OPTIMIZATION USING LEARNING AUTOMATA AND CONFIDENCE PROBABILISTICS

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Outline

- Introduction
- Learning automata
- Confidence probability
- Hybrid scheme
- Simulation experiments
- Conclusions

Motivations

Applications in process engineering

- identification, control, monitoring, optimization

Stochastic learning automata

- optimisation of multimodal functions in noisy environments
- in a feedback loop with the environment (on-line)
- very few assumptions (function, noise, search space)
- simple machines
- proofs on optimality and convergence available
- convergence to optimal action
- IEEE Trans SMC-B **32**(6)

Confidence probabilistics

- optimization in discrete event systems
- optimization based on a given set of data (off-line), optimal # samples
- based on statistics and probabilistics
- Gaussian noise
- complex computations, or approximations
- ordinal optimization

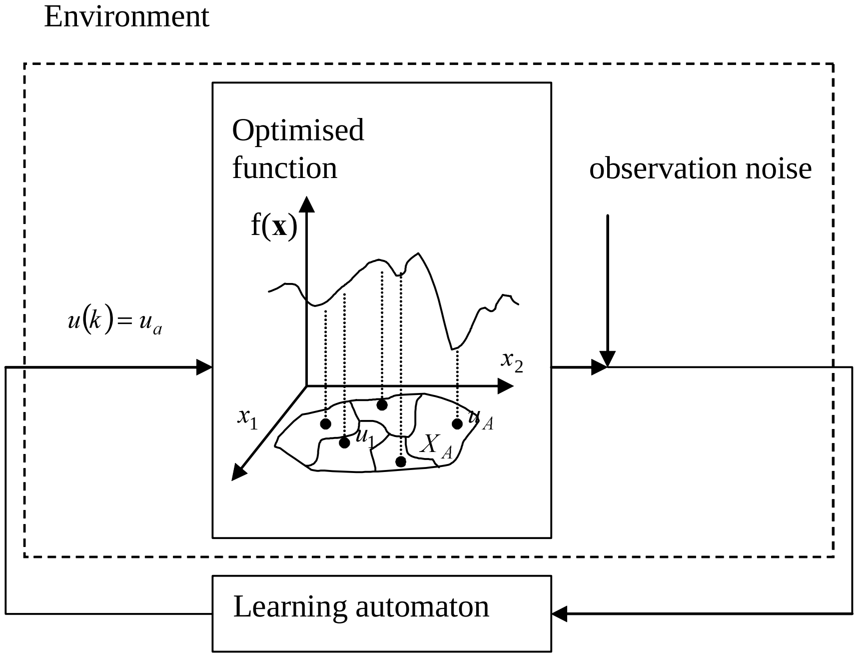
Efficiency in terms of loss

- on-line: profit

Efficiency in terms of samples

- on-line: time

Operation of a learning automaton



1. Select an action **u**(i) from the action set $\mathcal{U}$ based on the probability distribution **p**.
2. Calculate the normalized env. response $\overline{\xi}_n$
3. Adjust the probability vector **p** (reinforcement).
4. Return to step 1.

Reinforcement schemes

- linear / nonlinear; continuous / discretized; estimator; pursuit; teams; hierarchies; etc.
- ASELA (Absorbing Stochastic Estimator Learning Algorithm) (Papadimitriou et al. 2001)

$$\overline{\xi}_n^{\mathrm{ASELA}}(i) = \overline{\xi}_n(i) + \zeta$$

where $\zeta \sim N\left(0, \frac{\gamma^{\mathrm{ASELA}}}{c_n(i)}\right)$

$$i_n^{*,\mathrm{ASELA}} = \arg\min_i \overline{\xi}_n^{\mathrm{ASELA}}(i)$$

Probability update:

$$p_{n+1}(j) = \begin{cases} \max\left(0, p_n(j) - \frac{1}{N^{\mathrm{ASELA}}}\right) & j \neq i_n^{*,\mathrm{ASELA}} \\ 1 - \sum_{k \neq i^*} p_{n+1}(k) & \text{otherwise} \end{cases}$$

