New standpoints in long-term solar cycle evolution: A review

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2 authors:

I. G. Usoskin
University of Oulu

Kalevi Mursula
University of Oulu

NEW STANDPOINTS IN LONG-TERM SOLAR CYCLE EVOLUTION: A REVIEW

I. G. Usoskin[1] and K. Mursula[2]

[1]*Sodankylä Geophysical Observatory, POB 3000, FIN-90014 University of Oulu, Finland*
[2]*Department of Physical Sciences, POB 3000, FIN-90014 University of Oulu, Finland*
E-mail: Ilya.Usoskin@oulu.fi

ABSTRACT

The sunspot number series forms the longest directly observed index of solar activity and allows to trace its variations on the time scale of about 400 years since 1610. This time interval covers a wide range from seemingly vanishing sunspots during the Maunder minimum in 1645-1700 to the very high activity during the last 50 years. Although the sunspot number series has been studied for more than a century, new interesting features can still be found. This paper gives a review of the recent achievements and findings in long-term evolution of solar activity cycles such as determinism and chaos in sunspot cyclicity, cycles during the Maunder minimum, scenario of a great minimum, the phase catastrophe and the lost cycle in the beginning of the Dalton minimum in 1790s and persistent 22-year cyclicity. These findings shed new light on the underlying physical processes responsible for the sunspot activity and allow for better understanding of such empirical rules as Gnevyshev-Ohl rule and Waldmeier relations.

Key words: Solar activity, solar cycle.

1. INTRODUCTION

In order to study statistical properties of solar activity one needs some numerical characteristics related to the entire Sun (or its significant part) and reflecting its main activity features. Such characteristics are called indices of solar activity. Although there are many indices such as those based on faculae, flares, coronal holes, and electromagnetic radiation in different bands (e.g., 10.7 cm radio flux or the so-called green corona), the number of sunspot on the solar disc (so called sunspot activity) is the most famous and widely used index of solar activity. It is based on the longest series of continuous solar observations and reflects the varying strength of the hydromagnetic dynamo process which generates the solar magnetic field. Regular sunspot observations were started by Galileo in 1610 after the invention of telescope. Since that time, the observations were more or less regular covering nearly four hundred years by routine observations. Sunspot number series is the most used index of solar activity and probably the most analyzed time series in astrophysics.

The most pronounced feature of solar activity is the 11-year cycle, also called Schwabe cycle. This cycle dominates the sunspot activity during almost the whole observed time interval, but it is far from being a simple sinusoidal wave. Instead, it varies in amplitude, period (length) and shape on different time scales.

Although sunspot activity has been studied for more than a century and numerous books and reviews have been published in this area (e.g., Waldmeier 1961; Vitinsky 1965; Kuklin 1976; Vitinsky et al. 1986; Wilson 1994), some new interesting results related to the long-term variation of sunspot activity have appeared during the last few years. This paper aims to review some of these recent findings and suggestions and to provide a brief overview of the long-term solar cycle evolution. Because of the brevity of this paper, we have to leave some relevant topics, such as, e.g., the spatial distribution of sunspot activity (asymmetry of the latitudinal distribution, active longitudes of sunspot formations, etc.) beyond the scope of this review.

2. SUNSPOT ACTIVITY TIME SERIES

2.1. Wolf sunspot number (WSN) series

From sunspot observations one can measure the number of sunspot groups, G, and the number of individual sunspots in all groups, N, visible on the solar disc. Then the relative sunspot number can be defined as

$$R_z = k \cdot (10 \cdot G + N), \qquad (1)$$

where k denotes the individual correction factor which compensates differences in observational techniques and instrumentations used by different observers, and is used to normalize different observa-

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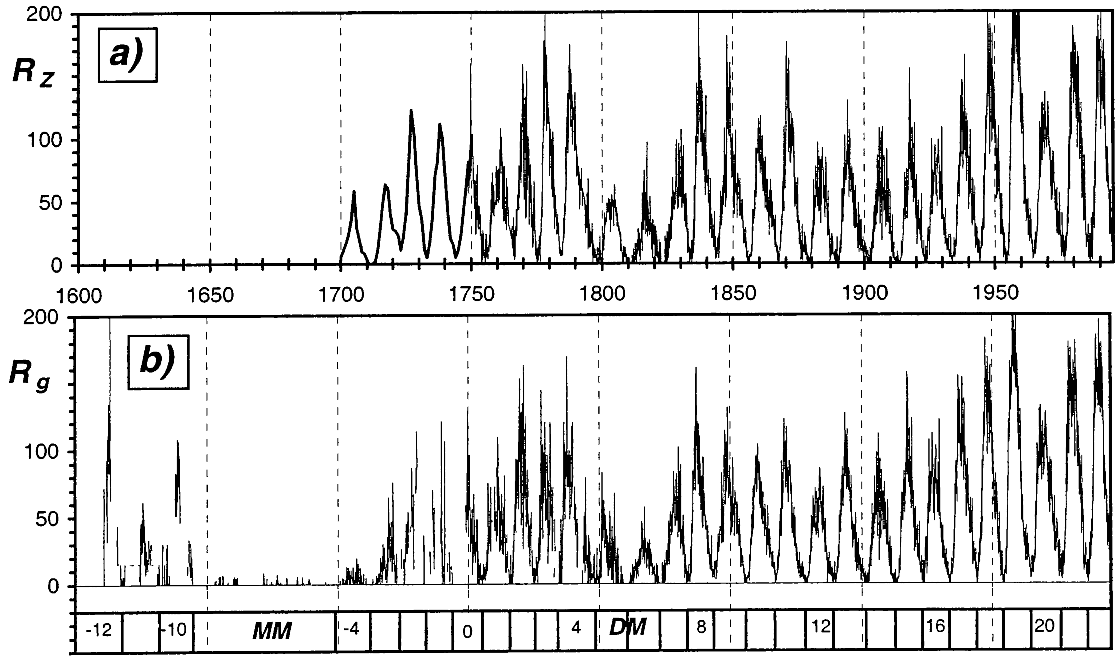

Figure 1. Sunspot activity since 1610. a) Monthly (since 1749) and yearly (1700–1749) Wolf sunspot number series. b) Monthly group sunspot number series. Standard (Zürich) cycle numbering as well as the Maunder (MM) and Dalton (DM) minima are shown in the lower panel.

