μ BRANCHING RATIOS

$\mu^- \to (e^- \overline{\nu}_e \nu_\mu \gamma)$/total (P2)

```
R1       27 EVENTS SEEN                       ASHKIN    59 CNTR
R1            1.4E-2   0.4E-2                 CRITTENDE 61 CNTR T(GAM) GT 10 MEV
R1           (3.3E-3) (1.3E-3)                CRITTENDE 61 CNTR T(GAM) GT 20 MEV
R1      862 EVENTS SEEN                       BOGART    67 CNTR T(GAM) GT 14.5 MEV
```

$\mu^+ \to (e^+ \overline{\nu}_e \nu_\mu)$/total (P3)

```
R2       FORBIDDEN BY ADDITIVE CONSERVATION LAW FOR LEPTON FAMILY NUMBER.
R2       MULTIPLICATIVE LAW PREDICTS THIS BRANCHING RATIO TO BE 1/2.
R2       FOR A REVIEW SEE NEMETHY 81.
R2            (0.25) OR LESS  CL=.90   EICHTEN   73 HLBC +
R2            (0.13)   (0.15)          BLIETSCHA 78 HLBC +- AVG. OF 4 VALUES
R2            (0.09) OR LESS  CL=.90   JONKER    80 CALO REPL. BY BERGSMA 83
R2           (-0.001)  (0.061)         WILLIS    80 CNTR +
R2    A        0.05  OR LESS  CL=.90   BERGSMA   83 CALO ANUMU E -->MU- ANUE
R2    A  BERGSMA 83 GIVES LIMIT ON INVERSE MUON DECAY CROSS SECTION RATIO
R2    A  SIG(NUBAR(MU) E- --> MU- NUBAR(E)) / SIG(NU(MU) E- --> MU- NU(E)),
R2    A  WHICH IS ESSENTIALLY EQUIVALENT TO R2 FOR SMALL VALUES LIKE THAT
R2    A  QUOTED.
```

$\mu \to (e\gamma)$/total (units 10^{-8}) (P4)

```
R3       FORBIDDEN BY LEPTON FAMILY NUMBER CONSERVATION
R3            (4.3)  OR LESS  CL=.90   FRANKEL1  63 OSPK
R3            (2.2)  OR LESS  CL=.90   PARKER    64 OSPK
R3            (2.9)  OR LESS  CL=.90   KORENCH1  71 OSPK +  DUBNA
R3            (0.10) OR LESS  CL=.90   SCHAAF    80 ELEC +  SIN
R3             0.017 OR LESS  CL=.90   KINNISON  82 SPEC +  LAMPF
R3            (0.10) OR LESS  CL=.90   AZUELOS   83 CNTR +
```

$\mu \to (3e)$/total (units 10^{-10}) (P5)

```
R4       FORBIDDEN BY LEPTON FAMILY NUMBER CONSERVATION
R4 F      (5000.)    OR LESS  CL=.90   PARKER    62 CNTR
R4 F      (1300.)    OR LESS  CL=.90   ALIKHANOV 62 OSPK
R4 F      (1500.)    OR LESS  CL=.90   FRANKEL2  63 CNTR
R4 F      (1200.)    OR LESS  CL=.90   BABAEV    63 OSPK
R4 K        (62.)    OR LESS  CL=.90   KORENCH2  71 OSPK    DUBNA
R4 K        (19.)    OR LESS  CL=.90   KORENCHEN 76 SPEC +  DUBNA
R4 K         (1.6)   OR LESS  CL=.90   BERTL     84 SPEC +  SINDRUM
R4 K         (1.3)   OR LESS  CL=.90   BOLTON    84 CNTR    LANL
R4 K          0.024  OR LESS  CL=.90   BERTL     85 SPEC +  SINDRUM
R4 F     FOUR ABOVE EXPERIMENTS EVALUATED UPPER LIMITS ASSUMING A SECOND
R4 F     ORDER V-A NEUTRINO LOOP DIAGRAM. LIMITS NOT SIGNIFICANTLY CHANGED
R4 F     BY ASSUMING A CONSTANT MATRIX ELEMENT.
R4 K     THESE EXPERIMENTS ASSUME A CONSTANT MATRIX ELEMENT.
```

