Digital PID

Three terms (gain , integral and derivative) in PID control in continuous time are following

Proportional control: $u(t) = Ke(t)$

Integral control: $u(t) = \frac{K}{T_i} \int_0^t e(h)\,dh$

Derivative control: $u(t) = KT_d\,\dot{e}(t)$

The approximations of three terms that can be implemented into the digital controller are:

Proportional control: $u(k) = Ke(k)$

Integral control: $u(k) = u(k-1) + \frac{K}{T_i} Te(k)$

Derivative control: $u(t) = \frac{KT_d}{T_i}[e(k)-e(k-1)]$

Digital PID

The differential equation relating $u(t)$ and $e(t)$ *is*

$$\dot{u} = K(\dot{e} + \frac{1}{T_i} e + T_d \ddot{e})$$

By using the Euler method $\dot{e} = \frac{e(k) - e(k-1)}{T}$, $T = $ sample time

Control derivative: $\dot{u} = \frac{u(k) - u(k-1)}{T}$

Proportional part: $K\dot{e} = K \frac{e(k) - e(k-1)}{T}$

Integral part: $K/T_i e(k)$

Derivative part: $f(k) = \frac{e(k) - e(k-1)}{T} = \dot{e}(k)$

$$\ddot{e}(k) = \dot{f}(k) = \frac{(e(k) - e(k-1)) - (e(k-1) - e(k-2))}{T^2} = \frac{e(k) - 2e(k-1) + e(k-2)}{T^2}$$

Digital PID

That results the form of the digital PID controller

$$u(k) = u(k-1) + K\left[(1 + \frac{T}{T_i} + \frac{T_d}{T})e(k) - (1 + 2\frac{T_d}{T})e(k-1) + \frac{T_d}{T}e(k-2)\right]$$