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Inhomogeneity and Nonlinearity Effects on Stop Bands of Alfvénic Ion Cyclotron Waves in Multicomponent Plasma

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We study the existence of stop bands for left-handed Alfvénic ion cyclotron waves propagating in the direction of magnetic field in a multicomponent plasma. Three effects are discussed: finite ion temperature, inhomogeneous magnetic field, and nonlinear wave amplitude. All of them affect the existence of stop bands and set critical bounds on the relevant physical parameters, particularly on the density of heavy ions. Using the model of a linearly varying, longitudinally inhomogeneous magnetic field, we calculate the critical lower bound on heavy ion density as a function of longitudinal inhomogeneity, and we discuss the difference between the truly inhomogeneous and essentially homogeneous situations. Typical values for the wave vector in the resonance region are derived. We also find the upper bound on the wave amplitude for the linear theory to be applicable, and we derive the condition for stop band formation in the nonlinear case. The obtained results are applied to the magnetospheric environment and are shown to lead to relevant modifications. For example, the formation of stop bands by thermal heavy ions is strongly dependent on the local inhomogeneity of the magnetic field.

1. INTRODUCTION

In recent years a number of experiments have investigated the role of heavy ions (mainly He^+ and O^+) in the generation and propagation of Alfvénic ion cyclotron waves (hereinafter to be called AICWs) in the frequency range of Pc 1–2 (0.1–5 Hz). New important phenomena, which could not be explained by the presence of one cold ion (proton) component only, have been observed in the geostationary orbit by the GEOS 1 and 2 and ATS 6 satellites [*Young et al.*, 1981*a*; *Mauk et al.*, 1981] and later in the plasmapause region by ISEE 1 and 2 [*Fraser et al.*, 1986]. Among these phenomena were the reversal of polarization and the splitting of the AICW spectrum into two branches, the high-frequency branch (higher than the gyrofrequency of He^+ or O^+) and the low-frequency branch (below the corresponding gyrofrequency).

The short-period micropulsations (Pc 1–2 and intervals of pulsations with diminishing periods, or IPDP) are ground signals of the magnetospheric AICWs, and, accordingly, their occurrence is dependent on the generation and propagation conditions of the AICWs. Unstructured Pc 1 pulsations observed at a high-latitude station are typically below 0.5 Hz [*Heacock and Akasofu*, 1973; *Hayashi et al.*, 1981], i.e., below the O^+ gyrofrequency. It is well known that structured Pc 1 pulsations appear mainly during low solar activity [*Fraser-Smith*, 1970, 1981; *Kawamura et al.*, 1983]. More recently, *Maltseva et al.* [1988] have shown that the IPDP type pulsations, which are due to the AICW activity during magnetic substorms, are also more frequent in the years of low solar activity. Furthermore, *Kawamura et al.* [1983] have reported that instead of Pc 1, longer-period Pc 2 pulsations tend to appear during high solar activity.

Ample evidence exists for the variation of heavy ion densities in terms of solar and geomagnetic activity. For example, *Young et al.* [1981*b*, 1982] have shown that the magnetospheric O^+ density may increase by a factor of about 8 when the geomagnetic activity increases from $Kp = 0$ to $Kp = 9$. The O^+ density also responds to solar activity: an increase by a factor of about 10 has been observed from the years of low solar activity to solar maximum years [*Stokholm et al.*, 1989]. According to *Young* [1983], the O^+ content may range from a few percent to more than 80% of the total ion density.

The strong influence of heavy ions on the generation and propagation of the AICWs has been demonstrated in a number of studies [*Young et al.*, 1981*a*; *Mauk et al.*, 1981; *Kozyra et al.*, 1984; *Perraut et al.*, 1984; *Nekrasov*, 1987*a*]. These studies together with the above discussed satellite and ground-based observations suggest that when the heavy ion density is relatively small, the AICWs generated in the magnetosphere can more freely propagate to the ground, while a larger heavy ion concentration forms stop bands more effectively, thus more often preventing the AICWs from propagating to the ground. Accordingly, it is very important to know the theoretical predictions for the formation of stop bands in a situation which corresponds to the observational conditions as well as possible.

