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Excitation of helium cyclotron harmonic waves during quiet magnetic conditions

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Abstract. A general approach to the generation of ion cyclotron harmonic waves observed on board the Akebono satellite in the deep plasmasphere is presented. It is shown that during quiet magnetic conditions the development of the hydrodynamic cyclotron instability with growth rate $\gamma \propto n_i^{1/2}$, where n_i is the number density of the hot heavy ions, is suppressed by the field-aligned inhomogeneity of the dipole magnetic field. The instability is, in this case, controlled by the weak resonant interaction of the waves and the trapped particles with growth rate $\gamma \propto n_i$. The waves are generated by a kinetic instability involving hot helium ions with a ring-like distribution. Such ions are present in the magnetosphere during quiet magnetic conditions. A simple analytical model of this instability accounting for the inhomogeneity of the ambient magnetic field is used. It is shown that the ULF wave observations during quiet times on board the Akebono satellite are in a reasonable agreement with the present theoretical approach.

1. Introduction

Wave generation can occur in the Earth's magnetosphere, for example, when wave packets repeatedly cross amplification regions where the hot particles have unstable velocity distributions. Such a situation may appear, for example, in the case of electromagnetic ion cyclotron wave generation [*Kennel and Petschek*, 1966]. This is realized owing to the magnetic focusing of Alfvén waves and their reflection from the conjugate ionospheres at the opposite ends of the magnetic field tubes. On the contrary, magnetosonic waves do not experience magnetic focusing. Their trajectories are more complicated curves. Actually, they rapidly escape the region of resonant interaction and disappear owing to dispersion. A favorable situation may appear in a specific region in the deep plasmasphere where a transverse waveguide for magnetosonic waves exists [see, e.g., *Guglielmi and Pokhotelov*, 1996]. The waves trapped in this region may interact with the resonant particles over a long period of time, and if the distribution of the particles is unstable, the wave amplitudes will grow.

The first observations of magnetosonic waves in the transverse waveguide were provided by *Russell et al.* [1970]. Later, these findings were confirmed in a number of satellite missions [e.g., *Gurnett*, 1976; *Perraut et al.*, 1978, 1982; *Laakso et al.*, 1990; *Olsen et al.*, 1987; *Gurnett and Inan*, 1988; *Kokubun et al.*, 1991; *Liu et al.*, 1994; *Kasahara et al.*, 1992, 1994]. The generation of these waves has been interpreted in terms of a hydrodynamic cyclotron instability of the toroidal waveguide when a loss cone or ring-like population of hot ions is present [*Guglielmi et al.*, 1975; *Perraut et al.*, 1978, 1982; *Pokhotelov et al.*, 1997]. The localization of the emissions as well as their direction of propagation and polarization were found to be in a reasonable agreement with the theory of magnetosonic wave generation in a toroidal waveguide inside the plasmasphere.

It should be noted that when considering the instability mechanism, two important effects were not taken into account in the above mentioned papers. On one hand, the treatment of the instability mechanism was, for simplicity, restricted to flute perturbations, which implies that the derivative of the wave field $\mathbf{E}$ along the longitudinal coordinates (along the background magnetic field) vanishes, i.e., $d\mathbf{E}/dz = 0$. Such a simplification does not allow us to explain in a self-consistent way the polarization characteristics of the observed waves since the ratio of the amplitudes of the right-hand and left-hand modes loses its physical sense. These features may only be explained if the effects of finite $d\mathbf{E}/dz$ are taken into account while considering the instability mechanism. On the other hand, the ambient mag-

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netic field $\mathbf{B}_0$ was assumed to be homogeneous, i.e., $d\mathbf{B}_0/dz = 0$. This is certainly not the case in the real magnetosphere. In what follows we will demonstrate that under certain conditions this simplification is justified if the growth rate of the instability is sufficiently large and particles do not have enough time to feel the magnetic field inhomogeneity. Such a situation was analyzed by *Perraut et al.* [1978, 1982] using the GEOS-1 data at rather high altitudes, and it was analyzed in the deep plasmasphere during magnetic storms by *Pokhotelov et al.* [1997] when the waves are driven by hot hydrogen or oxygen ions with ring-like distributions of perpendicular velocities.

ULF emissions related to the harmonics of the helium ion gyrofrequency were observed by *Kokubun et al.* [1991] during quiet magnetic conditions. In this case the waves are subject to slow growth, and the field line inhomogeneity of the ambient magnetic field may be important. This can be seen by the following simple consideration. As we know, the wave emission is localized in the vicinity of the equatorial region, where the Earth's magnetic field $\mathbf{B}_0$ may be expanded according to the parabolic approximation $B_0 = B_{\text{eq}}[1+(9/2)(z/LR_E)^2]$, where L is the McIllwain parameter, R_E is the Earth's radius, and B_{eq} is the equatorial value of the geomagnetic field. If the wave growth is controlled by the linear growth rate γ_L, then during the time $1/\gamma_L$, a particle will be displaced a distance v_i/γ_L, where v_i stands for a characteristic velocity of the hot helium ions. The variation of the gyrofrequency during this displacement is then

$$\Delta\Omega_{\text{He}^+} = \Omega_{\text{He}^+} - \Omega^{\text{eq}}_{\text{He}^+} \simeq \frac{9}{2}\frac{\Omega^{\text{eq}}_{\text{He}^+} v_i^2}{(\gamma_L L R_E)^2}. \qquad (1)$$

The longitudinal inhomogeneity of the magnetic field can be neglected if $\gamma_L \gg \Delta\Omega_{\text{He}^+}$. For the hydrodynamic cyclotron instability $\gamma_L \propto (n_{\text{He}^+}/N_{\text{He}^+})^{1/2}$, where $n_{\text{He}^+}/N_{\text{He}^+}$ is the ratio of the hot and cold helium ion number densities. Then it follows from (1) that $\Delta\Omega_{\text{He}^+} \propto n^{-1}_{\text{He}^+}$, and thus the neglect of the longitudinal inhomogeneity of the magnetic field is justified only for relatively active magnetospheric conditions, when the density of the hot particles is sufficiently high, that is, when

$$\eta_{\text{He}^+} \equiv \frac{n^+_{\text{He}}}{N_{\text{He}^+}} \gg \eta^*_{\text{He}^+} = \frac{9}{2}\frac{r^2_{\text{He}^+}}{(LR_E)^2}\frac{v_i^2}{c^2_{AH}}\frac{m_{H^+}}{m_{\text{He}^+}}, \qquad (2)$$

where r_{He^+} stands for the ion Larmor radius of hot helium ions and $c_{AH} = B_0/(\mu_0 N_{H^+} m_{H^+})^{1/2}$ is the Alfvén velocity defined by the proton density N_{H^+} and mass m_{H^+}. For typical magnetospheric conditions in the deep plasmasphere this inequality is satisfied only for relatively disturbed magnetospheric conditions. On the contrary, at quiet times this inequality is not satisfied, and the above mentioned effects, which have been neglected in the previous studies, must be taken into account.