tions to each other. This R_z quantity, called the Wolf or Zürich sunspot number (called WSN henceforth), was introduced by Rudolf Wolf of Zürich Observatory. R_z is calculated for each day using only one observation made by the "primary" observer (judged as the most reliable observer during a given time) for the day. The primary observers were Staudacher (1749–1787), Flaugergues (1788–1825), Schwabe (1826–1847), Wolf (1848–1893), Wolfer (1893–1928), Brunner (1929–1944), Waldmeier (1945–1980) and Koeckelenbergh (since 1980). If observations by the primary observer are not available for a certain day, the secondary, tertiary, etc. observers are used. The hierarchy of observers is given in (Waldmeier 1961). The use of only one observer for each day aims to make R_z a homogeneous time series. On the other hand, such an approach ignores all other available observations. If no sunspot observations are available for some period, the data gap is filled, without notice in the final WSN series, using an interpolation between the available data and employing also some proxy data. There are also some uncertainties in the definition of G and N. Depending on the observational conditions (clouds, jitters of the atmosphere, etc.), e.g., some small spots can be missed and the separation of cores in mixed groups and spots may be difficult. These problems were discussed in detail, e.g., by (Vitinsky et al. 1986) who estimated such systematic uncertainties to be about 25 % in monthly sunspot numbers. The bulk of the WSN series is based on observations performed at the Zürich Observatory during 1849-1981 using almost the same technique. This part of the series is fairly stable and homogeneous. The official Wolf series starts in 1749 with solar cycle #1 (see Fig. 1a). Before 1749, only yearly R_z values are available. However, prior to the regular observations at the Zürich Observatory there were many gaps in data during 1749-1849 that were interpolated. Therefore, the WSN series is a combination of direct observations and interpolations for the period before 1849. This results in possible errors and inhomogeneity of the series for those times (see, e.g., Vitinsky et al. 1986; Wilson 1998; Letfus 1999, and references therein). The quality of the Wolf series before 1749 is rather poor and hardly reliable (Hoyt & Schatten 1998). Therefore, the WSN series can be analyzed only for the period since 1849 or, with caveats, since 1749.

2.2. Group sunspot number (GSN) series

A new series of sunspot activity called the group sunspot numbers (GSN – see Fig. 1b) has been introduced recently (Hoyt & Schatten 1998). The daily group sunspot number R_g is defined as follows:

$$R_g = \frac{12.08}{n} \sum_i k_i' G_i \qquad (2)$$

where G_i is the number of sunspot groups recorded by ith observer, k' is the observer's individual correction factor, n is the number of observers for the particular day, and 12.08 is a normalization number scaling R_g to R_z values for the period of 1874-1976. The R_g value takes more discrete values but it is more robust than R_Z since it does not include number of individual spots. The GSN series includes not only one "primary" observation but all available observations. This approach allows to estimate systematic uncertainties of the resulting R_g values: about 10% before 1640, less than 5% in 1640-1728 and in 1800-1849, 15-20% in 1728-1799, and about 1% since 1849 (see Fig. 5 in Hoyt & Schatten 1998).

The new GSN series includes all available archival records of sunspot observation. The new database compiled by Hoyt and Schatten consists of 455242 observations from 463 observers, about 80% more daily observations than the WSN series. It has been shown that the GSN series is more reliable and homogeneous than the WSN for the times before 1849, while the two series closely agree with each other for recent times (Hoyt & Schatten 1998; Letfus 1999). The main solar cycle characteristics as obtained from GSN series are similar to WSN series (Hathaway et al. 2002). The GSN series does not include interpolated data and therefore allows to evaluate the data coverage for each period and to estimate related errors. The GSN series covers the period since 1610 (starting with solar cycle # -12 according to Zürich numbering), covering thus a 140 years longer period than the official WSN series. It is particularly interesting that the period of the Maunder minimum (1645–1715) was surprisingly well covered with daily observations which allows for a detailed analysis of sunspot activity during this great minimum. On the other hand, GSN still contains uncertainties and possible inhomogeneities (see, e.g., Letfus 2000). However, a great advantage of this series is that these uncertainties can be estimated and taken into account. The appearance of the GSN series and the fact that all raw information (which is hidden in the WSN series) is available to estimate uncertainties of the results was fundamental for many recent discoveries about the long-term sunspot activity.

2.3. Indirect solar proxies

In addition to the regular direct solar observations, there are also indirect solar proxies which are used to study solar activity in the pre-telescopic era.

Visual observations of aurorae borealis form fairly regular series reflecting geomagnetic activity caused by the varying solar wind and transient phenomena (e.g., Silverman 1983; Křivský & Pejml 1988). Although the auroral record reflects coronal and interplanetary features rather than the momentary magnetic fields on the Sun's surface, there is a strong correlation between the long-term occurrence of sunspot numbers and the frequency of auroras. Unfortunately, auroral observations were not done systematically in the early years which makes it difficult to produce a homogeneous data set (see, e.g., Silverman 1992, 1998).

Another proxy of solar activity is formed by the data on cosmogenic radionuclides (e.g., ^{10}Be or ^{14}C) which are produced by cosmic rays in the Earth's atmosphere (e.g., Stuiver & Quay 1980; Beer et al. 1990). After a complicated transport in the atmosphere they are stored in natural archives such as polar ice, trees, marine sediments, etc. This process is affected also by changes in the geomagnetic field and climate. Cosmic rays suffer from heliospheric modulation due to the solar wind and the frozen-in solar magnetic field. The intensity of modulation depends on solar activity and, therefore, the cosmic ray flux and the ensuing cosmogenic isotope data inversely depend on solar activity. An important advantage of the cosmogenic data is that they are based upon quantities measured nowadays in laboratories. In contrast to fixed historical archival data (as sunspot or auroral observations) this approach allows to obtain homogeneous data sets with stable quality and to improve the quality of data with inventions of new methods, e.g., acceleration mass spectrometry. The cosmogenic isotope data are the only regular indicator of sunspot activity on the very long-term scale where they can, however, hardly resolve details of individual solar cycles.