In most theoretical studies on the subject, the absolute and

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convective AICW growth rates were calculated using the model of a uniform magnetic field [*Gomberoff and Neira*, 1983; *Kozyra et al.*, 1984; *Gendrin et al.*, 1984]. The longitudinal inhomogeneity was taken into account in some papers, but only within the limits set by the uniform magnetic field [e.g., *Mauk*, 1982]. The essential assumption made in those analytical calculations was to consider the contribution of the heavy ions to the dispersion relation as a small correction. However, this assumption is often not appropriate in a real observational situation, as is clear from the discussion above. Also, the influence of the nonlinear motion of the heavy ions on the AICW dispersion relation has so far not been taken into account.

In the present paper we discuss the effects of thermal motion (section 2), inhomogeneous magnetic field (section 3), and a large (nonlinear) wave amplitude (section 4) on the propagation of AICWs in a multicomponent plasma. Our main interest is in the situation where the heavy ion contribution to the dispersion relation for the low-frequency branch of AICWs is dominant in the cyclotron resonance region. We find the critical conditions on the plasma parameters for the magnetic field inhomogeneity or the nonlinear heavy ion velocity to be important in this region. The existence of the AICW stop bands in the different cases is studied and related to the heavy ion concentration. The theoretical results obtained for the critical heavy ion densities are then applied (section 5) to the magnetospheric environment and shown to lead to significant changes as compared to the corresponding estimates using the homogeneous magnetic field or linear theory.

2. Homogeneous Magnetic Field

As is discussed above, the presence of heavy ions can result in the splitting of the pulsation spectrum. The spectrum splitting of the left-handed waves is possible owing to the appearance of stop bands, usually discussed in terms of a cold multicomponent plasma in a homogeneous magnetic field [*Smith and Brice*, 1964; *Gomberoff and Neira*, 1983; *Kozyra et al.*, 1984; *Gendrin et al.*, 1984]. If the wave is propagating parallel to the magnetic field, the left- and right-handed branches of the dispersion relation are not connected. On the other hand, if propagation is oblique, the left- and right-handed branches are coupled at the crossover frequency, and the waves can be grouped in three classes [*Young et al.*, 1981*a*; *Rauch and Roux*, 1982].

Here we are interested in the low-frequency branch of the AICWs (class 1 waves in the terminology of *Rauch and Roux* [1982]) which are excited below the local heavy ion gyrofrequency and experience the cyclotron resonance when traveling toward the equator. These waves were shown by Rauch and Roux to remain left handed and be guided along the geomagnetic field lines. The transverse wave number exerts little influence on their dispersion. Hence, for the sake of simplicity, we shall consider in this paper the parallel wave propagation only.

The dispersion relation for the AICWs propagating along the homogeneous magnetic field has the following form in the cold plasma case [*Smith and Brice*, 1964]:

$$\frac{k^2c^2}{\omega^2} = -\sum_j \frac{\omega_{pj}^2}{\omega(\omega-\omega_j)} \tag{1}$$

where ω_{pj} and ω_j are the plasma frequency and the cyclotron frequency, respectively, of the species j ($j = e, p$, and i for electrons, protons, and heavy ions, respectively); ω and k are the wave frequency and the wave number, respectively; and c is the speed of light. Note that the condition $\omega - \omega_j \gg kv_{\|j}$ was used to obtain equation (1). Here $v_{\|j}$ is the thermal velocity along the magnetic field. Taking the electrons, protons, and only one species i of heavy ions into account in (1), we obtain

$$\left(1-\frac{\omega}{\omega_p}\right)\frac{k^2c_A^2}{\omega^2} = 1-\delta_i\left(\frac{m_i}{m_p}-1\right)\frac{\omega_i}{\omega-\omega_i} \tag{2}$$