The paper is organized in the following way. In section 2 we derive the wave equation for magnetosonic waves propagating at large angles to the external magnetic field in a toroidal waveguide. Assuming that the transverse wavelength is much shorter than the longitudinal one, we deduce a model wave equation. Section 3 is devoted to the study of a kinetic type instability in an inhomogeneous magnetic field. The transition to the hydrodynamic limit is presented. The application of the theory to the analysis of the Akebono data is given in section 4.

2. Reduced Wave Equation for the Magnetosonic Mode

Let us now consider a low-frequency wave propagating at a large angle to the external magnetic field $\mathbf{B}_0$ in an inhomogeneous multicomponent plasma. The plasma is taken to be composed of cold hydrogen and helium ions and a small fraction of hot ions. We choose a Cartesian system of coordinates with the z axis along the ambient magnetic field and the x axis along the waveguide axis. Making use of Faraday's and Ampere's laws, we write the equation describing small oscillations of the electromagnetic field as

$$\nabla\times(\nabla\times\mathbf{E}) - (\omega^2/c^2)\mathbf{E} - i\mu_0\omega\sum_j e_j N_j \mathbf{v}_j = i\mu_0\omega\mathbf{j}, \qquad (3)$$

where $\mathbf{E}$ is the wave electric field, ω is the wave frequency, e_j, N_j, and $\mathbf{v}_j$ are the electric charge, number density, and velocity of the cold species, respectively, and μ_0 is the permeability of free space. The current density $\mathbf{j}$ on the right-hand side is induced by the hot ions.

Kokubun et al. [1991] reported results from the first eight months of Akebono data after the launch in February 1989. They observed multiband magnetosonic waves related to the helium-ion gyrofrequency and its harmonics during relatively quiet times. Following these measurements, we restrict our study to the frequency range $\Omega_{H^+} \gg \omega \geq \Omega_{\text{He}^+}$, where Ω_{H^+} and Ω_{He^+} are the proton and helium gyrofrequencies, respectively. In this approximation the electrons have an $\mathbf{E}\times\mathbf{B}_0$ drift across the ambient magnetic field and a fast motion along the magnetic field with the velocity $v_{ez} = -(ie/\omega m_e)E_z$, where m_e is the electron mass and e is the magnitude of the electronic charge. On the other hand, owing to the high mobility of the electrons, we can neglect the motion of the protons and the helium ions along the magnetic field. The velocities of the protons and the helium ions perpendicular to the magnetic field have the following standard form:

$$\begin{aligned} \mathbf{v}_{\perp H^+} &= \frac{\mathbf{E}\times\hat{\mathbf{z}}}{B_0} - i\frac{\omega}{\Omega_{H^+}}\frac{\mathbf{E}}{B_0} \\ \mathbf{v}_{\perp \text{He}^+} &= \frac{i\Omega_{\text{He}^+}}{\omega B_0}\frac{[\mathbf{E} + i\frac{\Omega_{\text{He}^+}}{\omega}(\mathbf{E}\times\hat{\mathbf{z}})]}{1-\frac{\Omega^2_{\text{He}^+}}{\omega^2}}, \end{aligned} \qquad (4)$$

where $\hat{\mathbf{z}} = \mathbf{B}_0/B_0$ is the unit vector along the geomagnetic field.

Similar to *Le Quéau et al.* [1993] and *Pokhotelov et al.* [1997], we substitute expressions (4) into (3) (thus restricting ourselves in the following to the case of two dominant ions, protons, and helium ions) and obtain

$$\frac{c^2}{\omega^2}\frac{d}{dz}\left[\frac{1}{1-\frac{n_\perp^2}{\varepsilon_\parallel}}\frac{dE_x}{dz}\right] + \varepsilon_\perp E_x + igE_y = -\frac{ij_x}{\varepsilon_0\omega} - \frac{c}{\varepsilon_0\omega^2}\frac{d}{dz}\left(\frac{n_\perp}{n_\perp^2-\varepsilon_\parallel}j_z\right), \quad (5)$$

$$\frac{c^2}{\omega^2}\frac{d^2E_y}{dz^2} + (\varepsilon_\perp - n_\perp^2)E_y - igE_x = -\frac{ij_y}{\varepsilon_0\omega}, \quad (6)$$

and

$$\frac{in_\perp c}{\omega}\frac{d}{dz}E_x + (n_\perp^2 - \varepsilon_\parallel)E_z = \frac{i}{\varepsilon_0\omega}j_z. \quad (7)$$

Here $n_\perp = k_\perp c/\omega$ is the refractive index ($\perp$ means perpendicular to $\mathbf{B}_0$), ε_0 is the permittivity of free space, and the dielectric constants $\varepsilon_\perp$, g, and $\varepsilon_\parallel$ are defined by

$$\varepsilon_\perp = \frac{c^2}{c_{AH}^2}\frac{\omega^2-\omega_{bi}^2}{\omega^2-\Omega_{\mathrm{He}^+}^2} \qquad g = -\frac{c^2}{c_{AH}^2}\frac{\omega(\omega_{co}-\Omega_{\mathrm{He}^+})}{\omega^2-\Omega_{\mathrm{He}^+}^2}$$

$$\varepsilon_\parallel = 1 - \frac{\omega_{pe}^2}{\omega^2}, \quad (8)$$

where $\omega_{bi} = \Omega_{\mathrm{He}^+}(1+\mu_{bi})^{1/2}$ is the bi-ion frequency, $\omega_{co} = \Omega_{\mathrm{He}^+}(1+\mu_{bi})$ is the cutoff frequency, $\mu_{bi} = (N_{\mathrm{He}^+}m_{\mathrm{He}^+}/N_{H^+}m_{H^+})$ is the ratio of the cold helium and proton mass densities, and $\omega_{pe} = (N_e e^2/\varepsilon_0 m_e)^{1/2}$ is the Langmuir frequency. The quasi-neutrality condition is $N_e = N_{H^+} + N_{\mathrm{He}^+} + n_{\mathrm{He}^+} + n_{H^+}$, where the hot hydrogen and helium number densities n_{H^+} and n_{He^+} are assumed to be much smaller than the densities of the cold plasma species, that is, the plasma quasineutrality is basically sustained by the cold species, $N_e \simeq N_{H^+} + N_{\mathrm{He}^+}$.

