LONG-TERM LONGITUDINAL ASYMMETRIES IN SUNSPOT ACTIVITY: DIFFERENCE BETWEEN THE ASCENDING AND DESCENDING PHASE OF THE SOLAR CYCLE

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Abstract. We study the longitudinal distribution of sunspot activity in 1917 – 1995 using vector sums of sunspot areas. The vector sum of sunspots of one solar rotation gives a total vector whose amplitude characterizes the size of longitudinal asymmetry and whose phase describes the location of the momentarily dominating longitude. We find that when the phase distributions are calculated separately for the ascending phase and maximum (AM) on the one hand and for the declining phase and minimum (DM) on the other hand, they behave differently and depict broad maxima around roughly opposite longitudes. While the maximum of the phase distribution for the AM period is found around the Carrington longitude of 180°, the maximum for the DM period is at the longitude of about 0°. This difference can be seen in both solar hemispheres, but it is more pronounced in the southern hemisphere where the phase distribution has a particularly clear pattern. No other division of data into two intervals leads to similar systematic differences.

1. Introduction

While the latitudinal distribution of solar activity and its development over the solar cycle are well understood by now, the longitudinal distribution of solar activity remains, despite several investigations, still far from being well understood. The main question is whether the longitudinal distribution of solar activity is systematically asymmetric. This important problem is directly connected with the nature of the solar dynamo since a systematically non-uniform longitudinal distribution of active regions would require non-axisymmetric dynamo models (Benevolenskaya *et al.*, 2001; Neugebauer *et al.*, 2000).

The longitudinal asymmetry of solar activity is connected with the possible existence of preferred or active longitudes: relatively narrow (20° – 60°) ranges of solar longitude where various forms of solar activity occur more often than in other longitudes. The regions of the new and old cycle were found to be clustered at particular longitudes during the transition from solar cycle (SC) 22 to SC 23 (Benevolenskaya *et al.*, 1999). This suggests that active longitudes exist in the Sun and play a significant role in the development of the solar cycle. The active longitudes have been found in different types of cool stars (Berdyugina and

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Tuominen, 1998), indicating that longitudinal asymmetry is an inherent property of stellar dynamos. It has also been shown that the non-axisymmetric mode is always present in the solar photospheric magnetic field and that it dominates during the solar maximum (Ruzmaikin *et al.*, 2001). Preferred longitudes have also been found in the solar wind and in the interplanetary magnetic field (Neugebauer *et al.*, 2000). An interesting possibility is that the longitudinal asymmetry is due to a tilted relic magnetic field in the Sun (or other stars) which can explain the north-south asymmetry of solar activity, the appearance of active longitudes, and the 22-year cycle in sunspot activity (see, e.g., Cowling, 1945; Sonett, 1982; Bravo and Stewart, 1995; Kitchatinov, Jardine, and Collier Cameron, 2001; Mursula, Usoskin, and Kovaltsov, 2001).

The existence of active longitudes has been investigated by different techniques and different indices of solar activity. (For a review see, e.g., Vitinsky, Kopecký, and Kuklin, 1986; Benevolenskaya *et al.*, 2001.) These studies have sometimes yielded contradictory and even opposite results, such as those concerning the rotation rate of active longitudes. Some studies have come to the rather unexpected conclusion that the longitudes where sunspots preferentially occur, do not experience differential rotation but rotate more or less rigidly, approximately with the Carrington period of 27.2753 days (Vitinsky, Kopecký, and Kuklin, 1986; Benevolenskaya *et al.*, 2001).

It was observed already a long time ago (Waldmeier, 1955) that the 27-day periodicity related to a nonuniform longitudinal distribution of solar activity is typically maintained during many solar rotations. Since the average sunspot lifetime is considerably shorter than this, the persistence of the 27-day periodicity can be produced if sunspots appear repeatedly in the same range of preferred longitudes. Indeed, Trotter and Billings (1962) found persistent sunspot activity in certain longitudes during 1954–1961. However, they did not detect any persistence from one cycle to the next. Bumba and Howard (1969) suggested that subsurface sources producing active regions over a longitude zone of some tens of degrees rotate with a synodic period of about 27 days. The patterns in the magnetic field synoptic maps persisting for at least 10 rotations showed average rotation rates corresponding to the $5°–10°$ latitude range regardless of the phase of the cycle. Complexes of solar activity during the ascending phase of SC 21 were investigated using synoptic maps of photospheric magnetic fields (Gaizauskas *et al.*, 1983). It was shown that they rotate at a constant rate which may coincide with the Carrington rate but may be slower or faster. Comparing active longitudes in SCs 18–20 Maris (1972) has shown that their locations did not change during the time interval of 2–3 cycles. He suggested that the law of differential rotation does not hold for active longitudes. Vitinsky, Kopecký, and Kuklin (1986) found that the active longitudes were preserved for at least two solar cycles, depicting a quasi-rigid rotation.