Here $\delta_i = n_i/n$ is the relative concentration of the heavy ions ($n = n_e = n_p + n_i$ is the total plasma density), m_i is the heavy ion mass, and $c_A = B/(4\pi nm_p)^{1/2}$ is the Alfvén velocity. It is seen from (2) that the presence of heavy ions results in the appearance of a stop band (where $k^2 < 0$) in the frequency range $\omega_i \le \omega \le \omega_i + \Delta\omega_i$, where the width $\Delta\omega_i$ is as follows:

$$\Delta\omega_i = \delta_i\left(\frac{m_i}{m_p}-1\right)\omega_i$$

Note that there is no critical lower bound for the heavy ion density above which stop bands would only be formed, since any nonvanishing amount of heavy ions can now give rise to a stop band. This unphysical artefact, due to the presence of the pole in (2), is a consequence of neglecting the thermal motion of the heavy ions. The thermal heavy ion motion removes these singularities via the Doppler effect, which qualitatively can be taken into account by replacing the term $\omega - \omega_i$ in (2) by $ikv_{\|i}$. Supposing that the heavy ions give the main contribution to the dispersion relation in the frequency range $\omega - \omega_i \lesssim k_r v_{\|i}$, we obtain the following approximate expression for the complex wave number in this range:

$$k = k_r - i\aleph \approx \left(\delta_i\frac{m_i}{m_p}\frac{c_A}{v_{\|i}}\right)^{1/3}\frac{\omega_i}{2c_A}(\sqrt{3}+i) \tag{3}$$

This formula is applicable if

$$\delta_i > \delta_i^h = \left(\frac{m_i}{m_p}\right)^{1/2}\left(\frac{m_i}{m_p}-1\right)^{-3/2}\frac{v_{\|i}}{c_A} \tag{4}$$

whence $|k| = (k_r^2 + \aleph^2)^{1/2} > \omega/c_A$. In this case the stop band will be present in the frequency range $\omega_i + k_r v_{\|i} \lesssim \omega \le \omega_i + \Delta\omega_i$.

On the other hand, if the concentration of the heavy ions is small, i.e., $\delta_i < \delta_i^h$, these ions make a small contribution to the dispersion relation. Then the stop band is absent, and we have in this case

$$k_r \approx \frac{\omega}{c_A}\left(1-\frac{\omega}{\omega_p}\right)^{-1/2} \qquad \aleph \approx -\frac{1}{2}\left(\frac{m_i}{m_p}-1\right)\delta_i\frac{\omega_i}{v_{\|i}} \tag{5}$$

Accordingly, the above mentioned artefact is removed and we have found a lower bound δ_i^h for the heavy ion concentration, below which the stop band cannot exist. Note also the heavy ion mass dependence of (4), which implies that, the heavier the ions, the smaller the concentration needed to create the stop band. The influence of thermal effects on the stop band formation in a homogeneous magnetic field has been considered numerically by *Ball* [1987].

3. Inhomogeneous Magnetic Field

The magnetic field, particularly in the magnetosphere, is not really homogeneous. We will now study how the above treatment of the AICWs has to be modified in an inhomogeneous magnetic field. In order to have definite quantitative estimates, let us parameterize the longitudinally inhomogeneous magnetic field by a linear form $B(s) = B_o(1 - s/H)$, where s is a coordinate along the magnetic field and H is the characteristic length of the longitudinal inhomogeneity. Even so simple a form is enough to lead to nontrivial changes in the treatment of AICWs and to give a first-order approximation to the more complicated and realistic structure of the magnetic field.

Now, because of the change of the local cyclotron frequencies, a wave with a given frequency in the range $\omega < \omega_i(s)$ ($\omega_p(s) > \omega > \omega_i(s)$), propagating in the direction of decreasing (respectively, increasing) magnetic field, will arrive at the stop band. Let us further assume that the wave frequency corresponds to the heavy ion cyclotron frequency at the origin of the coordinate, i.e., $\omega = \omega_i(0)$. (It has been shown by *Nekrasov* [1987*b*] that if the energetic ions have a bi-Maxwellian velocity distribution, the AICWs are dominantly generated out of the equatorial region and can thus resonate with the heavy ions when propagating toward the equator.) Then equation (2) can be rewritten in the following form:

$$\left[1 - \frac{\omega}{\omega_p(s)}\right]\frac{k^2c_A^2(s)}{\omega^2} = 1 + \delta_i\left(\frac{m_i}{m_p} - 1\right)\left(1 - \frac{H}{s}\right) \quad (6)$$

It is evident that the stop band is situated (now in the coordinate space) at $0 \leq s \leq s_H$, where the width s_H is as follows:

$$s_H = \delta_i\left(\frac{m_i}{m_p} - 1\right)\left[1 + \delta_i\left(\frac{m_i}{m_p} - 1\right)\right]^{-1} H$$