The magnetic field components of the wave can be expressed by the electric field components. For the analysis of wave polarization it is customary to introduce the right-hand $B_R = B_x - iB_y$ and left-hand $B_L = B_x + iB_y$ components, which are connected with electric field components by the relations

$$B_R = \frac{1}{\omega}\frac{d(E_x - iE_y)}{dz} + \frac{in_\perp}{c}E_z$$

$$B_L = \frac{1}{\omega}\frac{d(E_x + iE_y)}{dz} - \frac{in_\perp}{c}E_z. \quad (9)$$

A further simplification can be made owing to the high mobility of electrons along the field line resulting in the condition $n_\perp^2/\varepsilon_\parallel \to 0$. Using this fact, we can neglect the contribution of the field-aligned current induced by the hot particles in (5). Owing to the same reason, the contribution of the field-aligned electric field in (9) is small.

The experimental data show that the wave vector of the magnetosonic mode has an average angle of about 70° to the ambient magnetic field for most cases [*Kokubun et al.*, 1991; *Kasahara et al.*, 1994; *Liu et al.*, 1994]. Thus the wave equations (5)–(7) may be further simplified considering $\delta = (c/\omega n_\perp) d\ln E/dz$ as a small parameter, $\delta \ll 1$. Expanding these equations in powers of δ, we thus find in leading order

$$(2\varepsilon_\perp - n_\perp^2)\frac{c^2}{\omega^2}\frac{d^2E_x}{dz^2} + [\varepsilon_\perp(\varepsilon_\perp - n_\perp^2) - g^2]E_x = \frac{g}{\varepsilon_0\varepsilon_\perp\omega}(\varepsilon_\perp j_y + igj_x), \quad (10)$$

where derivatives of higher order have been neglected. Flute perturbations ($d/dz = 0$) in a cold plasma (when $j_{x,y} = 0$) yield the well-known dispersion relation for the extraordinary mode [*Pokhotelov et al.*, 1997]:

$$D(\omega) = n_\perp^2 - \frac{c^2}{c_{AH}^2}\frac{\omega^2-\omega_{co}^2}{\omega^2-\omega_{bi}^2} = 0.$$

When deriving (10), it was assumed that the derivatives of perturbations are greater than those of unperturbed values: $\partial\ln E_x/\partial z \gg (\partial\ln B_0/\partial z, \partial\ln N/\partial z)$.

Combining this inequality with the condition $\delta \ll 1$, we find that it is necessary that $k_\perp \gg k_\parallel \gg (LR_E)^{-1}$. This gives the validity condition for the "straight-field line approximation" that will be used here.

3. Evaluation of the Growth Rate

Let us start with the standard expression for the perturbed distribution function,

$$f_i = -\frac{e}{m_i}\int(\mathbf{E} + \mathbf{v}\times\mathbf{B})\frac{\partial F}{\partial\mathbf{v}}dt', \quad (11)$$

where $\mathbf{E}$ and $\mathbf{B}$ are the perturbations of the electric and magnetic fields, F is the equilibrium distribution function for the hot helium ions, and the integration is carried out along the trajectories of the particles.

Integrating (11) by a method described by *Kaladze et al.* [1976], we obtain

$$f_i = \frac{e}{m_i}\sum_{n=-\infty}^{\infty}\exp(i\xi\sin\alpha - in\alpha) \times \int_{-\infty}^{t} dt' S_n(t,t')\frac{\partial F}{\partial v_\perp}\left(\frac{nJ_n}{\xi}E_x + iJ_n' E_y\right), \quad (12)$$

where $S_n(t,t')$ is defined by

$$S_n(t,t') = \exp[i\omega(t-t') - in\int_{t'}^{t}\Omega_{\mathrm{He}^+}(t'')dt'']. \quad (13)$$

Here J_n is the Bessel function, $\xi = k_\perp v_\perp/\Omega_{\mathrm{He}^+}$, α is the pitch angle between the velocity vector and the ambient magnetic field, and the prime denotes the derivative with respect to ξ.

We then write the currents as follows:

$$j_x = -\frac{e^2}{m_{\mathrm{He}^+}} \sum_n \int_{-\infty}^{t} \int_0^{\infty} v_\perp^2 dv_\perp$$

$$\times \int_{-\infty}^{\infty} dv_\parallel\, S_n(t,t') \frac{\partial F}{\partial v_\perp} \frac{nJ_n}{\xi} \left(\frac{nJ_n}{\xi} - \frac{\varepsilon_\perp}{g} J_n' \right) E_x(t'), \tag{14}$$

and

$$j_y = \frac{ie^2}{m_{\mathrm{He}^+}} \sum_n \int_{-\infty}^{t} \int_0^{\infty} v_\perp^2 dv_\perp$$

$$\times \int_{-\infty}^{\infty} dv_\parallel\, S_n(t,t') \frac{\partial F}{\partial v_\perp} J_n' \left(\frac{nJ_n}{\xi} - \frac{\varepsilon_\perp}{g} J_n' \right) E_x(t'). \tag{15}$$