$\mu \to (e 2\gamma)$/total (units 10^{-7}) (P6)

```
R5       FORBIDDEN BY LEPTON FAMILY NUMBER CONSERVATION
R5         (160.)    OR LESS  CL=.90   FRANKEL1  63 OSPK +
R5 P        (40.)    OR LESS  CL=.90   POUTISSOU 74 CNTR +  LBL
R5   A      (0.5)    OR LESS  CL=.90   BOWMAN    78 CNTR DEPOMMIER 77 DATA
R5   B       0.084   OR LESS  CL=.90   AZUELOS   83 CNTR +
R5 P     POUTISSOU 74 LIMIT APPLIES TO SUM OF ALL NEUTRINOLESS MU+ DECAYS.
R5   A   BOWMAN 78 ASSUMES INT. LAGRANG. LOCAL ON SCALE OF INVERSE MU MASS.
R5   B   AZUELOS 83 USES PHASE SPACE DISTRIBUTION OF BOWMAN 78. SEE ABOVE.
```

$\mu \to (e^- e^+ e^- \overline{\nu}_e \nu_\mu)$/total (units 10^{-5}) (P7)

```
R6 L      3   (1.5)     (1.0)          LEE       59 HBC  +
R6 G      1   (2.)                     GUREVICH  60 EMUL +
R6 C      7   (2.2)     (1.5)          CRITTENDE 61 HLBC +    E(E+E-)>10MEV
R6  B7443     3.4        0.4           BERTL     85 SPEC +  SINDRUM
R6 L     IN THE THREE LEE 59 EVENTS, THE SUM OF ENERGIES S=E(E+)+E(E-)+E(E+)
R6 L     WAS S=51 MEV, 55 MEV, AND 33 MEV .
R6 G     GUREVICH 60 INTERPRET THEIR EVENT AS EITHER VIRTUAL OR REAL PHOTON
R6 G     CONVERSION.  E+ AND E- ENERGIES NOT MEASURED .
R6 C     CRITTENDEN 61 COUNT ONLY THOSE DECAYS WHERE TOTAL ENERGY OF EITHER
R6 C     (E+,E-) COMBINATION IS >10 MEV )
R6  B    BERTL 85 HAS TRANSVERSE MOMENTUM CUT PT> 17 MEV/C.
R6  B    STAT ERROR (0.2) AND SYST ERROR (INCREASED BY US) ADDED IN QUADR.
```

LIMIT ON $\mu^- \to e^-$ CONVERSION

FORBIDDEN BY LEPTON FAMILY NUMBER CONSERVATION

$\sigma(\mu^-\ {}^{32}S \to e^-\ {}^{32}S)/\sigma(\mu^-\ {}^{32}S \to \nu_\mu\ {}^{32}P^*)$

```
RE          (4. E-10)OR LESS  CL=.90   BADERTSCH 77 STRC    SIN
RE           0.7E-10 OR LESS  CL=.90   BADERTSCH 80 STRC    SIN
```

$\sigma(\mu^-$ Cu $\to e^-$ Cu$)/\sigma(\mu^-$ Cu $\to$ capture$)$

```
RF           1.6E-8  OR LESS  CL=.90   BRYMAN    72 SPEC
```

$\sigma(\mu^-$ Ti $\to e^-$ Ti$)/\sigma(\mu^-$ Ti $\to$ capture$)$

```
RG           1.6E-11 OR LESS  CL=.90   BRYMAN    85 SPEC    TRIUMF
```

LIMIT ON $\mu^- \to e^+$ CONVERSION

FORBIDDEN BY TOTAL LEPTON NUMBER CONSERVATION

$\sigma(\mu^-\ {}^{32}S \to e^+\ {}^{32}Si^*)/\sigma(\mu^-\ {}^{32}S \to \nu_\mu\ {}^{32}P^*)$

```
RP1         (1.5E-9) OR LESS  CL=.90   BADERTSCH 78 STRC    SIN
RP1          0.9E-9  OR LESS  CL=.90   BADERTSCH 80 STRC    SIN
```

$\sigma(\mu^-\ {}^{127}I \to e^+\ {}^{127}Sb^*)/\sigma(\mu^-\ {}^{127}I \to$ anything$)$

```
RP2  A       0.3E-9  OR LESS  CL=.90   ABELA     80 CNTR RADIOCHEMICAL TECH.
RP2  A  ABELA 80 IS UPPER LIMIT FOR MU- E+ CONVERSION LEADING TO PARTICLE-
RP2  A  STABLE STATES OF SB127. LIMIT FOR TOTAL CONVERSION RATE IS HIGHER
RP2  A  BY A FACTOR LESS THAN 4 (G. BACKENSTOSS, PRIVATE COMM.)
```