Approximation of confidence probability

- Consider $s_n(i)$ as random variables $S_n(i)$ with time-varying distributions, n = 1,2,...
- Confidence probability:

$$CP_n = \Pr\left(\bigcap_{j=1, j\neq i_n^*}^{I} S_n(i_n^*) < S_n(j) \right)$$

where $i_n^* \in \mathcal{J} = \arg\min_{i\in \mathcal{J}} s_n(i)$

- Lower bound approximation for CP_n:
  - o ACP (Chen 1996)

$$\underline{CP}_n(i_n^*) = \prod_{j=1, j\neq i_n^*}^{I} \Pr(S_n(i_n^*) < S_n(j))$$

  - o Chernoff bounds (Chen et al. 1999)

$$\Pr(S_n(i_n^*) < S_n(j)) \geq 1 - \exp\left(\frac{-\delta_n^2(i_n^*, i)}{2\left(\frac{v'_n(i_n^*)}{c_n(i_n^*)} + \frac{v'_n(i)}{c_n(i)} \right)} \right)$$

where $\delta_n(i_n^*, i) = (s_n(i_n^*) - s_n(i))$

On-line estimates

- counter of selections: $c_n(i)$
- sample mean of ξ: $s_n(i)$
- sample variance of ξ: $v'_n(i)$
- variance of $s_n(i)$: $v_n(i)$

Maximation of conf. probability

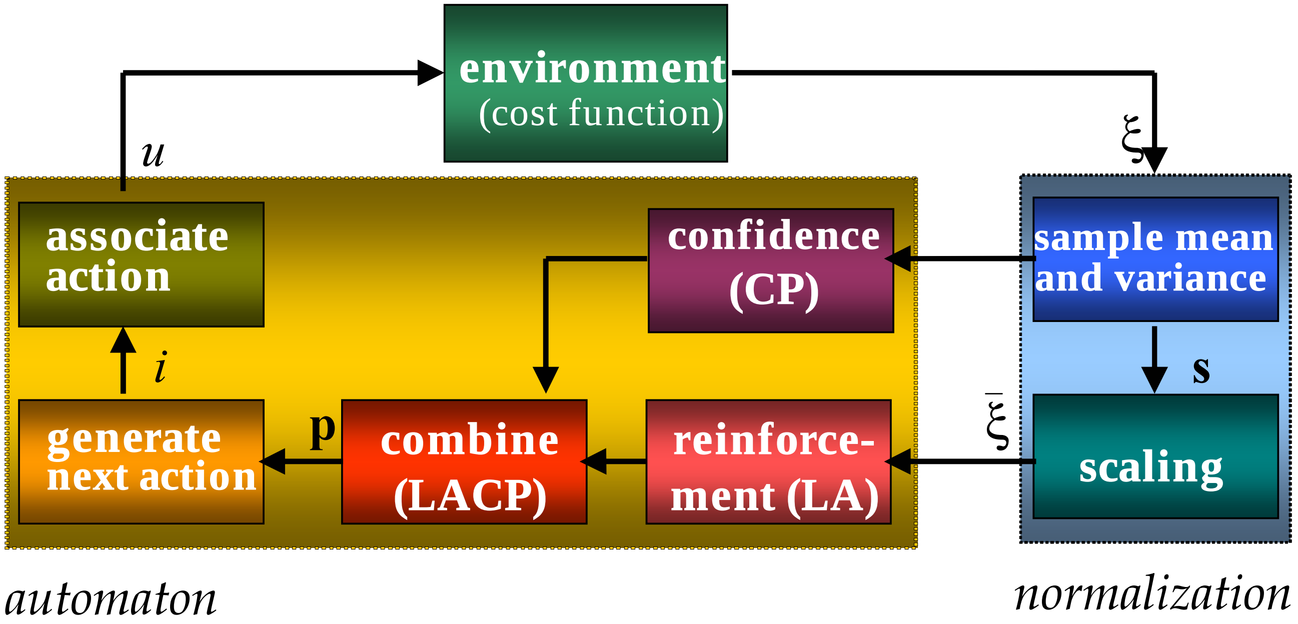
- Assume that the jth action is selected next
- Estimate of $S_{n+1}(i) | i_{n+1} = j$:

$$\hat{S}_{n+1|n}(i, j) = \begin{cases} N\left(s_n(i), \frac{v'_n(i)}{c_n(i)+1} \right) & i = j \\ N(s_n(i), v_n(i)) & \text{otherwise} \end{cases}$$