Substituting (14) and (15) into (10), we obtain

$$-\left(1 + \frac{g^2}{\varepsilon_\perp^2}\right) \frac{c^2}{\omega^2} \frac{d^2 E_x}{dz^2} + D(\omega) E_x =$$

$$-\frac{ig^2 e^2}{\varepsilon_0 \omega \varepsilon_\perp^2 m_{\mathrm{He}^+}} \sum_n \int_{-\infty}^{t} \int_0^{\infty} v_\perp^2 dv_\perp$$

$$\times \int_{-\infty}^{\infty} dv_\parallel\, S_n(t,t') \frac{\partial F}{\partial v_\perp} \left(\frac{nJ_n}{\xi} - \frac{\varepsilon_\perp}{g} J_n' \right)^2 E_x(t'). \tag{16}$$

If the magnetic field is homogeneous, then (13) reduces to

$$S_n(t,t') = \exp[i(\omega - in\Omega_{\mathrm{He}^+})(t - t')]. \tag{17}$$

Substituting this to (16), we finally have

$$D(\omega) = \frac{n_{\mathrm{He}^+} c^2}{N_{\mathrm{He}^+}} \frac{\Omega_{\mathrm{He}^+}^3 n^2 (\omega_{co} - \Omega_{\mathrm{He}^+})(\omega^2 - \omega_{bi}^2)}{\omega^3 (\omega - n\Omega_{\mathrm{He}^+})(\omega^2 - \omega_{co}^2)}$$

$$\times \int dv_\perp dv_\parallel \left(\frac{\xi J_n'}{n} - \frac{g}{\varepsilon_\perp} J_n \right)^2 \frac{\partial F}{\partial v_\perp}. \tag{18}$$

Equation (18) coincides with the corresponding expression by *Pokhotelov et al.* [1997] that was calculated for the case of hot oxygen ions. Near the helium harmonics $\omega \approx n\Omega_{\mathrm{He}^+}$ we find the following approximate estimate for the growth rate:

$$\gamma_L \simeq \left(\frac{n_{\mathrm{He}^+}}{N_{H^+}} \right)^{1/2} \frac{c_{AH}}{v_T} \left(\frac{m_{\mathrm{He}^+}}{m_{H^+}} \right)^{1/2} \Omega_{\mathrm{He}^+}, \tag{19}$$

where v_T is the thermal velocity of hot helium ions.

In a longitudinally inhomogeneous magnetic field it is customary to introduce the energy $\varepsilon = v^2/2$ and magnetic moment $\mu = \mathbf{v}^2/2B_0$ ($\mathbf{v}$ is the particle velocity), which are integrals of motion.

Moreover, we can change to an integration over energies and magnetic moments according to the relation

$$\int_0^{\infty} v_\perp dv_\perp \int_{-\infty}^{-\infty} dv_\parallel = \sum_\sigma B_0 \int \frac{d\varepsilon d\mu}{v_\parallel}. \tag{20}$$

Here $v_\parallel$ means the modulus of the parallel velocity, and we have introduced the index σ, which has the value $+1$ for particles moving in the positive direction along the field line and -1 for those moving in the opposite direction. When calculating the right-hand side of the wave equation, we must sum over σ.

In a longitudinally inhomogeneous magnetic field, particles are trapped between the mirror points and bounce with a period $\tau = \oint dz/v_\parallel$, where $v_\parallel = [2(\varepsilon - \mu B_0)]^{1/2}$. Owing to this periodicity, we can make a transition from the infinite integral in (16) to a finite integral over one bounce motion, using the following considerations. For this purpose the integral in (16) may be represented as a sum of integrals where each integral is taken along the time during one bounce; that is,

$$\int_{-\infty}^{t} \{...\} dt' = \int_{t-\tau}^{t} \{...\} dt' + \int_{t-2\tau}^{t-\tau} \{...\} dt' + ...$$

$$+ \int_{t-(n+1)\tau}^{t-n\tau} \{...\} dt' + ... \tag{21}$$

Making the substitution $t' = t'' - n\tau$ in each nth integral and taking into account that

$$S_n(t, t' - n\tau) = S_n(t,t') \exp[in \int_0^{\tau} (\omega - n\Omega_i) dt],$$

we then obtain

$$\int_{-\infty}^{t} \{...\} dt' = (1 + \exp i \int_0^{\tau} (\omega - n\Omega_i) dt$$

$$+ \exp 2i \int_0^{\tau} (\omega - n\Omega_i) dt + ...) \int_{t-\tau}^{t} \{...\} dt'$$

$$\equiv [1 - \exp i \int_0^{\tau} (\omega - n\Omega_i) dt]^{-1} \int_{t-\tau}^{t} \{...\} dt'. \tag{22}$$

The integral over t' on the right-hand side can be transformed as follows:

$$\int_{t-\tau}^{t} \{..\} dt' = \int_{t-\tau}^{-\tau/2} \{..\} dt' + \int_{-\tau/2}^{0} \{..\} dt' + \int_0^{t} \{..\} dt' =$$

$$i\{\exp[-i\sigma M(z_1, z)] \frac{1}{\sin M(z_1, z_2)} \int_{z_1}^{z_2} \{..\} \cos M(z_2, z') \frac{dz'}{v_\parallel}$$

$$+ i\sigma \int_{z_1}^{z} \frac{dz'}{v_\parallel} \{..\} \exp[i\sigma M(z', z)]\} [1 - \exp i \int_0^{\tau} (\omega - n\Omega_i) dt], \tag{23}$$

where $M(z_1, z_2) \equiv \int_{z_1}^{z_2} (\omega - n\Omega_i) dz/v_\parallel$. The mirror points z_1 and z_2 are defined from the equation $v_\parallel = 0$. Here we used the fact that (because the number of particles moving in both directions are equal)

$$E_x[z(t)] = E_x[z(\tau - t)]. \qquad (24)$$

We then obtain

$$D(\omega)E_x - C(\omega)\frac{d^2E_x}{dz^2} = \frac{2e^2}{\varepsilon_0 \omega m_{\mathrm{He}^+}} \sum_n \frac{n^2\Omega^2_{\mathrm{He}^+}}{k_\perp^2} \int\int \frac{d\varepsilon d\mu}{v_\parallel} G \left(\frac{\xi J_n'}{n} - \frac{g}{\varepsilon_\perp} J_n \right)^2$$