$\sigma(\mu^-$ Cu $\to e^+$ Co$)/\sigma(\mu^-$ Cu $\to \nu_\mu$ Ni$)$

```
RP3         (2.2E-7) OR LESS  CL=.90   CONFORTO  62 OSPK
RP3          2.6E-8  OR LESS  CL=.90   BRYMAN    72 SPEC
```

LIMIT ON (μ^+,e^-) BOUND STATE CONVERSION TO (μ^-,e^+)

FORBIDDEN BY LEPTON FAMILY NUMBER CONSERVATION

$R_g = g_C/g_F$

```
MC       WHERE GF = 1.16637E-5 GEV**(-2) IS THE FERMI CONSTANT AND
MC       GC IS AN EFFECTIVE COUPLING (DIMENSIONS GEV**(-2)) FOR A
MC       FOUR-FERMION INTERACTION ASSUMED TO BE RESPONSIBLE FOR THE
MC       CONVERSION OF THE (MU+,E-) BOUND STATE TO (MU-,E+).
MC
MC            42    OR LESS  CL=.95   MARSHALL  82 CNTR
```

NOTE ON MUON DECAY PARAMETERS

(by F. Scheck, University of Mainz, West Germany and K. Mursula, Nordita, Copenhagen, Denmark)

The muon decay parameters describe the momentum spectrum (ρ and η), the asymmetry (ξ and δ), and the polarization of the electron (ξ', ξ'', α, β, α', β') in the process $\mu^- \to e^- + \nu_\mu + \overline{\nu}_e$. Assuming a local, lepton-number-conserving, derivative-free, four-fermion interaction, the matrix element in charge-*changing* form may be written as[1]

$$\frac{G_F}{\sqrt{2}} \Big\{ \sum_1^2 h_{ik} \langle \overline{e} | 1[1+(-)^{i-1}\gamma_5] | \nu_e \rangle \times \langle \overline{\nu}_\mu | 1[1+(-)^{k-1}\gamma_5] | \mu \rangle + \sum_1^2 g_{ik} \langle \overline{e} | \gamma^\alpha [1+(-)^{i-1}\gamma_5] | \nu_e \rangle \times \langle \overline{\nu}_\mu | \gamma_\alpha [1+(-)^{k-1}\gamma_5] | \mu \rangle + \sum_1^2 f_{ii} \tfrac{1}{2} \langle \overline{e} | \sigma^{\alpha\beta} [1+(-)^{i-1}\gamma_5] | \nu_e \rangle \times \langle \overline{\nu}_\mu | \sigma_{\alpha\beta} [1+(-)^{i-1}\gamma_5] | \mu \rangle \Big\} \qquad (1)$$

The definitions of covariants and the sign conventions are the ones of Sachs and Sirlin (1975)[2] and Scheck (1978).[3] The connection to other charge-changing and charge-retention forms is worked out in Mursula and Scheck (1985).[4] Note that for massless particles the covariants chosen above project onto states of definite helicity. In the standard model, $g_{22} = 1$ and all other coupling constants vanish.

All electron observables can be expressed, in a model-independent way, as functions of the ten standard real constants (see references above) a, b, c, a', b', c', α, β, α', and β'. The rate is proportional to $A = a + 4b + 6c$. The above decay parameters depend on nine of these constants only:

Stable Particle Full Listings

$$\rho - \tfrac{3}{4} = \tfrac{3}{4}[-a + 2c]/A \,,$$

$$\eta = [\alpha - 2\beta]/A \,,$$

$$\delta - \tfrac{3}{4} = \frac{9}{4}\,\frac{[a' - 2c']/A}{1 - [a + 3a' + 4(b + b') + 6c - 14c']/A} \,,$$

$$1 - \xi\frac{\delta}{\rho} = 4\,\frac{[(b + b') + 2(c - c')]/A}{1 - [a - 2c]/A} \,, \qquad (2)$$

$$1 - \xi' = [a + a' + 4(b + b') + 6(c + c')]/A \,,$$

$$1 - \xi'' = [-2a + 20c]/A \,,$$

$$\alpha/A,\ \beta/A,\ \alpha'/A,\ \beta'/A,$$

the last four of which are obtained from the transverse components of the electron polarization. These real constants are easily related to bilinear combinations of coupling constants in any form of the interaction. For the case of the form (1), they are given by [note the scale factor $G_F/\sqrt{2}$ in Eq. (1)],