Waldmeier (1955) assumed that there is a distinct pattern in the location of active longitudes, and pointed out 3 possible modes: to each primary sunspot group with heliolatitude b and heliolongitude l corresponds a group with approximate coordinates of either (1) $-b$, l, (2) $-b$, $l + 180°$, or (3) b, $l + 180°$. Vitinsky, Kopecký, and Kuklin (1986) argued that the existence of any of these options is doubtful since only a few sunspots are ordered accordingly. However, they noted that during 9 solar cycles (12–21) at least one pair of antipodal (mode 2) active longitudes separated by about 180° was observed in opposite solar hemispheres. The most rare mode was the 3rd mode, i.e., two longitudes separated by 180° in the same hemisphere. However, presently there are several investigations where the tendency of active longitudes to occur antipodally (mode 2) could be seen (see later discussion).

In this paper we discuss the average longitudinal distribution of sunspots over the time interval from 1917–1995. In the next section we discuss the data and method used in the paper. Sections 3 and 4 present the phase distributions using the vector summing of sunspots over each solar rotation and all sunspots, respectively. In Section 5 we discuss and interpret the results and in Section 6 we give our conclusions.

2. Data and Method

The main aim of this study is to investigate the longitudinal structure of solar activity and its long-term change. As a quantitative measure of the longitudinal asymmetry of sunspot distribution we use the vector sum of sunspot activity (Vernov *et al.*, 1979; Vernova *et al.*, 2002) which takes into account the area and the Carrington longitude of each sunspot group. The ith sunspot group on a day k of a Bartels rotation ($k = 1, 2, \ldots 27$) in question is presented as a polar vector $\mathbf{s}_i(t_k)$ in the heliographic plane whose length equals the sunspot area and whose phase corresponds to the Carrington longitude of the group. Then a vector sum is calculated using all sunspot groups observed during each day of the Bartels rotation under consideration: $\mathbf{S} = \sum_{i,k} \mathbf{s}_{ik}$. A sunspot group which lives for several days will be counted equally many times in $\mathbf{S}$. Thus large, long-lived sunspot groups give the main contribution to $\mathbf{S}$. Whereas the modulus of $\mathbf{S}$ can be considered as a a measure of longitudinal asymmetry, its direction points to the Carrington longitude which dominates during the given Bartels rotation. In particular, $\mathbf{S}$ is zero if sunspots have no preferred direction in longitude. Hereafter we will call the modulus (absolute value) of the vector $\mathbf{S}$ the longitudinal asymmetry (LA) of sunspot distribution and the phase angle of $\mathbf{S}$ the phase of longitudinal asymmetry.

The absolute value of the longitudinal asymmetry was considered in our previous study (Vernova *et al.*, 2002). In this paper we focus our attention on the phase of the $\mathbf{S}$ vector. The time interval studied (1917–1995) and the data sources (Greenwich Royal Observatory for 1917–1954, Pulkovo Observatory for 1955–1995) are the same as in our earlier work (Vernova *et al.*, 2002). Also as earlier, we have calculated the phase values separately for the northern and southern solar hemispheres. We note that using two different sunspot data sets does not affect the results of the LA analysis since the large and long-lived sunspots give the major contribution to LA, and these are reliably recorded in both data sets. Calculations made for the overlapping time interval have shown only small differences in the two LA parameters.

3. Phase Histograms of Vectorial Sunspot Activity

The time series analysis of the phase of vector **S** and its long-term change is rather difficult because of large fluctuations and because the values close to 0° and 360° cannot be viewed very easily although the corresponding regions of activity are situated close by. Therefore we have calculated histogram distributions of the phase. (The histogram binning was taken to be 20.) Moreover, we have taken into account only those phase values for which the corresponding amplitude of the vector **S** was sufficiently large. This has reduced the sample only by about 15%. The lower limit was needed since the very small amplitude values correspond to times that are practically spotless or have an almost symmetric sunspot distribution. In either case the phase is not very significant.

Figure 1 depicts the phase histogram for the full disk from the whole data interval 1917–1995, as well as for the northern and southern hemispheres separately for the same interval. While the distribution for the northern hemisphere is fully random, the total disk histogram and, especially, the southern hemisphere histogram depict a weakly asymmetric distribution. When separating the histograms further with respect to the four phases of the solar cycle (minimum, ascending phase, maximum, descending phase), we have found considerable similarity between the ascending phase and maximum on one hand and between the descending phase and minimum on the other hand. This similarity is shown for the southern hemisphere in Figure 2 where we have also added the best fitting second order polynomials in each case in order to distinguish the convex (maximum close to 180° of Carrington longitude) or concave (maximum close to 0/360° longitude) nature of the distribution. Similar but slightly less systematic curves were also found for the northern hemisphere.

Accordingly, we grouped the whole data set into two parts, one (to be called AM) consisting of the ascending phases and maxima of the solar cycles included in the data set, the other (DM) consisting of the descending phases and minima. Figure 3 depicts the phase histograms for these AM and DM groups in the northern and southern hemispheres separately. One can see that while the histograms for AM periods are convex they are concave for the DM periods. These basic patterns are the same in both hemispheres although they are more systematic in the southern hemisphere. Thus the observed shapes of the histograms depicted in Figure 3 imply that the two parts of the solar cycle have, on an average, roughly opposite phase distributions.

