



Digital Control Theory

Student notes: lectures and exercises

written by

Jenő Kovács, D.Sc.(Tech.)

1996-2003

Systems Engineering Laboratory
Department of Process Engineering
University of Oulu

PL 4300 Linnanmaa, 90014 Oulun yliopisto
tel.: +358 (0)8 553 2432
fax: +358 (0)8 553 2439

Introduction

The present book, **Digital Control Theory**, has been developed for undergraduate studies in the field of systems and control engineering. The book aims to summarise the essential topics on digital control theory. It is assumed that the reader is familiar with the basics of systems and control engineering, the continuous-time control design and the basic understanding of Z transformation.

The book is based on the graduate course, “Digitaalinen Sääntöteoria 47453S”, given at the Systems Engineering Laboratory, University of Oulu in the period of 1996-2002.

The book serves as a textbook: describes the theoretical background deep enough to understand and execute the related exercises. It has three main parts: 1) introduction to the computer controlled systems and the sampling phenomenon, 2) description of the process modelling in discrete-time domain and finally 3) introduction of several discrete-time design methods.

The following references were used in completing the book:

G. Franklin, J.D. Powell & M. Workman (1998): Digital Control of Dynamic Systems. Addison-Wesley, third edition.

I.D. Landau (1990): System Identification and Control Design. Prentice-Hall, Inc.

K. Ogata (1995): Discrete-time Control Systems. Prentice-Hall, Inc., second edition.

K. Ogata (1995): Solution manual – Discrete-time Control Systems. Prentice-Hall, Inc., second edition.

Ch.L. Phillips, (1990): Digital control system analysis and design. Englewood Cliffs (N.J.) Prentice-Hall Inc., second edition.

K.J. Åström & B. Wittenmark (1997): Computer-Controlled Systems. Prentice-Hall, Inc., third edition.

K.J. Åström & B. Wittenmark (1997): Solutions Manual - Computer-Controlled Systems, Theory and Design. Prentice-Hall, Inc., first edition.

September, 2003.

Jenő Kovács, D.Sc.(Tech.)

Content

Introduction		II/2.3 Transient behaviour of the sampled system	103
I Computer controlled systems	1	II/2.4 Approximation of continuous-time systems	111
I/1 Computer controlled system	1	II/2.4.1 Approximation versus Z transform	112
I/2 Sampling continuous-time signals	4	II/2.4.2 Frequency prewarping	113
I/2.1 Sampling theorem	5	II/2.4.3 Response invariance approximation methods	114
I/2.2 Properties of periodic sampling	9	III Digital controller design methods	126
I/2.3 Choice of sampling period	15	III/1 Basic principles for design of digital controllers	127
I/3 Z transform	16	III/2 Structure of the digital PI controller	130
I/3.1 Z transform of elementary functions	17	III/3 Pole-placement design	142
I/3.2 Properties of Z transform	22	III/3.1 Regulation	143
I/3.3 Derivation of the Z transform	23	III/3.2 Zero steady-state error	143
I/3.4 Inverse Z transform	29	III/3.3 Tracking	144
I/3.5 Discrete models of sampled data systems	36	III/4 Pole-zero placement	158
I/3.6 Block-diagram algebra of discrete-time systems	50	III/4.1 Pole-zero placement with implicit reference model	163
II Discrete-time modelling and analysis	54	III/4.2 Pole-zero placement design with observer	176
II/1 Discrete-time modelling	56	III/4.3 Influence of the observer polynomial	177
II/1.1 Input-output approach: using Z transform	56	III/5 Deadbeat control design	182
II/1.2 Input-output approach: using shift-operator	58	III/6 Intersampling ripple	186
II/1.3 State-space approach	62	III/7 Time optimal (finite settling time) control design	189
II/1.4 Complex digital systems	70	III/8 Optimal design methods: minimum-variance control	200
II/1.5 Transformation between different system models	72	III/8.1 Models for disturbances	200
II/1.6 Controllable and observable canonical form	77	III/8.2 Minimum-variance control	203
II/2 Discrete-time analysis	87	III/8.3 General minimum-variance control	208
II/2.1 Controllability and observability	87	III/8.4 General minimum-variance control - R-S-T form	214
II/2.2 Stability of discrete-time systems	95	III/8.5 Control system design via pole placement in state-space	216

Appendix: Jury's stability	
Appendix: Linear equation set solution	ix
Appendix: Diophantine equation	Xiii
Appendix: How o calculate the desired characteristic polynomial?	xviii