Some fragmentary data on naked-eye observations of sunspots exist for quite early times, mostly from Oriental sources (see, e.g., Wittmann & Xu 1987; Yua & Stephenson 1988). Even though official Chinese chronicles are fairly reliable, these data are not straightforward to interpret and their observational methods are unknown. These data are also contaminated by meteorological or other phenomena as, e.g., only about 30% of Chinese naked-eye sunspot observations were confirmed by direct telescopic data after 1848 (Wittmann & Xu 1987; Letfus 2000). Another problem is that the records of naked-eye observations are fragmentary and strongly depend on the frequency of observations. On the other hand, this is a unique set of information of sunspot activity on the long-term scale.

There are also attempts to extend the sunspot series back in time using mathematical extrapolation of statistical properties of the WSN record (e.g., Nagovitsyn 1997; De Meyer 1998; Rigozo et al. 2001). Such models construct, e.g., a modulated carrier frequency or a multi-harmonic representation of the measured SN, which is then extrapolated backward in time. The disadvantage of this approach is that it is not a reconstruction based upon measured or observed quantities but rather a "prediction" of the SN based on extrapolation. Clearly such models cannot include periods exceeding the time span of observations upon which the extrapolation is based. Hence, the pre- or post-diction becomes increasingly unreliable with growing extrapolation time and its accuracy is hard to estimate. Some models suggest a compromise between the extrapolation and the proxy methods. E.g., Schove (1955) fitted the slightly variable but phase-locked carrier frequency, corresponding to the 11-year cycle, to fragmentary data from naked-eye sunspot observations or aurorae sightings.

3. CHARACTERISTICS OF THE SOLAR CYCLE

3.1. Notes on cycle definition

Usually the total number of spots on the solar disc is used to define the sunspot cycle. Since 1874, the location of sunspots on the solar disc is recorded on a routine basis at the Greenwich Solar Observatory. The latitude-time diagram of sunspot occurrence is known as the Maunder butterfly (see Fig. 2). Sunspots belonging to the new cycle appear first at higher latitudes and later the activity gradually moves to lower latitudes. This is known as the *Spörer law.* One can see that, when projected onto the time axis, two subsequent cycles overlap during a few years around the sunspot minimum (vertical lines in Fig. 2). Sunspots of one cycle exist during quite a long period of about 15-17 years.

Strictly speaking, one should separate the sunspots of the "old" and "new" cycle sunspots (between inclined lines in Fig. 2), thus taking the overlap into account. Accordingly, some scientists (see, e.g., Pelt et al. 2000; Li et al. 2002) have studied these extended cycles. However, the common practice of separating cycles by the minimum sunspot number times (vertical lines) is often a reasonable approximation (and the only possibility for times before 1874). In such a case sunspots in areas N21, S21 and L21 are omitted from cycle 21 (see Fig. 2). As a compensation, however, sunspots from areas N22, S22 and L20 are included in cycle 21 although they actually belong to the next and the previous solar cycles. Since the number of those sunspots that are not compensated or that are overcompensated is very small (smaller than 1%) compared to the total sunspot number (the so called intensity) of one cycle, it makes only little difference for the intensity to use the more correct sunspot numbers defined by the Maunder butterfly diagram rather than the standard definition, assuming that the cycles are of comparable amplitude. If, however, one cycle is essentially larger than the other, this may result in a distortion of lengths of both cycles as defined by the sunspot numbers. The smaller cycle would appear shorter than it really is while the bigger cycle would seem longer. Such a situation could appear around the Dalton minimum when a high cycle was followed by the tiny one.

As to the cycle length, it was shown (Mursula & Ulich 1998) that the cycle length as conventionally defined by the time interval between subsequent minima (min-min) or maxima (max-max) is very uncertain because of random

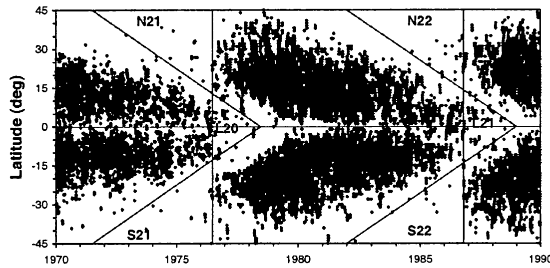

Figure 2. The Maunder butterfly diagram of sunspots for cycles 20-21. Vertical lines denote the times of official minimum, and inclined lined separate the cycles.

fluctuations with uncertainties extending up to more than an year. Instead these authors suggested to define the cycle length between the median times (when half of the total sum SN over the cycle is reached) which is much more stable. Cycle lengths defined from median times are accurate up to an uncertainty of a few days.

3.2. Cyclicities in solar activity

The idea of regular variations in sunspot numbers was first suggested by the Danish astronomer Christian Horrebow in 1770's on the basis of his sunspot observations in 1761-1769 (Gleissberg 1952; Vitinsky 1965). Unfortunately, his results were forgotten and the data lost. Later, in 1843, the amateur astronomer Schwabe established that the sunspot activity varies cyclically with the period of about 10 years. This was the beginning of the study of cyclic variations of solar activity. Here we separate three time scales in solar activity: short-term (below 5 years), mid-term (5-25 years) and long-term (above 25 years) variations. In this work we are primarily interested in the mid- and long-term time scales.

The most prominent cycle in the sunspot series and in all solar activity is the *11-year Schwabe cycle.* This cyclicity is a fundamental feature of solar activity. The 11-year cyclicity is known in many other solar, geomagnetic, space weather, climate, etc. parameters.

The long-term changes (trend) in the Schwabe cycle amplitude are known as the *secular Gleissberg* cycle (Gleissberg 1944). However, this Gleissberg cycle is not a cycle in the strict periodic sense but rather a modulation of the cycle envelope with the time scale of 60-120 years (e.g., Gleissberg 1971; Kuklin 1976). In simple models (see, e.g., Sonett 1983), sunspot activity is considered as an 11-year sinusoid which is amplitude modulated by the secular cycle. The background for the 11-year Schwabe cycle is the *22-year Hale magnetic polarity cycle.* Hale found that the polarity of sunspot magnetic field changes in both hemispheres when a new 11-year cycle starts

(Hale 1908). This relates to the reversal of the global magnetic field of the Sun with the period of 22 years. It is often considered that the 11-year Schwabe cycle is the modulus of the sign-alternating Hale cycle (e.g., Sonett 1983; Bracewell 1986; Kurths & Ruzmaikin 1990; De Meyer 1998; Mininni et al. 2001). A hierarchy of sunspot cycles is given in, e.g., (Mordvinov & Kuklin 1999).