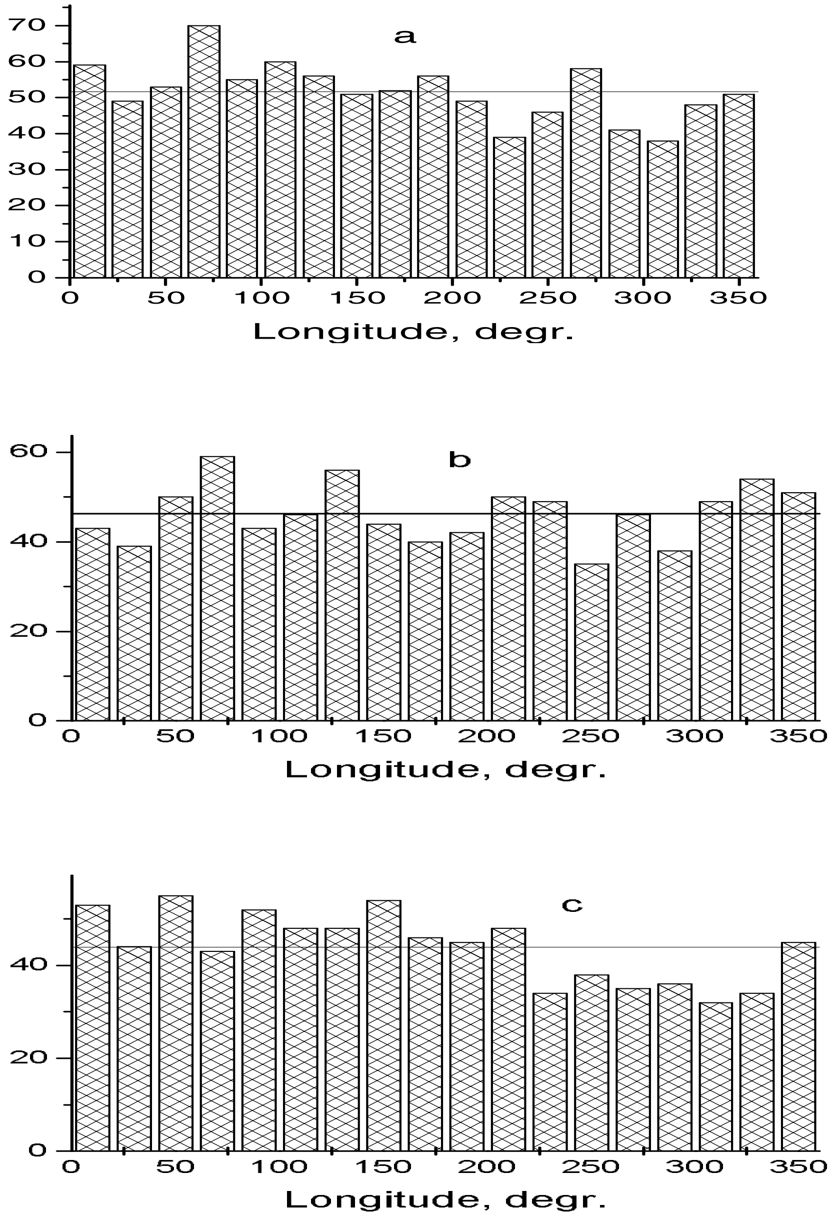

Figure 1. Phase distributions of the longitudinal asymmetry vector for 1917–1995: (a) full solar disk; (b) northern hemisphere; (c) southern hemisphere. *Horizontal lines* indicate the mean values of the corresponding distribution.