$$\times \left[\frac{\cos M(z_1, z)}{\sin M(z_1, z_2)} \int_{z_1}^{z_2} \frac{dz'}{v_\parallel} E_x(z') \cos M(z', z_2) - \int_{z_1}^{z} \frac{dz'}{v_\parallel} E_x(z') \sin M(z', z) \right], \qquad (25)$$

where $C(\omega) = c^2/\omega^2(1 + g^2/\varepsilon_\perp^2)$ and $G = \partial F/\partial\varepsilon + B_0^{-1}\partial F/\partial\mu$. It follows from (25) that the perturbation of the current is related to the perturbation of the electric field, although not locally as in the case of a longitudinally homogeneous magnetic field. It is therefore now quite difficult to find the eigenfrequencies of the oscillations. However, to establish the instability conditions, one does not need to know the frequency but only the sign of its imaginary part. In our case this can be found by means of integral relations. This situation is to a certain extent analogous to that used in the analysis of MHD instabilities. We can now take into account the processes of resonant interaction between particles and waves, and we must therefore introduce a complex functional. The real part of this functional corresponds to the total energy of the oscillations, and the imaginary part characterizes the energy balance between the wave and the resonant particles.

Integrating (25) over z, we obtain the integral equation

$$D(\omega) \int |E_x|^2 dz + C(\omega) \int \left| \frac{dE_x}{dz} \right|^2 dz = \frac{2}{\varepsilon_0 \omega k_\perp^2} \sum_n \frac{n^2(\Omega_{\mathrm{He}^+})^2}{m_{\mathrm{He}^+}} \int B_0 d\varepsilon d\mu G \left(\frac{\xi J_n'}{n} - \frac{g}{\varepsilon_\perp} J_n \right)^2$$

$$\times \left[\cot M(z_1, z_2) \left| \int_{z_1}^{z_2} \frac{dz'}{v_\parallel} E_x(z') \cos M(z', z_2) \right|^2 + \int_{z_1}^{z_2} \frac{dz'}{v_\parallel} E_x(z') \cos M(z', z_2) \int_{z_1}^{z_2} \frac{dz'}{v_\parallel} E_x^*(z') \sin M(z', z_2) - \int_{z_1}^{z_2} dz \int_{z_1}^{z} dz' E_x(z) E_x^*(z') \frac{\sin M(z', z)}{v_\parallel(z) v_\parallel(z')} \right], \qquad (26)$$

where the slowly varying values have been replaced by their equatorial ones whenever possible (the corresponding superscripts are omitted). In (26) the values of the dielectric constants and the arguments of the Bessel functions are also taken at the equator. In order to reveal the effects of the resonant interaction of the wave with the particles, we use the expansion

$$\cot M(z_1, z_2) = \frac{\omega_b}{\pi} \sum_{k=-\infty}^{\infty} \frac{1}{\omega - n\bar{\Omega}_{\mathrm{He}^+} - k\omega_b}, \qquad (27)$$

where $\omega_b = 2\pi/\tau$ stands for the bounce frequency. In (27) the denominator $1/(\omega - n\bar{\Omega}_{\mathrm{He}^+} - k\omega_b)$ must be interpreted in the Landau sense; that is,

$$\lim_{\nu \to 0} \frac{1}{\omega - n\bar{\Omega}_{\mathrm{He}^+} - k\omega_b} = P\left[\frac{1}{\omega - n\bar{\Omega}_{\mathrm{He}^+} - k\omega_b} \right] - i\pi\delta(\omega - n\bar{\Omega}_{\mathrm{He}^+} - k\omega_b), \qquad (28)$$

where P denotes the Cauchy principal value and the bar stands for the average value of the bounce motion:

$$\bar{\Omega}_{\mathrm{He}^+} \equiv \frac{\oint \frac{dz}{v_\parallel} \Omega_{\mathrm{He}^+}(z)}{\oint \frac{dz}{v_\parallel}}. \qquad (29)$$

Now supposing $\omega = n\Omega_{\mathrm{He}^+} + i\gamma_L$, where the helium gyrofrequency is taken at the equator, and using the expansions

$$D(\omega) \approx D(\omega)_{\omega = n\Omega_{\mathrm{He}^+}} + (\omega - n\Omega_{\mathrm{He}^+}) \left(\frac{\partial D}{\partial \omega} \right)_{\omega = n\Omega_{\mathrm{He}^+}} \qquad (30)$$

$$C(\omega) \approx C(\omega)_{\omega = n\Omega_{\mathrm{He}^+}} + (\omega - n\Omega_{\mathrm{He}^+}) \left(\frac{\partial C}{\partial \omega} \right)_{\omega = n\Omega_{\mathrm{He}^+}} \qquad (31)$$

we may obtain two equations which define the real and imaginary parts of (26). For the real part we have

$$D(n\Omega_{\mathrm{He}^+}) \int |E_x|^2 dz + C(n\Omega_{\mathrm{He}^+}) \int \left| \frac{dE_x}{dz} \right|^2 dz \approx 0, \qquad (32)$$

where the corresponding contributions from the hot particles are neglected (they are much smaller than the contributions of a cold plasma).

Calculation of the imaginary part of (26) gives the expression of the growth rate of the instability,

$$\gamma_L/\omega = -\frac{2c^2(\omega_{co} - \Omega_{\mathrm{He}^+})(n^2\Omega^2_{\mathrm{He}^+} - \omega_{bi}^2)}{N_{\mathrm{He}^+} n^2 (n^2\Omega^2_{\mathrm{He}^+} - \omega_{co}^2)} \times \frac{1}{\frac{\partial D}{\partial \omega} \int |E_x|^2 dz + \partial D(\omega)/\partial\omega \int |d^2E_x/d^2z|^2 dz]} \times \sum_{k=-\infty}^{\infty} \int d\mu d\varepsilon B_0 \omega_b G \left(\frac{\xi J_n'}{n} - \frac{g}{\varepsilon_\perp} J_n \right)^2$$

$$\left| \int_{z_1}^{z_2} \frac{dz'}{v_\parallel} E_x(z') \cos M(z', z_2) \right|^2 \delta(\omega - n\bar{\Omega}_{\mathrm{He}^+} - k\omega_b), \qquad (33)$$

where

$$(\omega\partial D/\partial\omega)_{\omega=n\Omega_{He^+}} = -2\frac{c^2}{c_{AH}^2}\left[1 + \frac{\omega_{bi}^2(\omega_{co}^2 - \omega_{bi}^2)}{(n^2\Omega_{He^+}^2 - \omega_{bi}^2)^2}\right], \quad (34)$$

and

$$(\omega\partial C/\partial\omega)_{\omega=n\Omega_{He^+}} = -2\frac{c^2}{\omega^2}\left[1 + 2\frac{(\omega_{co}^2 - \omega_{bi}^2)n^4\Omega_{He^+}^4}{(n^2\Omega_{He^+}^2 - \omega_{bi}^2)^3}\right]. \quad (35)$$