A possible *17-year cycle*, probably related to the Sun's spin-orbit dynamics, has been proposed recently (Juckett 1998; Mendoza 1999; Juckett 2000) continuing long-lasting attempts to connect the time pattern of sunspot series to periods of the planetary configuration in the solar system (see, e.g., Vitinsky et al. 1986, and references therein).

Longer (super-secular) cycles are found in cosmogenic isotope data. Most prominent are the 205–210-year De Vries (Suess) cycle, 600–700-year cycle and 2000–2400-year cycle.

3.3. Waldmeier relations

The 11-year solar cycle has an asymmetric shape with shorter ascending ($\approx$ 4-year in average) and longer ($\approx$ 7-year) descending phases, and the asymmetry is larger for shorter cycles, but the shape of individual cycles may vary. The cycle length varies from 8 to about 17 years in the Wolf sunspot series. The longest (17 years max-max length) is cycle #4 in WSN. The amplitude of cycles also changes greatly, up to 200 in monthly sunspot numbers. The so called Waldmeier relations relate the amplitude and the duration of different phases of a solar cycle as follows (Waldmeier 1935): (i) there is a strong negative correlation (the cross-correlation coefficient is $r = -0.83$ including cycles up to 22nd) between the duration of the ascending phase of a cycle and its amplitude; (ii) the relation between the duration of the descending phase and the cycle amplitude is weakly positive ($r = 0.41$). Together (i) and (ii) yield a weak negative relation between the amplitude and length of the solar cycle ($r = -0.35$). However, the (negative) relation is quite strong ($r = -0.65$) between the amplitude of one cycle and the length of the preceding cycle (e.g., Solanki et al. 2002) which is expected from the dynamo action (e.g., Charbonneau & Dikpati 2000). Surprisingly, a significant negative correlation ($r = -0.6$) between the i^{th} cycle amplitude and $(i-3)^{th}$ cycle length was also found (Solanki et al. 2002), but the latter relation is valid only after the Dalton minimum.

3.4. Great minima and phase catastrophes of solar activity

Sometimes the regular time evolution of solar activity is intervened by periods of greatly depressed activity called great minima. The last great minimum (and the only one covered by direct solar observations) was the famous Maunder minimum during 1645–1715 (Eddy 1976, 1983). Other great minima in the past known from cosmogenic isotope data include Spörer minimum in about 1450-1550, Wolf minimum around 12^{th} century, etc. Great minima are an enigma for the solar dynamo theory. It is intensely debated what is the mode of the solar dynamo during such periods and what causes such minima (e.g., Sokoloff & Nesme-Ribes 1994; Schmitt et al. 1996). Sometimes the Dalton minimum (about 1790-1820) is also considered as a great minimum. However, sunspot activity was not greatly depressed and still showed the Schwabe cyclicity during the Dalton minimum. As suggested, e.g., by (Schüssler et al. 1997) this can be a separate, intermediate between the great minimum and normal activity, state of the dynamo. The Dalton minimum is often connected to the so-called phase catastrophe of solar activity evolution (see, e.g., Vitinsky et al. 1986; Kremliovsky 1994) which occurred in the beginning of the Dalton minimum. The phase catastrophe is shown in Fig. 3a using a 2D projection (the method of time delayed components) of the sunspot activity phase evolution, similar to Fig. 2 of (Kremliovsky 1994), Fig. 38 of (Vitinsky et al. 1986) and Fig. 5 of (Serre & Nesme-Ribes 2000). The phase catastrophe (1790–1798) is the period when the solar cycle evolution was not cyclic but roughly linear with greatly reduced evolution rate along the phase trajectory. A peculiarity in the phase evolution of sunspot activity around 1800 was also noted by Sonett (1983) who ascribed it to a possible error in Wolf data and by Wilson (1988) who reported on a possible misplacement of SN minima for cycles 4-6 in WSN series.

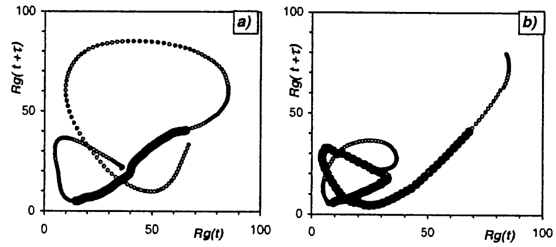

Figure 3. 2D projection of the phase evolution of sunspot activity around the Dalton minimum using the group sunspot numbers. Black dots denote the phase catastrophe in 1790-1798. a) The standard sunspot number evolution with the clear phase catastrophe. b) The same period but using the newly suggested lost cycle in 1793-1800 which removes the phase catastrophe.

3.5. Randomness in solar activity

The sunspot number series contains some noise which is larger than the observational uncertainties and this noise is thus a part of the real data. It is important to note that this noise is not white but rather "colored" or correlated noise (e.g., Ostryakov & Usoskin 1990b; Oliver & Ballester 1996; Frick et al. 1997), i.e., the variance of noise depends on the level of sunspot activity. However, after normalization of the noise to the current average level of sunspot activity, the distribution of such dimensionless noise is nearly Gaussian, implying the existence of random fluctuations (e.g., Usoskin et al. 2001b).

Earlier it was common to describe sunspot activity as a multiharmonic process with several basic harmonics (e.g., Vitinsky 1965; Sonett 1983; Vitinsky et al. 1986, and references therein). Fluctuations of the observed sunspot numbers were believed to be due to noise which is added to the regular part and plays no role in the solar cycle evolution. This approach is oversimplified, depends on the chosen reference time interval and does not adequately describe the long-term evolution (see, e.g., Rozelot 1994). The fact that purely mathematical/statistical models cannot give good predictions of solar activity (as will be discussed later) implies that the nature of solar cycle is not a multi-periodic or other purely deterministic process, but random (chaotic or stochastic) processes play an essential role in sunspot formation.