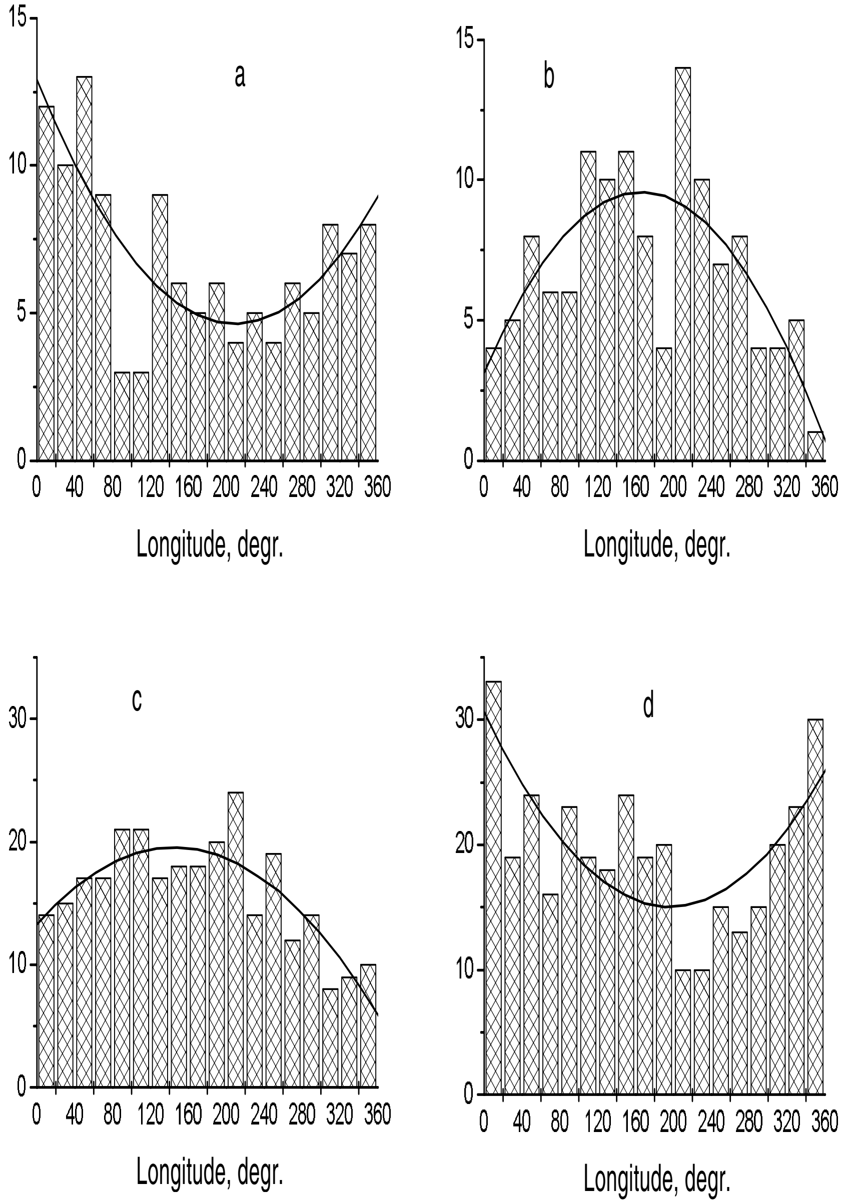

Figure 2. Phase distributions of the longitudinal asymmetry vector in the different phases of the solar cycle for the southern hemisphere: (a) minimum; (b) ascending phase; (c) maximum; (d) descending phase. *Curves* indicate the best-fitting second-order polynomial.

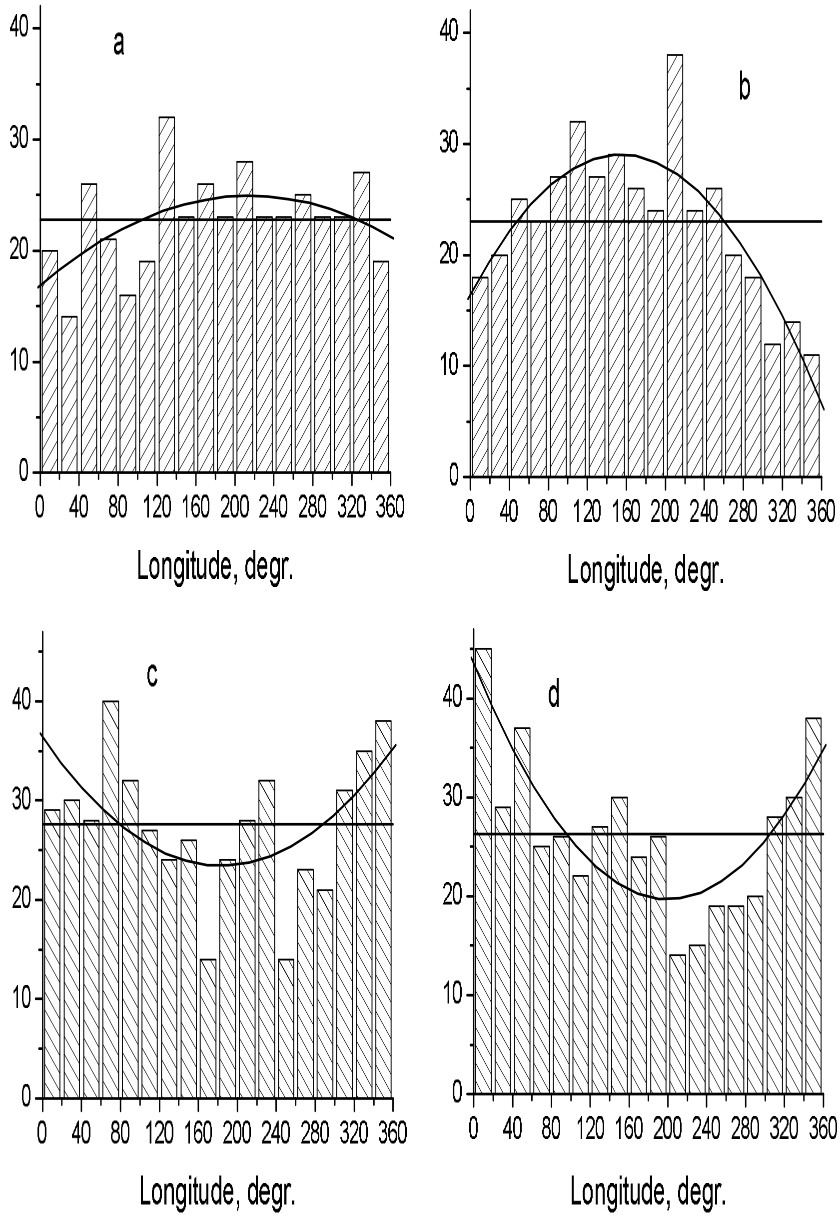

Figure 3. Phase distributions of the longitudinal asymmetry vector for the two AM and DM periods: (a) AM in the northern hemisphere; (b) AM in the southern hemisphere; (c) DM in the northern hemisphere; (d) DM in the southern hemisphere. *Curves* indicate the best-fitting second-order polynomial.