It should be noted that only the terms on the right-hand side of (26), which contain the delta functions, contribute to the imaginary part of (26). The expression for the growth rate may be further simplified if one compares the first and second terms in the square brackets in the denominator of (33). Their ratio is of the order of $1/\delta^2 \gg 1$. Thus we can neglect the contribution of the second term in this expression, and the growth rate may be reduced to the form

$$\gamma_L/\omega \approx -\frac{2c^2(\omega_{co} - \Omega_{He^+})(n^2\Omega_{He^+}^2 - \omega_{bi}^2)}{N_{He^+}n^2(n^2\Omega_{He^+}^2 - \omega_{co}^2)\partial D(\omega)/\partial\omega \int |E_x|^2 dz} \times \sum_{k=-\infty}^{\infty} \int d\mu d\varepsilon B_0\omega_b G\left(\frac{\xi J_n'}{n} - \frac{g}{\varepsilon_\perp}J_n\right)^2 \times \left|\int_{z_1}^{z_2} \frac{dz'}{v_\parallel}E_x(z')\cos M(z', z_2)\right|^2 \times\delta(\omega - n\bar{\Omega}_{He^+} - k\omega_b), \quad (36)$$

where $\omega\partial D(\omega)/\partial\omega < 0$.

From the expression for the growth rate (36) it follows that similarly to the case with disturbed conditions, there is a possibility for the growth of both the super-Alfvénic and the sub-Alfvénic branches. However, in contrast to the case of the hydrodynamic instability the growth rate (36) is defined by the interaction of the wave with the resonant particles due to the resonant condition $\omega = n\bar{\Omega}_{He^+} + k\omega_b$. The growth rate of this kinetic instability is proportional to the relative density of hot and cold ions, and it is much weaker than that in the case of hydrodynamic instability.

Now let us consider the necessary conditions for the instability. It follows from (36) that the sign of the growth rate depends on the sign of the integral I,

$$I = \sum_{k=-\infty}^{\infty} \int d\mu d\varepsilon B_0\omega_b G\left(\frac{\xi J_n'}{n} - \frac{g}{\varepsilon_\perp}J_n\right)^2 \times \left|\int_{z_1}^{z_2} \frac{dz'}{v_\parallel}E_x(z')\cos M(z', z_2)\right|^2 \delta(\omega - n\bar{\Omega}_{He^+} - k\omega_b). \quad (37)$$

In order to obtain the qualitative estimation of the growth rate we note that in the region of the wave localization we can use the parabolic approximation of the external magnetic field. In this region the magnetic field may be represented in the form

$$B_0 = B_{eq}\left[1 + \frac{9}{2}\left(\frac{z}{LR_E}\right)^2\right], \quad (38)$$

where B_{eq} corresponds to the equatorial value of the Earth's magnetic field.

Using (38), we may calculate ω_b and $M(z, z_2)$,

$$\omega_b = \frac{3\sqrt{\mu B_{eq}}}{LR_E}, \quad (39)$$

$$M(z, z_2) = \frac{\omega - n\Omega_{He^+}^{eq}}{\omega_b}\left(\frac{\pi}{2} - \arcsin\frac{z}{z_2}\right) - \frac{9n\Omega_{He^+}^{eq}}{2\omega_b}\left(\frac{z_2}{LR_E}\right)^2 \times\left[\frac{\pi}{2} - \arcsin\frac{z}{z_2} + \frac{z}{z_2}\sqrt{1 - \frac{z^2}{z_2^2}}\right], \quad (40)$$

where the particle mirror points $z_{2,1}$ are determined by $z_{2,1} = \pm(\sqrt{2}LR_E/3)\sqrt{(\varepsilon - \mu B_{eq})/\mu B_{eq}}$.

For simplicity we consider the case when the helium ions are localized in the vicinity of the top of the field line. This corresponds to the case of strongly trapped particles, i.e., $v_\parallel \ll v_\perp$. Substituting (39) and (40) in (37), we obtain the expression for I in the form

$$I = |E_x^{eq}|^2 B_{eq} \sum_{p=-\infty}^{\infty} \int d\mu d\varepsilon\omega_b G_{eq}\left(\frac{\xi J_n'}{n} - \frac{g}{\varepsilon_\perp}J_n\right)^2 \times J_p^2(n\lambda)\delta(\omega - n\bar{\Omega}_{He^+} - 2p\omega_b), \quad (41)$$

where

$$\lambda = \frac{\Omega_{He^+}^{eq}}{4\omega_b}\frac{\varepsilon - \mu B_{eq}}{\mu B_{eq}}. \quad (42)$$

From (41) it follows that only even bounce resonances give contributions in (41).

For the occurrence of the instability it is necessary that $I > 0$. Such a situation may appear in cases of "bump-on-tail," "loss cone," or "ring-like" distributions of the hot helium ions. All these distributions have necessary free energy for the excitation of waves. Because we do not know the actual form of the distribution function, we may choose the simplest case of ring-like distribution of helium ions,

$$F = \frac{n_{He^+}\delta(v - v_0)}{2v_0^2}, \quad (43)$$

where $v = \sqrt{2\varepsilon}$ and v_0 is the helium ion "ring" velocity. It should be noted that these ring-like distribution functions for magnetospheric ions were systematically observed when magnetosonic waves were detected [*Perraut et al.*, 1982]. Then the expression for I reads

$$I = -\frac{n_{\mathrm{He}^+}\pi^2 \mid E_x^{eq} \mid^2 \Lambda^2}{2n\Omega_{\mathrm{He}^+}^{eq} v_0^2} \sum_p \left[J_p^2(n\lambda_p) \frac{\partial}{\partial \xi_0} \xi_0 \left(\frac{\xi_0 J_n'}{n} - \frac{g}{\varepsilon_\perp} J_n \right)^2 - 2n\lambda_0 J_p(n\lambda_p) J_p'(n\lambda_p) \left(\frac{\xi_0 J_n'}{n} - \frac{g}{\varepsilon_\perp} J_n \right)^2 \right]. \quad (44)$$

and

$$\lambda_p = \frac{\omega - n\Omega_{\mathrm{He}^+}^{eq} - 2p\omega_b}{2n\omega_b}, \quad \xi_0 = \frac{k_\perp v_0}{\Omega_{\mathrm{He}^+}^{eq}}, \quad \Lambda = LR_E \frac{\sqrt{2}}{3}. \quad (45)$$