The above results for the typical wave number in a homogeneous magnetic field (equations (3) and (5)) remain valid even in the case of a longitudinally inhomogeneous magnetic field if the inhomogeneity is weak. More specifically, this is true if the concentration of the heavy ions is sufficiently large to fulfil the inequality $2\aleph s_r > 1$, where

$$s_r = \sqrt{\pi}(Hv_{\|i}/\omega_i)^{1/2}$$

is the width of the cyclotron resonance region of the heavy ions [*Nekrasov*, 1987*a*]. If we now use the above result (equation (3)) for the wave vector in the case where heavy ions dominate to form a stop band, this leads to the following lower bound for the heavy ion concentration, which must be valid in addition to (4):

$$\delta_i > \delta_i^w = \frac{m_p}{m_i}\left(\frac{c_A^4}{H^3\omega_i^3 v_{\|i}}\right)^{1/2} \quad (7)$$

Note that the heavier the ions are, the larger the lower bound is, and thus the more local inhomogeneities matter. Naturally, δ_i^w vanishes as H tends to infinity and the homogeneous magnetic field is restored. Similarly, a lower bound for the heavy ion concentration could be found using the wave vector of (5) in the case where stop bands are not formed.

In the opposite case of a stronger inhomogeneity, i.e., when $2\aleph s_r < 1$, the longitudinal inhomogeneity of the magnetic field is essential, and the above results for the wave vector in the homogeneous case have to be modified. The characteristic value of the wave number in the cyclotron resonance region ($|s| \leq s_r$) can be obtained in this case from (2) by the following substitution [*Nekrasov*, 1987*a*]:

$$\omega - \omega_i \rightarrow \frac{i-1}{\pi}\left|\frac{d\omega_i}{ds}\right| s_r \quad (8)$$

(The sign of the imaginary part in (8) is chosen from the condition that the wave propagating in the direction of positive s is absorbed.)

Assuming that the heavy ions give the main contribution to the dispersion relation in the resonance region and using (2) and (8) and the above expression for s_r, we obtain a typical value for the wave number in the resonance region:

$$k \approx \left(\delta_i \frac{m_i}{m_p}\right)^{1/2}\left(\frac{H\omega_i}{v_{\|i}}\right)^{1/4}\frac{\omega_i}{c_A}\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right) \quad (9)$$

The concentration of the heavy ions must satisfy in this case the condition

$$\delta_i > \delta_i^s = \left(\frac{m_i}{m_p} - 1\right)^{-1}\left(\frac{v_{\|i}}{H\omega_i}\right)^{1/2} \quad (10)$$

This inequality must be compatible with the inequality opposite to that expressed in (7). Then the stop band exists (as far as $s_r < s_H$) and is located in the region $s_r \lesssim s \leq s_H$. Note also that, again, the heavier the ion, the smaller the density needed to form a stop band. Accordingly, the longitudinal inhomogeneity of the magnetic field can, in analogy to thermal motion, remove the unphysical singularity, and it requires a minimum concentration of heavy ions for a stop band to exist.

On the other hand, if $\delta_i < \delta_i^s$, the heavy ions make a small contribution to the dispersion relation. Then the stop band does not exist, and the wave number attains typically the following form in this case:

$$k_r \approx \frac{\omega}{c_A}\left(1 - \frac{\omega}{\omega_p}\right)^{-1/2}$$
$$\aleph \approx -\frac{\sqrt{\pi}}{4}\delta_i\frac{m_i}{m_p}\left(1 - \frac{m_p}{m_i}\right)^{1/2}\left(\frac{H\omega_i}{v_{\|i}}\right)^{1/2}\frac{\omega_i}{c_A} \quad (11)$$

4. Nonlinear Wave Effects

In the preceding sections we have assumed that the wave amplitude is sufficiently small that the linear theory of the AICWs could be used. However, in some events the observed AICW amplitudes in the Pc 1–2 range have been such that the validity of the linear theory is questionable. For example, an amplitude of about 3 nT was observed at the geostationary orbit [*Young et al.*, 1981*a*]. In such cases the existence of the stop bands for Pc 1–2 pulsations will also depend on their intensity, and, accordingly, nonlinear effects have to be taken into account.