As the next step we have combined the phase distributions for the southern hemisphere (Figures 3(b) and 3(d)) so that the distribution for the DM period (Figure 3(d)) was first shifted by 180° and then summed up with the corresponding distribution for the AM period (Figure 3(b)) to produce the combined distribution for the southern hemisphere depicted in Figure 4(b). The resulting modified histogram for the southern hemisphere depicts a very systematic (almost sinusoidal) behavior of the phase with a pronounced maximum at about 180°. Note that Figure 4(b) covers the data of the southern hemisphere from the whole interval of 79 years under consideration, yet the resulting histogram shows an extraordinary large and smooth variation. Figure 4(b) also depicts the best fitting sinusoid fitted to the histogram. The amplitude of the sinusoid is as large as 14.0 ± 1.4 against the average value of 49.3. The maximum of the fitting sinusoid is located at $184 \pm 6°$ longitude. The probability for the pattern to be due to random fluctuations is much less than 1%.

We have done the same treatment for the phase distributions in the northern hemisphere (Figures 3(a) and 3(c)) by first shifting the DM period distribution (Figure 3(c)) by 180° and then summing up with the AM period (Figure 3(a)) to produce the combined distribution for the northern hemisphere depicted in Figure 4(a). In accordance with the above noted difference to the southern hemisphere (Figure 3), the combined curve in the northern hemisphere (Figure 4(a)) depicts a less regular behavior in longitude than in the southern hemisphere (Figure 4(b)). Correspondingly, the amplitude of the best-fitting sinusoid is smaller (about 8.7 ± 2.7) and the statistical probability is weaker, about 1%. Also, the maximum of the sinusoid is displaced slightly later in longitude to $219 \pm 18°$, but still remains within 2 standard deviations from the maximum of the southern hemisphere. Finally, we would like to note that we have used also other ways to group data but all (except those presented here) resulted in essentially uniform distributions.

4. Phase Histograms of Scalar Sunspot Activity

In order to exclude the possibility that the above results are only an artifact of the vectorial sunspot technique, we have calculated similar histograms for the same time interval (1917–1995) without forming the vectorial sums **S** of sunspots for each solar rotation. Instead, each sunspot occurring at a given Carrington longitude on any day was considered as an independent event and added in the corresponding bin in the histogram. (A sunspot observed during n days was included n times.) Accordingly, we obtained phase distributions of 'scalar' sunspot activity in contradiction to the earlier presented phase distributions of vectorial sunspot activity based on the vector **S**.

The resulting histograms for the AM and DM periods, separately for the northern and southern hemispheres are plotted in Figure 5. These should be compared with the similar vectorial distributions presented in Figure 3. In agreement with the