$$\left.\begin{matrix} a \\ a' \end{matrix}\right\} = 16(|g_{12}|^2 \pm |g_{21}|^2) \pm |h_{11} + 6f_{11}|^2 + |h_{22} + 6f_{22}|^2 \,, \qquad (3)$$

$$\left.\begin{matrix} b \\ b' \end{matrix}\right\} = 4(|g_{11}|^2 \pm |g_{22}|^2) \pm |h_{12}|^2 + |h_{21}|^2 \,, \qquad (4)$$

$$\left.\begin{matrix} c \\ c' \end{matrix}\right\} = \frac{1}{2}\Big[\pm |h_{11} - 2f_{11}|^2 + |h_{22} - 2f_{22}|^2\Big] \,, \qquad (5)$$

$$\left.\begin{matrix} \alpha \\ \alpha' \end{matrix}\right\} = \left.\begin{matrix} \mathrm{Re} \\ \mathrm{Im} \end{matrix}\right\} 8[g_{21}(h^*_{22} + 6f^*_{22}) \pm g_{12}(h^*_{11} + 6f^*_{11})] \,, \qquad (6)$$

$$\left.\begin{matrix} \beta \\ \beta' \end{matrix}\right\} = \left.\begin{matrix} \mathrm{Re} \\ \mathrm{Im} \end{matrix}\right\} (-4)[g_{22}h^*_{21} \pm g_{11}h^*_{12}] \,. \qquad (7)$$

As the decay parameters (2) depend on $(b + b')$ but not on $(b - b')$, the constant h_{12} appears only in β and β', Eq. (7). However, g_{11} is found to be compatible with zero, so the decay parameters do not determine h_{12}. (The coupling constant h_{12} does occur in the rate parameter A and may, in principle, be obtained by comparing μ decay to other data.) By using Eqs. (2) and the experimental determinations of ρ, η, $\xi\delta/\rho$, δ, ξ', ξ'', α, β, α', and β', limits can be placed on the nine parameters a/A, a'/A, $(b + b')/A$, c/A, c'/A, α/A, α'/A, β/A, and β'/A. These are given in the Listings. These limits are easily translated into limits on specific coupling constants in Eq. (1), depending on what kind of extension of the standard model one wishes to test. Examples such as tests for right-handed interactions or for scalar/pseudoscalar effective couplings are given in Mursula and Scheck (1985).[4]

The limits on a, a', $\cdots$, can be recast into limits on the effective charge-retention coordinates g_S/g_V, g_P/g_V, g_A/g_V, g_T/g_V, ϕ_{VA}, and ψ_{VA}, which were used in the earlier literature (cf., e.g., DERENZO 69). The most recent values are found in BURKARD2 85.

Note that the radiative corrections are unambiguous only if h_{11}, h_{22}, g_{12}, g_{21}, f_{11}, and f_{22} vanish.

References

1. F. Scheck, in *Leptons, Hadrons, and Nuclei* (North Holland, Amsterdam, 1983).
2. A.M. Sachs and A. Sirlin, in *Muon Physics II*, eds. C.S. Wu and V. Hughes (Academic Press, New York, 1975), p. 49.
3. F. Scheck, Phys. Rep. **44**, 187 (1978).
4. K. Mursala and F. Scheck, Nucl. Phys. **B253**, 189 (1985); and K. Mursala et al., Nucl. Phys. **B219**, 321 (1983).

μ DECAY PARAMETERS

ρ PARAMETER **(V−A theory predicts ρ=0.75)**

```
RHO C          (0.741)  (0.027)          DUDZIAK    59 CNTR +  20-53 MEV E+
RHO  P9213      0.745    0.025           PLANO      60 HBC  +  WHOLE SPECTRUM
RHO  P  TWO PARAMETER FIT TO RHO AND ETA.
RHO C 2276     (0.751)  (0.034)          BLOCK      62 HEBC  - WHOLE SPECTRUM
RHO D          (0.64)   (0.04)           BARLOW     64 CNTR  - WHOLE SPECTRUM
RHO D          (0.661)  (0.016)          BARLOW     64 CNTR +  WHOLE SPECTRUM
RHO D          (0.867)  (0.035)          PONTECORV  64 CC    -
RHO D   RESULTS IN DOUBT.
RHO C 800K     (0.7503) (0.0026)         PEOPLES    66 ASPK +  20-53 MEV E+
RHO C 280K     (0.760)  (0.009)          SHERWOOD   67 ASPK +  25-53 MEV E+
RHO C 170K     (0.762)  (0.008)          FRYBERGER  68 ASPK +  25-53 MEV E+
RHO C   ETA CONSTRAINED =0. THESE VALUES INCORPORATED INTO A TWO PARAMETER
RHO C   FIT TO RHO AND ETA BY DERENZO 69.
RHO             0.7518   0.0026          DERENZO    69 RVUE
RHO          . . . . . . . . . .
RHO AVG         0.7517   0.0026  AVERAGE
```