Using the equation of motion, it can be shown [*Nekrasov*, 1987*c*] that if the condition

$$k_r v_{\perp i} \omega_i h \gg \frac{d\omega_i}{ds} \max\left\{ v_{\parallel i}; \frac{k_r v_{\perp i}^2}{2\omega_i} \right\} \quad (12)$$

is fulfilled, it is necessary to take into account the influence of the wave on the longitudinal velocity of the particles. Here $v_{\perp i}$ is the heavy ion velocity perpendicular to the external magnetic field **B** and $h = B_w/B$, where $B = |\mathbf{B}|$ and B_w is the magnetic field of the wave. If $v_{\perp i}$ is greater than the thermal velocity perpendicular to the magnetic field, we find from the equation of motion [*Nekrasov*, 1987*c*] that $v_{\perp i} \approx h^{1/3}\omega/|k|$. Then, in the linearly varying inhomogeneous external magnetic field the inequality (12) will take the following form:

$$h \gg h_i^m = \max\left\{ \left(\frac{v_{\parallel i}}{\omega_i H}\right)^{3/4}; \frac{1}{(2|k|H)^{3/2}} \right\} \quad (13)$$

Note that in the case opposite to (13), the linear theory considered in the previous sections is restored.

In the nonlinear case the treatment of the Doppler effect given in section 2 must be changed. The influence of the wave on the parallel velocity of the particles in the wave phase can be taken into account by replacing the term $\omega - \omega_i$ in (2) by $k_r v_W$, where v_W is the parallel velocity acquired by the particle in the wave. Such a substitution gives a qualitative description of the nonlinear interaction of the particles with a wave [*Galeev and Sagdeev*, 1973]. In the present case it may be shown [*Nekrasov*, 1987*c*] that $v_W \approx h^{2/3}\omega/|k|$. In analogy with (8), we obtain, after the substitution, the anomalous dispersion relation in the cyclotron resonance region to be as follows:

$$\left(1 - \frac{m_p}{m_i}\right)\frac{k^2 c_A^2}{\omega^2} \approx 1 + \frac{1+i}{2}\left(\frac{m_i}{m_p} - 1\right)\delta_i^u h^{-2/3} \quad (14)$$

Here δ_i^u is the concentration of the untrapped [*Karpman and Shklyar*, 1977] heavy ions absorbing the wave energy.

If the condition

$$h > h_i^s = \left[\frac{1}{2}\left(\frac{m_i}{m_p} - 1\right)\delta_i^u\right]^{3/2} \quad (15)$$

is valid, the stop band will be absent because the contribution of the heavy ions to the dispersion relation is small in this case. For the imaginary part of the wave number we then obtain the following expression:

$$\aleph \approx -\frac{m_i}{4m_p}\left(1 - \frac{m_p}{m_i}\right)^{1/2} \delta_i^u h^{-2/3} \frac{\omega_i}{c_A} \quad (16)$$

and k_r is determined by (11).

If the concentration of heavy ions is high enough to fulfil the inequality opposite to (15), the heavy ion contribution to the dispersion relation will be dominant in the resonance region. Then the complex wave number is as follows:

$$k \approx \left(\delta_i^u \frac{m_i}{m_p}\right)^{1/2} h^{-1/3} \frac{\omega_i}{c_A}\left(\cos\frac{\pi}{8} + i \sin\frac{\pi}{8}\right) \quad (17)$$

The width of the resonance region (where $k_r^2 \geq 0$) is approximately $s_r \approx h^{2/3}H$. The stop band will then exist in the interval $s_r \lesssim s \leq s_H$.

Thus a nonlinear treatment of the AICW stop bands leads to a constraint relating the heavy ion density and the wave amplitude. For high-amplitude waves and small concentrations, i.e., in the case of (15), the stop band will be absent. In the opposite case (equation (13) being still valid), the stop band exists, and its width for a nonlinear wave will depend on the wave amplitude. The absorption of the intense Alfvénic waves is inversely proportional to their amplitude, as seen in (16) and (17).

5. Consequences for Magnetospheric AICW

In this study we have shown, for example, that the inhomogeneity of the magnetic field may significantly change the AICW propagation conditions. In order to reach the ground, the wave has to propagate all through the field line, thus being subject to all local inhomogeneities between the wave amplification region and the ground. Therefore we think it is more important, especially in view of the ground-based observations of micropulsations, to study the effects of such inhomogeneities on the AICW propagation rather than their generation.