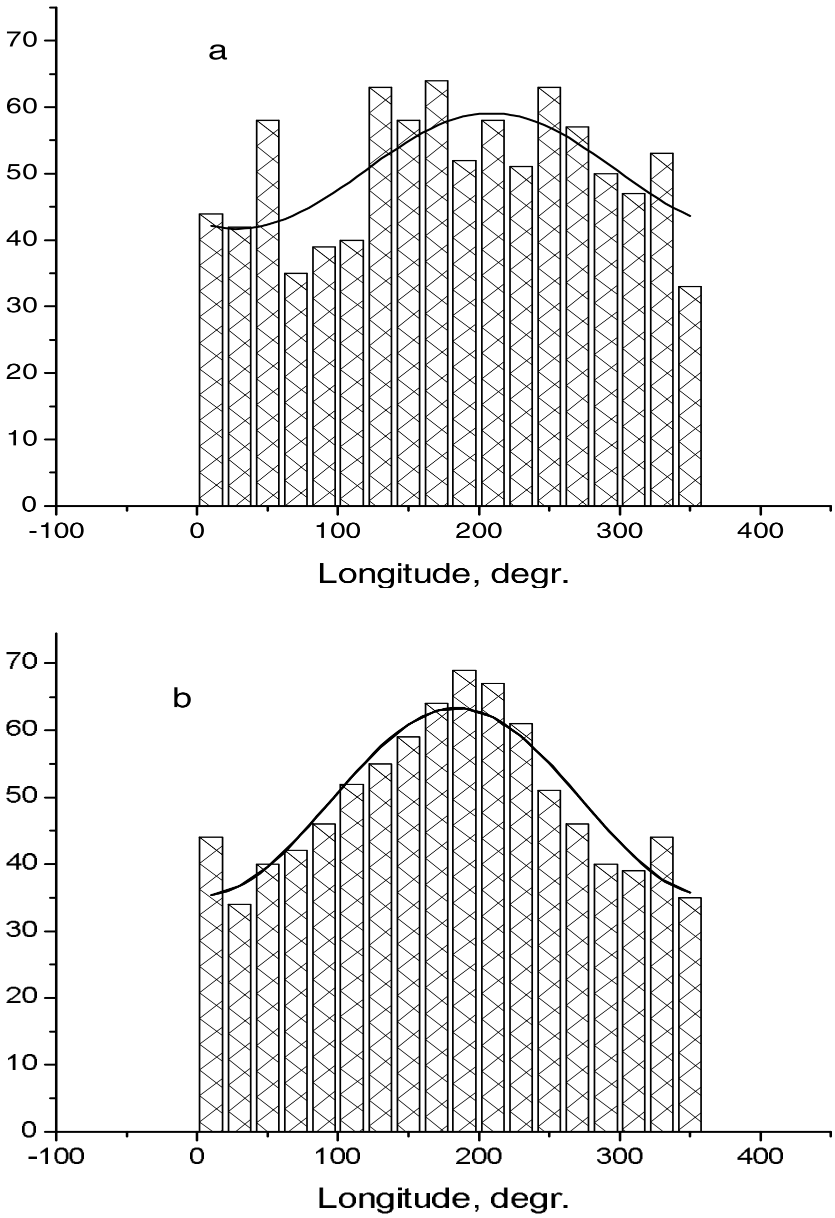

Figure 4. Phase distributions obtained by shifting the DM distribution by 180° and summing with the AM distribution: (a) northern hemisphere; (b) southern hemisphere. *Curves* indicate the best-fitting sinusoid.

previous treatment we have excluded very weak solar activity by setting a lower limit of 20 for the sunspot area of those sunspots that were included in Figure 5. Because of the long time interval (1917–1995) the amount of data is still very large, each of the histograms in Figure 5 containing about 30 000 events.

The same patterns that were observed above in the histograms based on the vectorial sunspots (Figure 3) can also be seen in Figure 5: the histograms for AM periods are convex, and concave for the DM periods. Again, although the difference between AM and DM periods is found in both hemispheres, it is more systematic in the southern hemisphere. The asymmetries are overall clearly smaller in Figure 5 than in Figure 3, reflecting the fact that calculating the vectorial sums of sunspot activity as described above indeed reduces sporadic, homogeneous sunspot activity and emphasizes the ordered, inhomogeneous activity. (As we have noted in our earlier paper (Vernova *et al.*, 2002), taking into account ordered sunspot activity only leads to important changes, e.g., in the relative heights of solar cycles, making them more equal to each other and displacing SC 19 as the largest cycle). However, although the longitudinal asymmetries in Figure 5 are smaller than in Figure 3, the distributions are significantly different from uniform and, consequently, the concave AM and the convex DM distributions differ from each other.

5. Discussion

The results presented in this study evidence the existence of a persistent, long-term asymmetry in the longitudinal distribution of sunspot activity during the last 8 solar cycles. The average longitudinal distributions of sunspots display broad maxima and minima roughly in opposite longitudes. We note that this long-term average pattern differs from the pattern of short-term active longitudes that are often relatively narrow (about 20°–60°) longitude ranges where sunspots appear preferably. The existence of significantly asymmetric long-term distributions favors the idea that the preferred longitudes rotate rather rigidly, on average, and approximately with the Carrington period.

The broad maxima of the longitudinal distributions observed here may be related to the observations made by Mordvinov and Plyusnina (2001). They investigated sunspots in 1818–2001 by means of wavelet analysis and concluded that during the whole interval sunspot activity was concentrated in relatively narrow intervals of longitude which persisted for several solar cycles. They suggested that zones of increased activity slowly drift and oscillate with respect to the Carrington reference system. While they did not observe a distinct regularity of the rotation rate change in different phases of the solar cycle, they stated that each phase has its own mode of rotation with period from 27 to 28 days. Individual cycles may have different modes of rotation for the same phases of the solar cycle.

A fundamentally new feature observed here is the fact that the average phase distributions in the AM period (ascending phase and maximum) have a maximum