Since early 1990's, many authors have considered solar activity as an example of low-dimensional deterministic chaos, described by the so called strange attractor (e.g., Kurths & Ruzmaikin 1990; Ostryakov & Usoskin 1990a; Morfill et al. 1991; Mundt et al. 1991; Rozelot 1995; Salakhutdinova 1999; Serre & Nesme-Ribes 2000). Randomness is a natural factor in the time series realization for such processes. However, parameters of the low-dimensional attractor were different when obtained by different authors because the analyzed data set is too short (Carbonell et al. 1993, 1994). Also, the results are dependent on the choice of filtering methods (Prince et al. 1992). Developing this apporach, (Mininni et al. 2000, 2001) suggested to consider sunspot activity as an example of a 2D Van der Pol relaxation oscillator with an intrinsic stochastic component.

Interesting results were obtained using the approach suggested in (Ruzmaikin 1997, 1998). The theory of magnetic flux emergence predicts a threshold for the buoyancy of magnetic tubes which results in sunspot formation (see, e.g., Schüssler et al. 1994; Caligari et al. 1998; Dikpati et al. 2002). The regular magnetic field generated by the dynamo in the bottom of the convection zone is below the threshold (see, e.g., Zeldovich et al. 1983; Schüssler et al. 1994). Accordingly, (Ruzmaikin 1997, 1998) suggested that a randomly fluctuating field plays an important role so that only a combination of the regular dynamo and random fields can exceed the buoyancy threshold. This phenomenological model qualitatively reproduces some features of the sunspot cycle. Developing the idea of Ruzmaikin, (Usoskin et al. 2001b) built a similar model of sunspot activity which includes a superposition of a regular 11-year oscillating dynamo related field, a weak permanent (relic) magnetic field and a randomly fluctuating component. This simple model was found to reproduce fairly well most of the fundamental features of sunspot cyclicity both for normal times and great minima. Note that no other model described above discussed the sunspot cycle behaviour during great minima. The fact that the model (Usoskin et al. 2001b) reproduces sunspot activity in two very different conditions with the same parameters of the random and permanent components only changing the level of dynamo was concluded to support the idea of the relic field in the Sun.

Not only phenomenological or basic principles models were used to understand the nature of randomness in sunspot activity. Corresponding theoretical dynamo models have also been developed which include stochastic processes (e.g., Schmalz & Stix 1991; Lawrence et al. 1995; Schmitt et al. 1996; Charbonneau & Dikpati 2000; Charbonneau 2001). E.g., (Charbonneau & Dikpati 2000) studied stochastic fluctuations in a Babcock-Leighton dynamo model and succeeded to qualitatively reproduce the known anti-correlation of amplitude vs. length of cycles. Their model also predicts a phase-lock of the Schwabe cycle, i.e., that the 11-year cycle is an internal "clock" of the Sun. Note that a significant fluctuating component (with the amplitude more than 100% of the regular component) is essential in their model. (Durney 2000; Charbonneau 2001) demonstrated that a Leighton-Babcock dynamo can be reduced to a one-dimensional iterative return map. This return map naturally gets the Gnevyshev-Ohl rule (in the form of cycle amplitude alteration) but the phase of the G-O rule is not locked and may occasionally suffer a random phase jump. Again, the presence of noise is essential in this model.

While the co-existence of regularity and randomness in sunspot series is obvious, their mutual relationships are not clear (see, e.g., Wilson 1994). The regular component of SN dominates during the normal activity times, while the sunspot occurrence was seemingly sporadic during the Maunder minimum. Moreover, the question is still open if randomness in sunspot data is due to chaotic or stochastic processes.

3.6. Predictability of sunspot activity

Randomness in the SN series is directly related to the predictability of solar activity. Forecasting of solar activity is a subject of intensive studies since long (e.g., Yule 1927; Newton 1928; Gleissberg 1948; Vitinsky 1965, and references therein). All prediction methods can be classified as regression (statistical) or precursor techniques or their combinations (Hathaway et al. 1999). Methods of the first class are based solely on the statistical properties of sunspot activity. Their prediction abil-

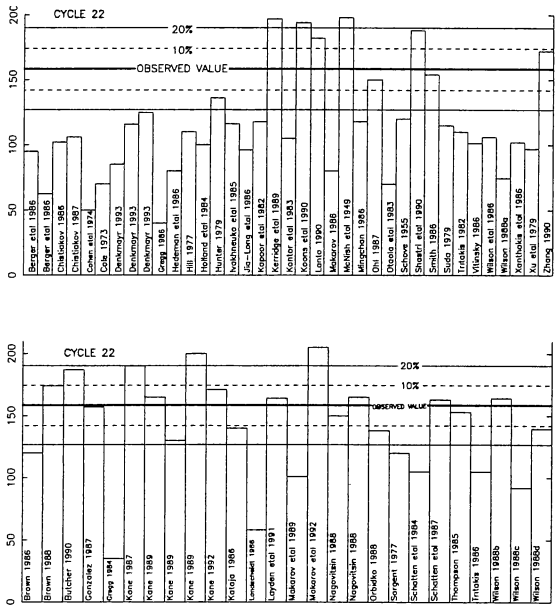

Figure 4. Predictions of the maximum sunspot number of solar cycle 22 (after Li et al. 2001) using (a) mathematical/statistical methods based solely on sunspot activity features and (b) precursor methods.

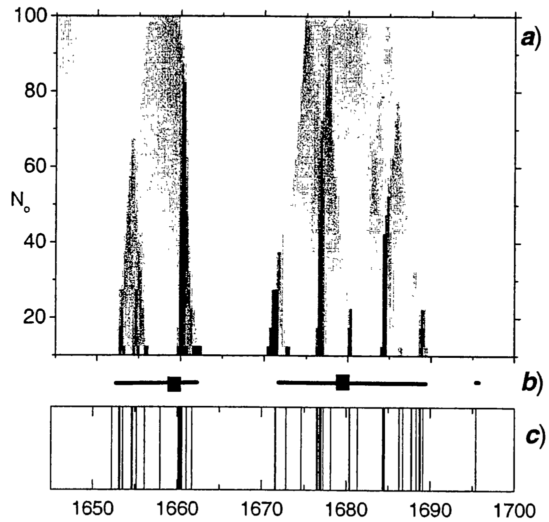

Figure 5. Occurrence of sunspot days during the Maunder minimum (after Usoskin et al. 2001a). a) Dynamical map of sunspot day concentration for different scales. b) Intervals of sunspot occurrence with the mass centers. c) Sunspot days during the deep Maunder minimum.

ity of the long-term activity is quite poor (see, e.g., reviews Conway 1998; Hathaway et al. 1999; Kane 2001; Li et al. 2001). E.g., only few predictions (3 out of 37 analyzed in Li et al. 2001) of the maximum sunspot number of cycle 22 appeared close (within 10%) to the observed value of 157.6 (see Fig. 4a). The situation is similar with the prediction of cycle 23: only 4 out of 37 predictions were close to the observed annual maximum sunspot number 119.6.