In order to make the implications of the paper more transparent we next give numerical estimates for some of the above derived formulas. In these estimates we use oxygen (O^+) as a sample heavy ion, but analogous results for any other heavy ions can easily be found from the original equations. All numerical estimates are given in a scaled form where the change of any parameter value can be calculated in a straightforward manner. The parameter values we use in these estimates are relevant in the magnetospheric environment and aim to correspond, to some accuracy, to the values around the geostationary orbit: $n \approx 10$ cm^{-3}, $B \approx 150$ nT, whence $c_A \approx 10^6$ m/s and $\omega_{O^+} \approx 1$ s^{-1}. The heavy ion energies are varied between 1 eV ("thermal") and 10 keV ("energetic"), taking 1 keV to be the scale for the energy in the equations.

For the scale value of the magnetic inhomogeneity parameter we will take $H \approx 10^7$ m, which is approximately one order of magnitude smaller than the length of the magnetic field line through the geostationary orbit ($L \approx 6.6$). The corresponding magnetic field gradient, which is inversely proportional to H, is larger than the local gradient close to the equator region of the geostationary field line but smaller than the gradient close to Earth. Therefore the chosen scale value for H well corresponds to the situation on the geostationary field line between the ground and the equator. It should also be noted that the local field gradient depends, for example, on magnetic activity.

Let us first consider thermal effects in the case of a homogeneous magnetic field. The relevant critical oxygen density can be expressed in the following form (see (4)):

$$\delta_{O^+}^h = 7.6 \times 10^{-3}(E_{\parallel O^+}/1\ \text{keV})^{1/2}(c_A/10^6\ \text{m/s})^{-1} \quad (18)$$

where, instead of the velocity $v_{\parallel i}$, we have used the corresponding longitudinal energy $E_{\parallel i} = \frac{1}{2}m_i v_{\parallel i}^2$. This and two other critical densities have been plotted in Figure 1 as a function of the oxygen parallel energy, using the above mentioned scale values for the other parameters. As a manifestation of the above discussion it can be seen that very small concentrations of thermal oxygen are enough to form a stop band in the homogeneous case. The critical density $\delta_{O^+}^h$ varies from 0.024% to 2.4% in the energy range considered.

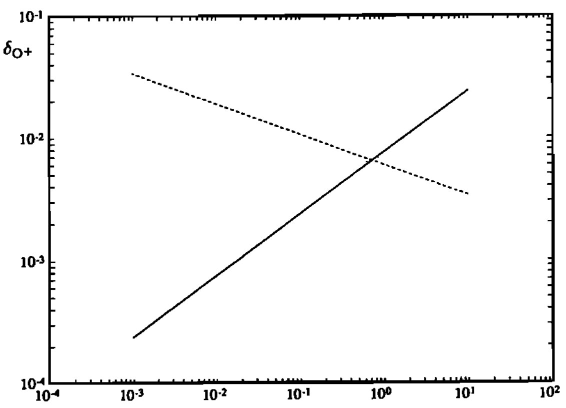

Fig. 1. The curves for the three critical relative oxygen densities as a function of the oxygen parallel energy (given in keV): $\delta_{O^+}^h$ (solid line), $\delta_{O^+}^w$ (dashed line), and $\delta_{O^+}^s$ (dotted line). The curves cross at the crossover energy E_{co} corresponding to the crossover density δ_{co}. Other parameters attain their scale values: $H = 10^7$ m, $c_A = 10^6$ m/s, and $\omega_{O^+} = 1$ s^{-1}.

In the inhomogeneous magnetic field the critical density δ_i^w given in (7) attains the following expression:

$$\delta_{O^+}^w = 6.0 \times 10^{-3}(c_A/10^6 \text{ m/s})^2(H/10^7 \text{ m})^{-3/2} \cdot (\omega_{O^+}/1 \text{ s}^{-1})^{-3/2}(E_{\|O^+}/1 \text{ keV})^{-1/4} \quad (19)$$

Note that this value naturally comes out to be of the order of magnitude which is observationally relevant in the magnetospheric environment. From Figure 1 one sees that a concentration of thermal oxygen ions in excess of about 3.4% is enough to restore the homogeneous case. Note that this number is more than 2 orders of magnitude larger than the limit derived from the thermal motion for the homogeneous case. This shows that inhomogeneity effects are particularly important for the thermal component of the heavy ion plasma.