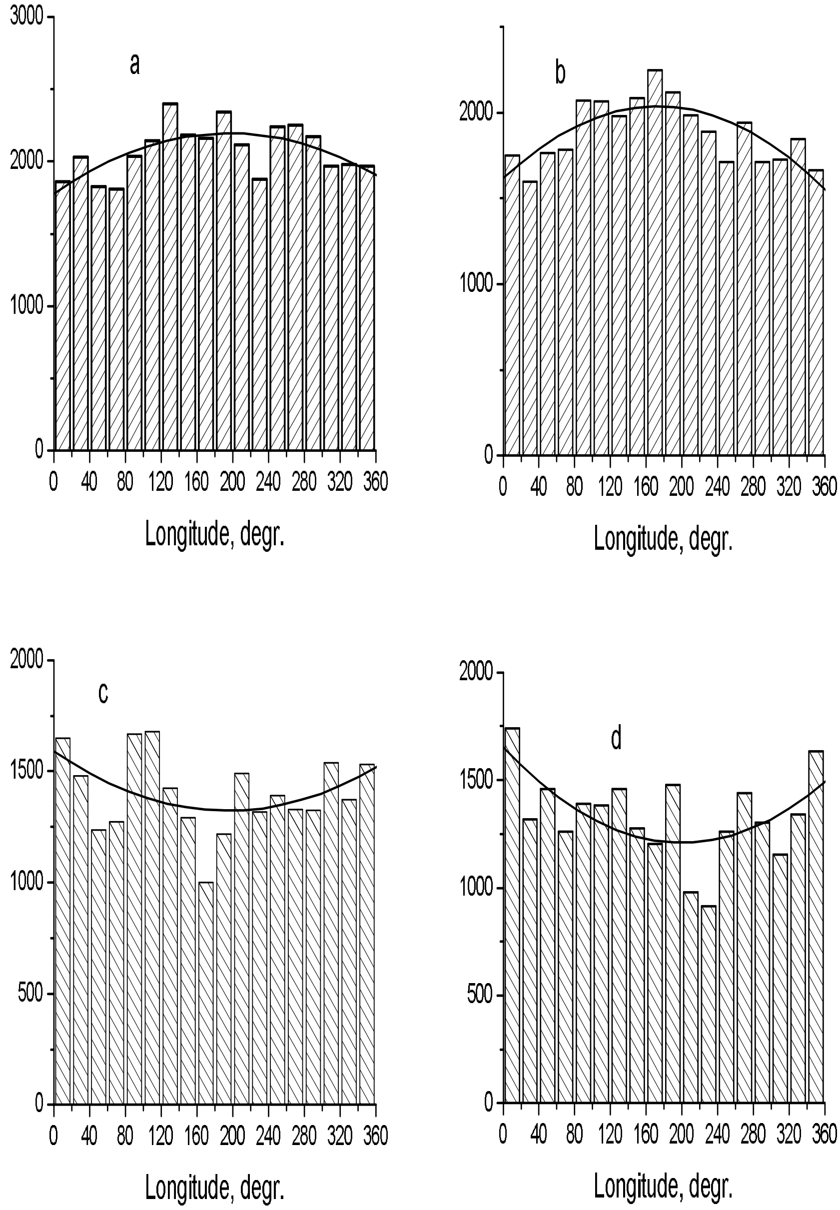

Figure 5. Phase distributions of all sunspots in 1917–1995 for the two AM and DM periods: (a) AM in the northern hemisphere; (b) AM in the southern hemisphere; (c) DM in the northern hemisphere; (d) DM in the southern hemisphere. *Curves* indicate the best-fitting second-order polynomial.

TABLE I

Longitudinal asymmetries and polarities in the different phases of the solar magnetic cycle.

Period	Solar hemisphere	Global magnetic field polarity	Polarity of the leading sunspot	Preferred longitude (in deg)
AM of	N	−	−	180
even cycle	S	+	+	180
AM of	N	+	+	180
odd cycle	S	−	−	180
DM of	N	+	−	0
even cycle	S	−	+	0
DM of	N	−	+	0
odd cycle	S	+	−	0

at about 180° of longitude, i.e., roughly 180° in difference to the maximum of the DM period (descending phase and minimum) which is at about 0/360°. The times separating the AM and DM intervals are important intervals in the solar cycle. The time between solar maximum and the beginning of the declining phase coincides with the inversion of Sun's global magnetic field. On the other hand, the time between the solar minimum and the ascending phase is related to the start of the new solar cycle and the change of the magnetic polarity of sunspots according to Hale's law. Accordingly, we have found that the 22-year solar cycle can be divided into four intervals (depicted in Table I) which have definite characteristics in longitudinal asymmetry. The common property of those intervals (the two AM periods) where the longitudinal maximum is at 180° is that the polarity of the global magnetic field and the polarity of the leading sunspot are the same (within one hemisphere). On the other hand, these polarities are opposite during those intervals (the two DM periods) where the maximum is at about 0°. Also, as earlier noted, the two hemispheres of the Sun develop rather synchronously, although the longitudinal asymmetries were systematically larger in the southern hemisphere.

It is interesting to compare these long-term results with the results obtained by different techniques. Three pairs of active longitudes separated approximately by 160° were found by Maris (1972) for 1964–1969. Analysis of the solar activity for 1962–1966 has shown that centers of activity have a tendency to develop on opposite sides of the Sun (Dodson and Hedeman, 1968). E.g., they observed proton-emitting regions located near longitudes 180° and 354° in 1966.

When studying the longitudinal distribution of the major solar flares, active zones separated by 180° were found, where the flare occurrence rate was much higher (Bai, 1987). These active zones were called 'hot spots' (Bai, 1988). For solar cycles 19–23 seven hot spot systems were established, three of them being double hot spots, i.e., separated by 180° (Bai, 2003). Some of the hot spots persisted as long as 3 solar cycles.

A long-lived complex of solar activity which appeared in the form of several consecutively developed sunspot groups was observed around 180° in the southern hemisphere between July 1991 and April 1992 (Bumba *et al.*, 1996). Based on an extensive investigation of this complex the authors suggest the following pattern for the development of solar complexes: 'Depending on the phase of the solar activity cycle, several complexes of activity may exist simultaneously on the Sun but their developments are always shifted in phase, never proceeding parallel. If there are only two of them, usually during the late declining phase of the cycle, they are formed at almost opposite heliographic longitudes and their evolutionary phases are also in opposite phases.' Mordvinov and Plyusnina (2001) observed two stable intervals of active longitudes separated by 180°. They found that during 10 of the 13 solar cycles they considered the active longitudes were centered at 270° longitude in the ascending phase, whereas at the maximum and thereafter the 90° longitude was more active. During SCs 14–16 they found the arrangement of the active longitudes to differ significantly from that observed both in previous and successive cycles, the maxima being situated at 0° and 180°. Moreover, it has recently been found that active longitudes are a persistent feature and depict an oscillation of activity (so-called flip-flop) at an average period of about 3.7 years (Berdyugina and Usoskin, 2003).