The prediction methods of the second class are based on a physical relation between the poloidal solar magnetic field, estimated in the beginning of a cycle by geomagnetic activity, with the toroidal field responsible for sunspot formation. They usually yield better predictions of a forthcoming cycle maximum than the statistical methods, e.g., 10 out of 24 precursor predictions analyzed in (Li et al. 2001) lie close to the observed value (Fig. 4b).

It has been shown using, e.g., such characteristics of the SN time series as the Lyapunov exponent (Ostryakov & Usoskin 1990a; Kremliovsky 1995) or wavelet entropy (Sello 2000) that the applicability of regression prediction methods does not exceed one solar cycle. This is related to a question if an internal "memory" exists in the solar dynamo which is expected in some dynamo models (see, e.g., Ossendrijver et al. 1996). If such a memory does exist, the sunspot activity could be predicted at least at these time scales. E.g., (Balthasar & Schüssler 1983, 1984) suggested, studying preferred longitudes of sunspot occurrence, that there is a memory of one-two cycles, but the presence of a memory in SN was not confirmed by (Oliver & Ballester 1998). On the other hand, recent results on a possible three-cycle relation within the SN time series (Orfila et al. 2002; Solanki et al. 2002) might imply that a kind of memory exists even on the time scale of several decades.

4. MAUNDER MINIMUM (1645–1700)

The Maunder minimum was well covered (more than 95% of days) by direct sunspot observations (Hoyt & Schatten 1996). On the other hand, sunspots appeared only rarely, during ≈2% of days, and seemingly sporadically making standard time series analysis methods fail (e.g., Frick et al. 1997). In such a case when daily sunspot numbers are small and large uncertainties of individual observations are possible, the most reliable information is whether a sunspot has been reported on a given day or not. This means that the sunspot activity was determined not by the number of spots on the solar disc but by the frequency of sunspot occurrence. This approach was used to study the distribution of spotless days vs. days with sunspots around solar minima (e.g., Harvey & White 1999). The occurrence of sunspot days during the Maunder minimum is shown in Fig. 5c. It is seemingly sporadic without indications of the 11-year cycle during the deep minimum (e.g., Letfus 2000). Using a technique developed for an analysis of sparsely occurring events, (Usoskin et al. 2000, 2001a) demonstrated that the sunspot occurrence is gathered into two large clusters (shaded in Fig. 5a) in 1652–1662 and 1672–1689 with the mass centers of these clusters in 1658 and 1679–1680 (Fig. 5b). Together with the sunspot maxima before (1640) and after (1705) the deep Maunder minimum, this implies a dominant 22-year periodicity in sunspot activity throughout the Maunder minimum. This dominant 22-year cyclicity also appear clearly as long spotless periods in 1645–1651, 1662–1671, and 1689–1695. A subdominant 11-year cycle emerges in SN towards the end of the Maunder minimum (Ribes & Nesme-Ribes 1993; Mendoza 1997; Usoskin et al. 2000) and becomes dominant again after 1700. This is in a general agreement with an earlier concept of "immersion" of 11-year cycles during the Maunder minimum (Vitinsky et al. 1986, and references therein). This concept means that full cycles cannot be resolved and sunspot activity only appears as pulses around the cycle maximum times.

It is also interesting to note that sunspots were only seen in the southern solar hemisphere during the end of the Maunder minimum (Ribes & Nesme-Ribes 1993; Sokoloff & Nesme-Ribes 1994) which implies a significant asymmetry of Sun's surface magnetic field.

The conclusion of the dominant 22-year cycle and a weak sub-dominant Schwabe cycle during the Maunder minimum (Usoskin et al. 2000, 2001) is in accordance with indirect solar proxy data: auroral occurrence (Křivský & Pejml 1988; Schlamminger 1990; Silverman 1992) depicts the 22-year variability during that period, as does the ^{14}C cosmogenic isotope concentration in tree rings (Kocharov et al. 1995; Peristykh & Damon 1998). On the other hand, another cosmogenic isotope (abundance of ^{10}Be in polar ice) shows a dominant 11-year cycle during the Maunder minimum (Beer et al. 1998). This may be, e.g., due to the effect of local climate on the ^{10}Be precipitation (e.g., Lal 1987; Beer et al. 1990; Steig et al. 1996). A detailed study is required to resolve this discrepancy.

The time behaviour of sunspot activity during the Maunder minimum yields the following general scenario (Vitinsky et al. 1986; Ribes & Nesme-Ribes 1993; Sokoloff & Nesme-Ribes 1994; Usoskin et al. 2000, 2001). Transition from the normal high activity to the deep minimum was sudden (within few years) without any apparent precursor. A 22-year cycle was dominant in SN occurrence during the deep minimum (1645–1700). The 11-year cycle was sub-dominant and became visible only towards the end of the minimum, starting to dominate the SN series after 1700. Recovery of sunspot activity from the deep minimum to normal activity was gradual passing through a period of nearly linear amplification of the 11-year cycle. It is interesting to note that such a qualitative evolution of a great minimum is predicted by the stochastically forced return map (Charbonneau 2001) which is a truncation of the

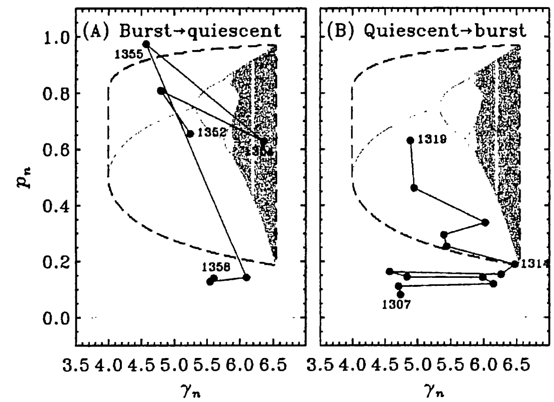

Figure 6. An onset (a) and offset (b) of a great minimum according to the stochastically forced return map simulations (after Charbonneau 2001). Values of p_n and γ_n are related to the intensity and dynamo number of a solar cycle.