The truly inhomogeneous situation is reached when the local heavy ion density is smaller than the critical density δ_i^w. In this case, the lower bound on the oxygen density for stop bands to be formed attains the following form:

$$\delta_{O^+}^s = 7.0 \times 10^{-3}(H/10^7 \text{ m})^{-1/2} \cdot (\omega_{O^+}/1 \text{ s}^{-1})^{-1/2}(E_{\|O^+}/1 \text{ keV})^{1/4} \quad (20)$$

As seen in Figure 1, this critical density is larger than the homogeneous estimate. Thus the lower bound on the heavy ion density for stop bands to exist is increased when taking the inhomogeneity of the magnetic field into account. For thermal energies this means a change by a factor of 5. Note also that since for thermal energies $\delta_{O^+}^w$ is about 30 times larger than $\delta_{O^+}^h$, the situation is truly inhomogeneous for a large range of heavy ion densities and the essentially homogeneous case is restored only at much higher densities.

The curves for the three critical densities cross each other at one point at the crossover energy E_{co} corresponding to the crossover density δ_{co} (see Figure 1). The crossover energy represents the upper energy bound for the validity of our inhomogeneity treatment. In fact the three critical densities cross each other at one point irrespective of the values of other parameters, as can easily be seen from (4), (7), and (10). The change of the crossover energy and the crossover density for oxygen as a function of H has been illustrated in Figure 2. The crossover energy depends on H^{-2}, and the crossover density on H^{-1}. As can be seen there, for H smaller than the scale value used (corresponding to the field gradient closer to Earth), the validity of our treatment and the range of inhomogeneity effects is extended to higher energies. On the other hand, even for H larger than the scale value (corresponding to the gradient around the equator), the treatment applies to a large range of thermal energies. This again demonstrates the necessity of taking the inhomogeneity of the magnetic field into account when discussing the existence of stop bands.

Let us finally evaluate the nonlinearity effects in more detail. The second term on the right-hand side of (13), the formula of the lower bound for the nonlinear amplitude, is, using the scale values for parameters, about 1.1% or less for oxygen, independent of its energy. Accordingly, the amplitude of 3 nT observed at the geostationary orbit may already be reaching the nonlinear regime. The first term of the same equation grows to be larger than the second term above the oxygen parallel energy of 52 eV, and it attains the value of 8.1% for energetic oxygen ions ($E_\| = 10$ keV). The lower bound implied by (13) altogether is depicted in Figure 3. As seen there, energetic particles sustain linearity for larger amplitudes than thermal ones.

The condition for stop bands, equation (15), can now be used to study the density of untrapped heavy ions necessary to form a stop band for a nonlinear AICW. From the chain of inequalities

$$h_{O^+}^s > h \gg h_{O^+}^m \quad (21)$$

one finds a lower bound on the oxygen density which is also depicted in Figure 3 as a function of oxygen parallel energy. If the oxygen density is larger than this lower bound, there is a range of nonlinear amplitudes for which a stop band is formed. This amplitude range is limited by (13) from below (also shown in Figure 3) and by (15) from above, as shown by (21). As can be seen in Figure 3, fairly small concentrations (less than 1%) of untrapped thermal oxygen ions are enough

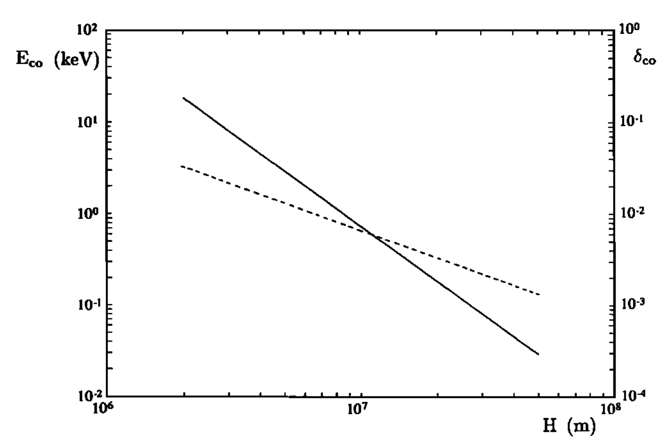

Fig. 2. The change of the crossover energy (solid line; scale given on the left ordinate in keV) and the crossover density (dashed line; scale given on the right ordinate) as a function of the longitudinal inhomogeneity parameter H (given in meters) around its scale value, $H = 10^7$ m. The H values for the geostationary dipole field line range from 0.2 to 5 times the scale value. Other parameter values are as in Figure 1.