6. Conclusions

Using the vector sum of sunspots calculated for each solar rotation we have observed persistent asymmetries in the longitudinal distribution of sunspots. The average sunspot distributions in 1917–1995 have wide maxima roughly around 0° and 180° of Carrington longitude. A new feature observed here is that the average phase distributions in the ascending phase and maximum on one hand and in the declining phase and minimum on the other hand have a roughly 180° phase difference. While in the ascending phase and maximum (the AM period) the maximum of the average phase distribution is situated at around 180°, in the declining phase and minimum (the DM period) the maximum is near 0°.

The existence of significantly asymmetric distributions favors the idea that the preferred longitudes rotate, on an average, rather rigidly and approximately with the Carrington period. The large amount of data used provides high statistical significance for the observed differences and asymmetries, especially in the southern hemisphere. While the vector method is more appropriate when studying longitudinal asymmetries, the same features can also be found using normal (scalar) longitude distributions of sunspots, thus verifying that the results are not artifacts of the vector method.

Also, we have found that the detected differences occur in both solar hemispheres, indicating that the two hemispheres of the Sun develop rather synchronously. However, the asymmetries were systematically larger in the southern hemisphere. Thus it is obvious that sunspot activity in the southern hemisphere is considerably more ordered than in the northern hemisphere.

Acknowledgements

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References

Bai, T.: 1987, *Astrophys. J.* **314**, 795.
Bai, T.: 1988, *Astrophys. J.* **328**, 860.
Bai, T.: 2003, *Astrophys. J.* **585**, 1114.
Benevolenskaya, E. E., Hoeksema, J. T., Kosovichev, A. G., and Scherrer, P. H.: 1999, *Astrophys. J.* **517**, L163.
Benevolenskaya, E. E., Kosovichev, A. G., Scherrer, P. H., and Lemen, J. R.: 2001, *Synoptic Structure of the Solar Corona and Active Longitudes in EUV*, http://soi.stanford.edu/ elena/.
Berdyugina, S. V. and Tuominen, I.: 1998, *Astron. Astrophys.* **336**, L25.
Berdyugina, S. V. and Usoskin, I. G.: 2003, *Astron. Astrophys.* **405**, 1121.
Bravo, S. and Stewart, G. A.: 1995, *Astrophys. J.* **446**, 431.
Bumba, V. and Howard, R.: 1969, *Solar Phys.* **7**, 28.
Bumba, V., Klvaňa, M., Kalman, B., and Garcia, A.: 1996, *Astron. Astrophys. Suppl. Ser.* **117**, 291.
Cowling, T. G.: 1945, *Monthly Notices Royal Astron. Soc.* **105**, 167.
Dodson, H. W. and Hedeman, E. R.: 1968, in K. O. Kiepenheuer (ed.), Structure and Development of Solar Active Regions', *IAU Symp.* **35**, 56.
Gaizauskas, V., Harvey, K. L., Harvey, J. W., and Zwaan, C.: 1983, *Astrophys. J.* **265**, 1056.
Kitchatinov, L. L., Jardine, M., and Collier Cameron, A.: 2001, *Astron. Astrophys.* **374**, 250.
Maris, D.: 1972, *Solnechnye Dannye*, No. 3, 86 (in Russian).
Mordvinov, A. V. and Plyusnina, L. A.: 2001, *Proc. Int. Conf. Sun During the Epoch of the Magnetic Field Change of Sign*, Pulkovo, 2001, 289 (in Russian).
Mursula, K., Usoskin, I. G., and Kovaltsov, G. A.: 2001, *Solar Phys.* **198**, 51.
Neugebauer, M., Smith, E. J., Ruzmaikin, A., Feynman, J., and Vaughan, J. H.: 2000, *J. Geophys. Res.* **105**, 2315.
Ruzmaikin, A., Feynman, J., Neugebauer, M., and Smith, E. J.: 2001, *J. Geophys. Res.* **106**, 8363.
Sonett, C. P.: 1982, *Geophys. Res. Lett.* **9**, 1313.
Trotter, D. E. and Billings, D. E.: 1962, *Astrophys. J.* **136**, 1140.
Vernov, S. N., Charakhchyan, T. N., Bazilevskaya, G. A., Tyasto, M. I., Vernova, E. S., and Krymsky, G. F.: 1979, *Proc. 16th Intern. Cosmic Ray Conf., Kyoto* **3**, 385.
Vernova, E. S., Mursula, K., Tyasto, M. I., and Baranov, D. G.: 2002, *Solar Phys.* **205**, 371.

Vitinsky, Yu. I., Kopecký, M., and Kuklin, G. V.: 1986, *Statistics of the Sunspot Producing Activity of the Sun*, Moscow (in Russian).
Waldmeier, M.: 1955, in: *Ergebnisse und Probleme der Sonnenforschung.* 2 Auflage, Leipzig.