Babcock-Leighton dynamo model (see Fig. 6): the onset of a great minimum occurs within one cycle (points 1355–1356 in Fig. 6a) while the transition from the minimum to normal activity takes place through a gradual increase of cycle amplitudes (points 1313 onwards in Fig. 6b).

5. 22-YEAR CYCLE AND RELIC FIELD

Although the 22-year magnetic cycle is the basis for the 11-year Schwabe cycle, a 22-year cycle is not expected in the unsigned SN if the dynamo process is symmetric with respect to the changing polarity. On the other hand, even a quick look at Fig. 1 reveals an alteration of cycle amplitudes. (Gnevyshev & Ohl 1948) studied the intensity of solar cycles and showed that solar cycles are coupled in pairs of a less intensive even-numbered cycle followed by a more intensive odd cycle. This is called the Gnevyshev-Ohl (G-O) rule which is much more stable than its simplified form known as the cycle amplitude alteration. Using WSN, the G-O rule works only since cycle 10 and fails for cycle pairs 4-5 and 8-9 (Gnevyshev & Ohl 1948; Wilson 1988; Storini & Sykora 1997). When using GSN, the G-O rule is valid since the Dalton minimum (Fig. 7) and, in the reverse order, even before that (Mursula et al. 2001). It is possible that the G-O rule will be broken for the recent cycle pair 22–23 (Komitov & Bonev 2001). A phase jump of the G-O rule is possible, e.g., in the framework of a reduced Leighton-Babcock dynamo model (Charbonneau 2001). A careful analysis of the GSN series reveals (Mursula et al. 2001) that there exists a persistent 22-year cyclicity in the sunspot data since the Maunder minimum (see Fig. 7b). Taking into account also the dominant 22-year cyclicity in SN during the Maunder min-

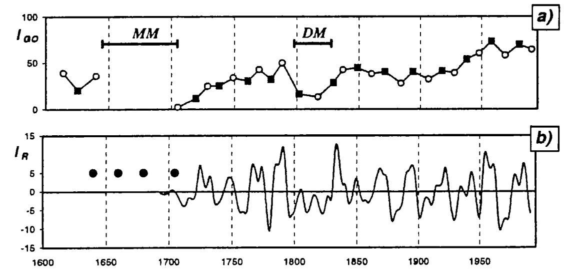

Figure 7. Illustration of the 22-year cycle in sunspot activity (after Mursula et al. 2001). a) Sunspot cycle intensities for odd (black squares) and even (open circles) sunspot cycles according to the official Zürich numbering. b) Band-pass filtered intensity of GSN series. The times of the 22-year cycle maxima before and during the Maunder minimum (Usoskin et al. 2000) are noted by black circles.

imum (Usoskin et al. 2000) and the obvious cycle amplitude alteration before the Maunder minimum (both are shown by black dots in Fig. 7b), one can see that the 22-year periodicity is present in sunspot activity throughout the entire interval of about 400 years of solar observations. During the Maunder minimum this 22-year activity was dominant over the Schwabe cycle, and its maxima around the Maunder minimum are well in phase with the continuous curve in Fig. 7, implying that it is persistent and phase-locked. In particular there is no phase reversal across the Dalton minimum, contrary to the G-O rule based on standard cycle numbering. The peak-to-peak amplitude of the 22-year cycle is roughly constant (see Fig. 7b) and independent of the solar activity level, i.e. its amplitude does not correlate with the current level of solar activity. The 22-year cycle is the underlying pattern behind the G-O rule.

This 22-year cycle with a stable phase and a constant amplitude independent on the solar activity level was interpreted by (Mursula et al. 2001) as a systematically asymmetric oscillation of the magnetic field in the convection zone. Such asymmetry can be naturally explained if one assumes a weak constant magnetic field in the bottom of the convection zone. A relic magnetic field can survive in the Sun due to the high conductivity in the solar interior (Cowling 1945; Sonett 1983). Some evidences favouring this idea have been presented earlier (Sonett 1983; Bravo & Stewart 1995; Boruta 1996). Due to a strong amplification by the dynamo fluid motions in the convection zone, such a weak constant field can interact with the poloidal/toroidal dynamo field and play a role in the formation of a sunspot cycle (Levy & Boyer 1982; Sonett 1983; Boyer & Levy 1984). This hypothesis explains (Mursula et al. 2001; Usoskin et al. 2001b) the main pattern of the 22-year cycle: persistent phase and constant amplitude during the normal activity times and its dominance during the Maunder minimum.

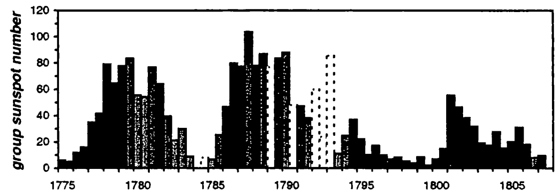

Figure 8. Semiannual GSN data at the beginning of Dalton minimum (after Usoskin et al. 2002). White, light grey, dark grey and black shading denotes unreliable (< 6 observation days during the corresponding 6 months), poorly reliable (6-12 days), reliable (13-24 days), and highly reliable (> 24 days) values.