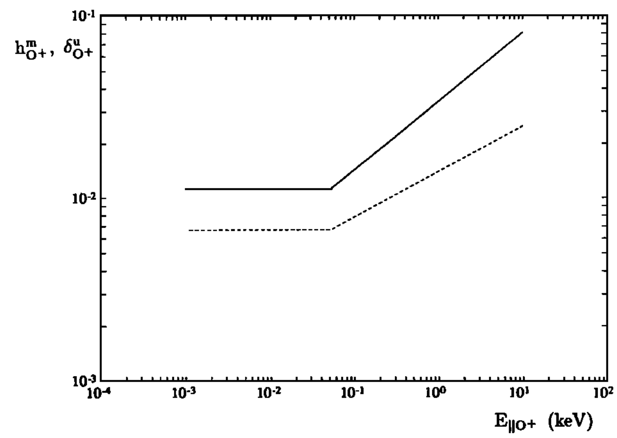

Fig. 3. The lower bounds on the normalized nonlinear amplitude (solid line) and on the density of untrapped oxygen ions to form stop bands (dashed line) as a function of the oxygen parallel energy (given in keV). The other parameter values are as in Figure 1.

to form nonlinear stop bands. Since the energetic particles are more effective in sustaining linearity, the critical oxygen density also grows with energy accordingly.

6. Conclusions

In this paper we have studied the changes in the propagation of Alfvénic ion cyclotron waves in a multicomponent plasma due to finite ion temperature, longitudinally inhomogeneous magnetic field, and nonlinear wave amplitude. We have discussed the conditions for the formation of stop bands in qualitatively different situations and derived the corresponding critical values for the density of heavy ions in the plasma.

We first studied the thermal motion of heavy ions in the homogeneous magnetic field and showed that stop bands are only formed for heavy ion densities above a critical lower bound. Then, using a linear model for the longitudinally inhomogeneous magnetic field, we derived the corresponding critical heavy ion densities for the formation of stop bands both in a situation where the effect of the magnetic field inhomogeneity is negligible (essentially homogeneous case) and, on the other hand, in a truly inhomogeneous situation. Typical qualitative values of the wave number were also presented for both cases.

Applying these results to the magnetospheric plasma, we have found that inhomogeneity effects are very important in the formation of stop bands, especially for thermal heavy ions. A typical situation for the three different critical oxygen densities is depicted in Figure 1 (parameter values corresponding to those around the geostationary orbit are used). The critical oxygen densities derived from the magnetic field inhomogeneity arguments are larger, particularly for thermal heavy ions, than the critical density derived from the finite temperature in the homogeneous magnetic field. This already underlines the necessity of taking inhomogeneities into account when discussing the stop band formation. For a large range of heavy ion densities, stop bands are formed in a truly inhomogeneous situation ($\delta^s_{O^+} < \delta < \delta^w_{O^+}$). The homogeneous case is restored only at higher densities ($\delta > \delta^w_{O^+}$).

When discussing the changes to the linear theory due to large wave amplitudes we derived a lower bound on the nonlinear amplitude, and the condition for the existence of stop bands in the nonlinear situation. Stop bands can be formed only for a limited range of amplitudes. While the very condition for nonlinearity gives the lower bound to this range, the condition of stop band formation sets the upper bound. This upper bound depends on the density of untrapped heavy ions.

The lower bound for the normalized wave amplitude to be nonlinear is shown in Figure 3 as a function of oxygen parallel energy (the same plasma environment was used as mentioned above). As seen there, the nonlinear regime for oxygen ions with thermal energies is reached if the wave amplitude is more than about 1.1% of the background field. Figure 3 also shows the lower bound on the density of untrapped oxygen ions to have stop bands for a range of nonlinear amplitudes of the Alfvénic ion cyclotron wave. For a large range of energies the value of this critical density is only of the order of 1–2%.

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