrightarrow 2;\, h\to -h)\right\}, \tag{A.4}$$

$$\left(\frac{d\sigma}{dt}\right)_{SP-T}=-\frac{G_0^2}{2\pi s^2}\left[g_1(s)-g_2(s,t)\right]\left\{(1-h)\mathrm{Re}\left(h_{11}f_{11}^*\right)+(1\leftrightarrow 2;\, h\to -h)\right\}. \tag{A.5}$$

In contrast to the SP − T interference term (above), the SP − VA and VA − T interference terms become negligible as soon as $s \gg m_\mu m_e$. They are★

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}t}\right)_{\mathrm{SP-VA}} = -\frac{G_0^2}{2\pi s^2} u m_e m_\mu \left\{ \mathrm{Re}\left(g_{22} h_{21}^* + g_{12} h_{11}^* \right)(1-h) + (1 \leftrightarrow 2;\, h \rightarrow -h) \right\}, \tag{A.6}$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}t}\right)_{\mathrm{VA-T}} = -\frac{G_0^2}{\pi s^2} 3 u m_e m_\mu \left\{ \mathrm{Re}\, g_{12} f_{11}^* (1-h) + (1 \leftrightarrow 2;\, h \rightarrow -h) \right\}. \tag{A.7}$$

The second term in each of the curly brackets is obtained from the first by exchanging the indices 1 and 2 on the couplings and by replacing the helicity h by $-h$. In fact, as h is now known to be equal to -1 within 0.4%, the second terms all vanish and our formulae (A.1) become very simple. In this case, again, the coupling h_{12} does not appear in the expressions for the cross section.

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★ Eq. (A.7) of ref. [1] is misprinted: It should have a factor 4 in the denominator, not 8.