Let us consider the excitation of waves with the frequencies equal to the harmonics of gyrofrequencies of strongly trapped helium ions, $\omega = n\Omega_{\mathrm{He}^+}^{eq}$. In this case, (44) reduces to

$$I = -\frac{n_{\mathrm{He}^+}\pi^2 \mid E_x^{eq} \mid^2 \Lambda^2}{2n\Omega_{\mathrm{He}^+}^{eq} v_0^2} [1+\Theta_b] \frac{\partial}{\partial \xi_0} \xi_0 \left(\frac{\xi J_n'}{n} - \frac{g}{\varepsilon_\perp} J_n \right)^2 \quad (46)$$

where $\Theta_b \equiv \sum_{p=1}^{p_{\max}} J_p^2(p)$ and the value of $p_{\max}$ is determined from the resonant condition and equals $p_{\max} = n\Omega_{\mathrm{He}^+}^{eq}/2\omega_b$. For the actual conditions this value is large. Thus we have to retain a large number of terms in the sum over p.

For the occurrence of the instability it is necessary that $I > 0$. From (46) one may see that the growth rate is negative for small arguments, $\xi_0 = k_\perp v/\Omega_{\mathrm{He}^+}^{eq} \ll 1$, and becomes positive at sufficiently high ring velocity v_0. Then it becomes oscillating for large ξ_0. These oscillations reflect interferences between Bessel functions. We consider them as unphysical because they would not appear with smooth distribution functions. In order to obtain an explicit expression for a marginal instability, let us consider the limiting case of a high n numbers when $n \gg 1$. In this case the expression (46) reduces to

$$I = -\frac{n_{H_e^+}\pi^2 \mid E_x^{eq} \mid^2 \Lambda^2 g^2}{2n\Omega_{\mathrm{He}^+}^{eq} v_0^2 \varepsilon_\perp^2} [1+\Theta_b](J_n^2 + 2\xi_0 J_n J_n'), \quad (47)$$

The instability occurs when $J_n^2 + 2\xi_0 J_n J_n' < 0$.

Now we analyze the role of bounce resonances. For that purpose we calculate Θ_b, taking into account that in the sum over p the terms with $p \gg 1$ give the principal contribution. Making use of asymptotic representation of Bessel function $J_m(x)$ for large values of index and argument, i.e., $m \gg 1$ and $x \gg 1$ [e.g., *Gradshteyn and Ryzhik*, 1980],

$$J_m(x) = \left(\frac{2}{x}\right)^{1/3} Ai \left[\left(\frac{2}{x}\right)^{1/3} (m - x) \right], \quad (48)$$

where $Ai(x)$ is the Airy function of the first kind defined as $Ai(x) = \pi^{-1} \int_0^\infty \cos(t^3/3 + xt)$. Using the formula [*Gradshteyn and Ryzhik*, 1980],

$$\int_0^\infty x^{\mu-1} \cos(ax) dx = \frac{\Gamma(\mu)}{a\mu} \cos\left(\frac{\mu\pi}{2}\right), \quad (49)$$

we find that

$$J_m(m) \simeq \frac{\Gamma(1/3) 2^{-2/3}}{\pi 3^{1/6} m^{1/3}} \qquad m \gg 1. \quad (50)$$

Replacing in Θ_b the sum by integration over p, we finally obtain $\Theta_b \simeq \left(n\Omega_{\mathrm{He}^+}^{eq}/\omega_b\right)^{1/3}$. Since for real conditions in the Earth's magnetosphere, $\Omega_{\mathrm{He}^+}^{eq}/\omega_b \simeq LR_E/\rho_{\mathrm{He}^+} \simeq 10^2$, the account of bounce resonances leads to the amplification of the growth rate by a factor of 5.

Combining (36) and (47), we now obtain the following estimate for the growth rate of the instability:

$$\gamma/\omega \simeq \frac{n_{\mathrm{He}^+}}{N_{\mathrm{He}^+}} \frac{c_{AH}^2}{v_0^2} \left(\frac{n\Omega_{\mathrm{He}^+}}{\omega_b}\right)^{1/3}. \quad (51)$$

The factor $(n\Omega_{\mathrm{He}^+}/\omega_b)^{1/3}$ arises owing to the influence of the bounce effects, and it leads to a slight amplification of the growth rate by a factor of 5. Certainly, it would be desirable to present a complete analysis of the growth rate (36) in some real geometry, for example, in dipole approximation of the geomagnetic field. In order to provide a proper analysis of the growth rate of the instability in dipole geometry, it is better to use numerical methods. However, this is an elaborate problem and will be presented in a separate paper.

4. Discussion and Conclusions

The analysis presented above shows that the theory of the cyclotron instability during quiet conditions must be corrected by taking into account the longitudinal inhomogeneity of the geomagnetic field. Under quiet conditions there exists a weaker kinetic instability with a growth rate proportional to the ratio of hot and cold ion densities. The growth of the waves arises owing to the existence of bounce cyclotron resonances with trapped particles. The bounce resonances lead to an additional increase in the wave growth of the order of $(n\Omega_{\mathrm{He}^+}/\omega_b)^{1/3}$. However, this additional increase does not turn out to be very large for the actual conditions in the deep plasmasphere.