η PARAMETER **(V−A theory predicts η=0)**

```
ETA P 9213  (-2.0)     (0.9)             PLANO      60 HBC  +  WHOLE SPECTRUM
ETA P   TWO PARAMETER FIT TO RHO AND ETA- PLANO 60 DISCOUNTS VALUE FOR ETA
ETA C 800K   (0.05)    (0.5)             PEOPLES    66 ASPK +  20-53 MEV E+
ETA C 280K  (-0.7)     (0.6)             SHERWOOD   67 ASPK +  25-53 MEV E+
ETA C 170K  (-0.7)     (0.5)             FRYBERGER  68 ASPK +  25-53 MEV E+
ETA C   RHO CONSTRAINED =0.75.
ETA   6346   -0.12      0.21             DERENZO    69 HBC  +  1.6-6.8 MEV E+
ETA  85.3M    0.011     0.085            BURKARD2   85 CNTR +  9-53 MEV E+
ETA DB5.3M  (-0.012)   (0.016)           BURKARD2   85 CNTR +  9-53 MEV E+
ETA  B  STATISTICAL (0.081) AND SYSTEMATIC ERRORS ADDED IN QUADRATURE.
ETA  B  MEASURED VALUE OF ETA IS AN ENERGY AVERAGE.
ETA D   ALPHA=ALPHA-PRIME=0 ASSUMED.
ETA          . . . . . . . . . .
ETA AVG      -0.007     0.079   AVERAGE
```

ξ'' PARAMETER

```
XPP  B326K    0.65      0.36             BURKARD1  85 CNTR +  BHABHA + ANNIHIL
XPP  B  BURKARD1 85 MEASURE (XI''-XI*XI')/XI AND XI'
XPP  B  AND SET XI=1.
```

(ξ PARAMETER)*(μ LONGITUDINAL POLARIZATION)

```
XI
XI                          (V-A THEORY PREDICTS XI=1, LONG.POL.=1)
XI       9K  (0.97)    (0.05)            BARDON    59 CNTR    BROMOFORM TARGET
XI     8354  (0.93)    (0.06)            PLANO     60 HBC  +  8.8 KGAUSS
XI   A       (0.903)   (0.027)           ALI-ZADE  61 EMUL +  27 KGAUSS
XI   A  DEPOLARIZATION BY MEDIUM NOT KNOWN SUFFICIENTLY WELL.
XI      66K  (0.975)   (0.030)           GUREVICH  64 EMUL REPL.BY AKHMANOV 68
XI           (0.975)   (0.015)           AKHMANOV  68 EMUL    140 KGAUSS
```

(ξ PARAMETER)*(μ LONGITUDINAL POLARIZATION)*δ/ρ

```
XID         (0.9959)OR MORE  CL=.90   CARR      83 SPEC +  11 KGAUSS
XID  S       0.9966 OR MORE  CL=.90   STOKER    85 SPEC +  MU-SPIN ROT
XID  S  STOKER 85 FIND (XI*PMU*DELTA/RHO)>0.9955 AND >0.9966, WHERE FIRST
XID  S  LIMIT IS FROM NEW MU-SPIN ROTATION DATA AND SECOND IS FROM
XID  S  COMBINATION WITH CARR 83 DATA.  (DELTA/RHO)=1.0 IN V-A THEORY.
```

δ PARAMETER **(V−A theory predicts δ=0.75)**

```
DEL  8354     0.78      0.05             PLANO     60 HBC  +  WHOLE SPECTRUM
DEL           0.782     0.031            KRUGER    61
DEL   490K    0.752     0.009            FRYBERGER 68 ASPK +  25-53 MEV E+
DEL     VOSSLER 69 HAS MEASURED THE ASYMMETRY BELOW 10 MEV
DEL          . . . . . . . . . .
DEL AVG       0.7551    0.0085  AVERAGE
```