6. DALTON MINIMUM AND THE LOST CYCLE

The Dalton minimum (DM) around 1800 is a period when SN was reduced to the maximum values of about 30. The beginning of the Dalton minimum is quite exceptional in the SN series. The years 1790-1794 were very poorly covered by sunspot observations (Fig. 8), probably because of the unstable political situation in Europe after the French revolution in 1789. The WSN series was interpolated during those years leading to abnormally long cycle #4 (about 14, 17 and 14.5 years using min-min, max-max or median cycle length definitions). A strong anomaly in the cycle evolution during 1790's was found by many authors. E.g., Sonett (1983) suspected an error in the WSN series, and Wilson (1988) found that cycle minima were likely misplaced for cycles 4, 5 and noted that "clearly, something is amiss in Hale cycle 3". Sello (2000) studied the wavelet entropy (a measure of disorder in the time series) of SN series. The absolute maximum of entropy falls onto the cycle 4 which implies high disorder in the SN data during that time. It has earlier been suggested that this anomaly is related to a phase catastrophe of solar cycle evolution due to the very long (> 10 years) descending phase of cycle # 4 when the SN evolution was not cyclic but linear (see Fig. 3a), probably due to the linear interpolation over sparse data points (Vitinsky et al. 1986; Kremliovsky 1994). Moreover, the fact that the G-O rule suffered the phase reversal at this time implies that the cycle numbering could be distorted circa 1800. Usoskin et al. (2001b) suggested that one small cycle could have been lost in 1790's because of sparse and unreliable observations during that time. The new cycle (1793–1800) would be the first and smallest cycle of the Dalton minimum. The suggested time profile of SN during this interval is shown in Fig. 9. This would remove all the problems related to the phase catastrophe, making the solar activity evolution cyclic again (see Fig. 3b). It also makes the scenario of the Dalton minimum similar to that of the Maunder minimum: sudden descend of a normal high cycle to the lowest level without a precursor, followed by a gradual restoration of the activ-

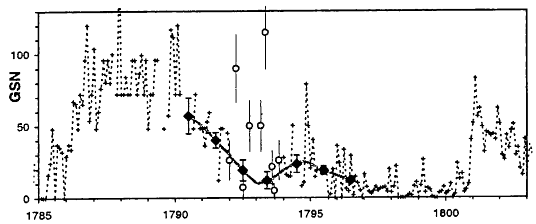

Figure 9. Group sunspot numbers circa 1800 (after Usoskin et al. 2003). GSN values outside 1792–1793 are shown by the dashed curve, while open dots with error bars depict the estimated monthly means and their standard errors in 1792–1793. The solid diamonds present the estimated weighted annual averages in 1790–1796 with the spline fit to them.

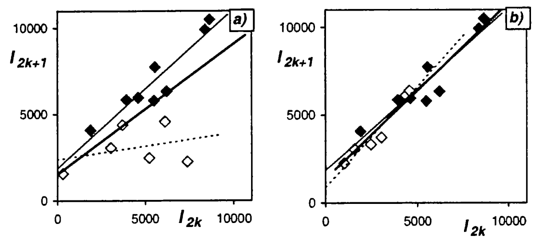

Figure 10. Illustration of the Gnevyshev-Ohl rule: intensities of odd sunspot cycles vs. even cycles (after Usoskin et al. 2001b). a) Standard cycle numbering; b) Including the new cycle. Open (filled) diamonds correspond to the interval before (after) the Dalton minimum. Dotted, thin and thick solid lines give the linear correlation before and after the Dalton minimum, and for the entire period (1610–1996), respectively.

ity level. The new cycle restores the order of the G-O rule of cycle pairing making it valid now since 1610 (see Fig. 10). Also, the new cycle does not distort the cycle length distribution or Waldmeier relations (Usoskin et al. 2002).

Unfortunately, the absence of latitudinal or magnetic polarity information of sunspots during the period under question makes it impossible to verify the suggested lost cycle directly, and one has to use indirect evidences. Usoskin et al. (2002) demonstrated that, e.g., data on visual aurorae depict a peak in 1796 supporting the idea of the lost cycle. Also, direct measurements of the geomagnetic field declination yield a similar pattern, giving further support to the idea (Mursula et al. 2003). The sensitivity of cosmogenic ^{10}Be and ^{14}C isotopes is not enough to resolve the small cycle (Usoskin et al. 2002). Recently Krivova et al. (2002) called the existence of the new cycle in question claiming that the suggested new minimum in 1793 contradicts with SN statistics. However, as argued in (Usoskin et al. 2003), the SN statistical features during 1792–1793 are typical for sunspot activity during the times of sunspot minima, thus favoring the additional minimum in 1793.

7. SUMMARY

In this brief review we have discussed some recent achievements, mostly experimental and phenomenological, in the study of long-term solar cycle evolution. These new findings became possible, thanks to a tremendous work of Hoyt & Schatten (1998), due to the recently completed series of group sunspot numbers with all raw information available. The theoretical basis is due to recent developments of the dynamo and magnetic flux buoyancy theories. Below we list some interesting recent findings.

- It has been demonstrated that randomness is an essential intrinsic factor of sunspot cycle, although its nature is not yet completely understood. This implies a stochastic or chaotic component which limits the predictability of the solar cycle. An essential random component (either stochastic or chaotic) is found necessary in recent theoretical models.
- The cyclic nature of sunspot activity during the Maunder minimum has been re-analyzed. A 22-year cycle was dominant during the deep Maunder minimim (1645–1700), and the 11-year cycle was gradually emerging towards the end of the minimum period.
- A persistent 22-year cycle with stable phase and roughly constant amplitude is present throughout the whole sunspot number series. The relation between the 11-year and the 22-year cycles (the latter was dominant during the Maunder minimum and sub-dominant with constant amplitude during the normal activity times) has been interpreted as a result of the mean field dynamo in the presence of a weak constant relic (fossil) magnetic field in the Sun.
- It has been suggested that one small solar cycle could have been lost in 1790's because of extremely sparse sunspot observations. Introduction of this new cycle removes the phase catastrophe in the beginning of the Dalton minimum and restores the order of the Gnevyshev-Ohl rule of cycle pairing throughout the whole 400-year long sunspot number series. Although the existence of this cycle cannot be confirmed directly, there is other independent, indirect evidence favoring such an idea.
- A unified scenario of a great minimum has been suggested: a sudden suppression of sunspot activity to the lowest level (immersion of the cycles) followed by a gradual, nearly linear restoration of the activity level through emergence of cycles. Such a behaviour is in agreement with some stochastically forced dynamo models.

These results should be accounted for and explained by realistic dynamo and magnetic flux emergence theories.

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