The main aim of this paper was to formulate the general criteria for the generation of ULF waves with discrete spectrum in the deep plasmasphere. Certainly, much work has to be done, for example, by using numerical methods in order to obtain a more quantitative agreement with the observations. Another problem may be connected with the possible spreading of the resonance peaks in the vicinity of helium gyrofrequencies due to thermal effects of the background plasma and due to the dependence of the bounce frequency on energy and pitch angle. However, the corresponding frequency shift caused by the thermal effects is much narrower than the width of the resonance peak and thus can be neglected. (This can be proven similarly as in the work of *Guglielmi et al.* [1975].) Also, the broadening of emission lines caused by the corresponding integration over phase space does not affect much since the ratio of the helium gyrofrequency to the bounce frequency is large, $\Omega^{eq}_{\mathrm{He}^+}/\omega_b \approx LR_E/r_{\mathrm{He}^+} \approx 10^2$, under realistic magnetospheric conditions.

When deriving the expression for the growth rate (36), it was assumed that the wave growth is slower than the typical time of bounce motion, i.e., $\gamma_L \ll \omega_b$. According to the estimation (51), we have $\gamma_L/\omega_b \simeq n^{4/3}(r_{\mathrm{He}^+}/LR_E)^{2/3}$, where for the ratio $n_{\mathrm{He}^+}/N_{\mathrm{He}^+}$ we have substituted its boundary value $\eta^*_{\mathrm{He}^+}$ from the expression (2). Thus the necessary inequality follows if $r_{\mathrm{He}^+}/LR_E \ll 1/n^2$, which is certainly satisfied for the actual observations since according to *Kokubun et al.* [1991], the observed harmonic numbers are $n \leq 10$ in most cases.

The ULF emission observations by *Kokubun et al.* [1991] during relatively quiet magnetic conditions demonstrated their close correlation with the He^+ gyrofrequency and its harmonics. According to our model, these oscillations can result from a loss cone or ring-like resonant instability with $\gamma_L \propto n_{\mathrm{He}^+}$, leading to the excitation of ULF emissions near the helium ion cyclotron frequency and its harmonics. During intense magnetic storms, a high density of ionospheric O^+ ions is observed, and the minimum value of the *Dst* index shows a remarkable correlation with $\eta_{O^+} \equiv n_{O^+}/N_{O^+}$. The latter can attain values as large as 70° during intense storms [*Thorne and Horne,* 1997]. A change to a stronger hydrodynamic instability with $\gamma \propto (n_{O^+})^{1/2}$ is then possible. These instabilities may generate both sub-Alfvénic and super-Alfvénic waves near the helium/oxygen gyrofrequency harmonics. A typical example of multiband emissions observed during a great storm on March 24, 1991, was presented by *Liu et al.* [1994]. The waves experience complex nonlinear interactions, which can be studied by means of a detailed data analysis. However, it is important to note that the super-Alfvénic and sub-Alfvénic modes have a different nonlinear behavior. This can be understood by noting that the conditions for three-wave interaction ($\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3$ and $\omega_1 + \omega_2 = \omega_3$) are automatically satisfied for the sub-Alfvénic mode but not for the super-Alfvénic mode. This leads to a different scenario for the wave energy cascading in different frequency ranges. On one hand, above the cutoff frequency the nonlinear evolution is controlled only by the induced scattering of waves with thermal plasma. Owing to the conservation of the total energy, this leads to the accumulation of wave energy in the low harmonics as was observed in the Akebono data. On the other hand, below the bi-ion frequency the waves are subject to a parametric decay instability.

One of the important features of observations is wave polarization. This was studied by analyzing the $\mid B_R \mid / \mid B_L \mid$ ratio. From (9) it follows that this ratio is larger than unity for the super-Alfvénic mode and less than unity for the sub-Alfvénic mode. The magnetosonic waves registered by *Kokubun et al.* [1991] during quiet conditions generally satisfied the condition $\mid B_R \mid \geq \mid B_L \mid$. However, in some cases it was noted that $\mid B_L \mid$ was larger than $\mid B_R \mid$. Thus the probability for the appearance of a super-Alfvénic mode was more favorable.

It would be important to provide model calculations of the growth rate (36) with the specific distribution functions obtained from the actual satellite data. Unfortunately, the low-energy particle (LEP) instrument of Exos D [*Mukai et al.*, 1990] was not operated at low latitudes because of radiation problems. Moreover, it is yet unclear by which mechanism the heavy ions, for example, the oxygen or helium ions, are accelerated to ring current energies during enhanced magnetic activity. They may be accelerated by perpendicular heating, which leads to the generation of ion conics [*Sharp et al.*, 1977; *Gorney et al.*, 1981; *Le Quéau et al.*, 1993] or by a potential drop between the ionosphere and the magnetosphere [*Shelley et al.*, 1976; *Peterson et al.*, 1993]. An alternative mechanism proposed recently by *Thorne and Horne* [1994] involves the cyclotron resonant interaction with intense electromagnetic ion cyclotron waves.

The most complete analysis of ELF emissions with discrete spectrum observed on board the Akebono satellite was offered by *Kasahara et al.* [1992, 1994]. They have examined ELF wave data of 221 orbits tracked at Kagoshima Space Center in Japan from April 1989 to October 1990. The ELF emissions were classified into two types: One (type A) is the ordinary ion cyclotron wave below the local proton cyclotron frequency with a clear cutoff above the local cyclotron frequencies of heavy ions such as helium or oxygen, and the other (type B) was assumed to be a magnetosonic wave observed also above the local ion cyclotron frequencies. The subject of our study corresponds to the type B emission.

Kasahara et al. [1992, 1994] estimated the refractive index of magnetosonic waves by using one component of the electric field observed by a 60-m tip-to-tip wire antenna and three components of the magnetic field observed by three orthogonal search coil antennas. In general, they found that the refractive index for type B emission was between 100 and 300, which fits well with our assumptions. The waves were found to propagate in the azimuthal direction close to the equatorial plane, as was assumed in our study. In addition, *Kasahara et al.* [1994] made a three-dimensional ray-tracing study in a cold plasma model including the effect of multicomponent ions. They suggested that the propagation perpendicular to the geomagnetic meridian plane is possible as the result of density gradient such as plasmapause. Statistical analysis presented by *Kasahara et al.* [1994] shows that emissions under study appear both at low- and high-K_p indices, supporting the idea that wave generation may occur both during quiet and disturbed conditions.

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