- Approximate $\hat{CP}_{n+1|n}(i_n^*, j)$
- 'Optimal' action

$$i_{n+1} \in \mathcal{J}_{n+1\ |n}^{**} = \arg\max_{j\in \mathcal{J}} \underline{\hat{CP}}_{n+1|n}(i_n^*, j)$$

Combined scheme



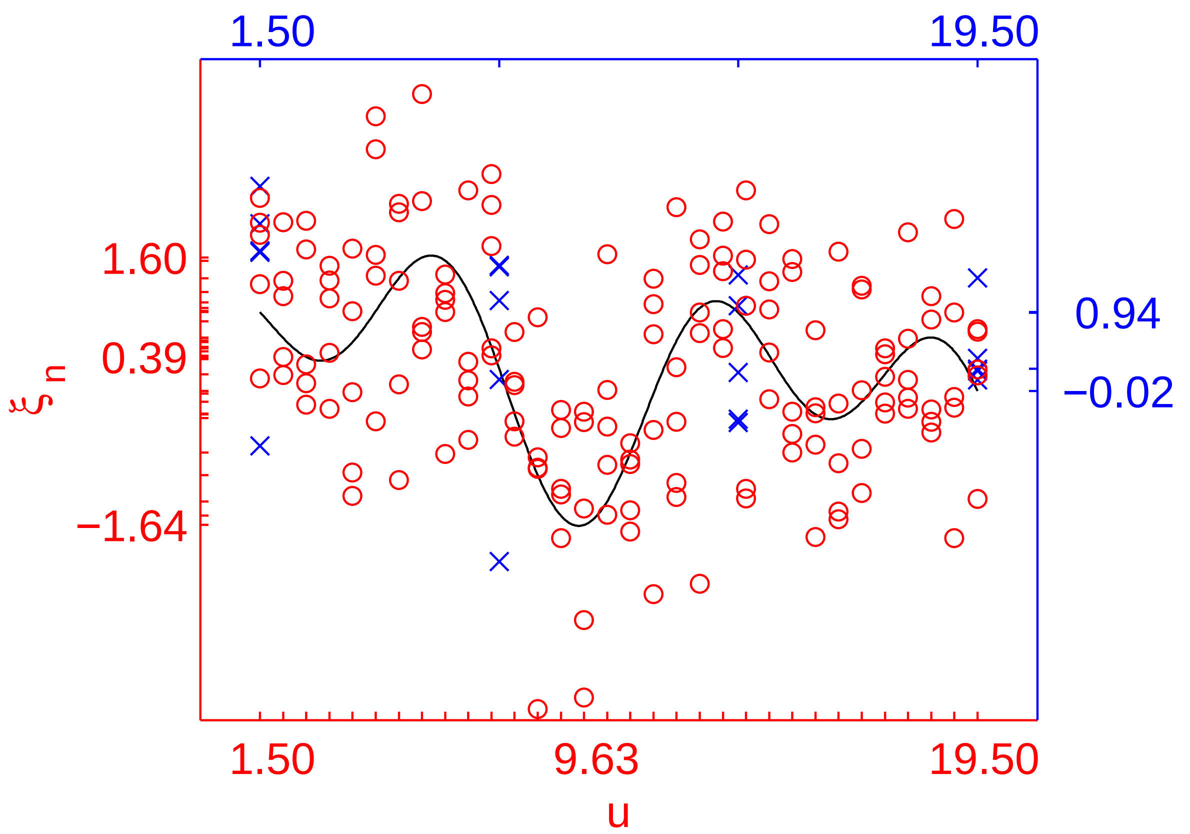
Constrain action probabilities

- action(s) with smallest sample mean: J_n^*
- action(s) that maximize increase in CP: $J_{n+1|n}^{**}$

$$q_{n+1}(i) \geq \max\left(\frac{p_n^{**}}{\mathrm{card}\, J_{n+1}^{**}}, p_{n+1}(i)\right) \forall i \in J_{n+1}^{**},\quad J_{n+1}^{**} = \left\{J_n^*, J_{n+1|n}^{**}\right\}$$

where $p_n^{**} = \theta_1\left(1 - \underline{CP}_n\left(i_n^*\right)\right) + \theta_2$; Use $\mathbf{p}_{n+1} \leftarrow \mathbf{q}_{n+1}$.

Numerical Experiments

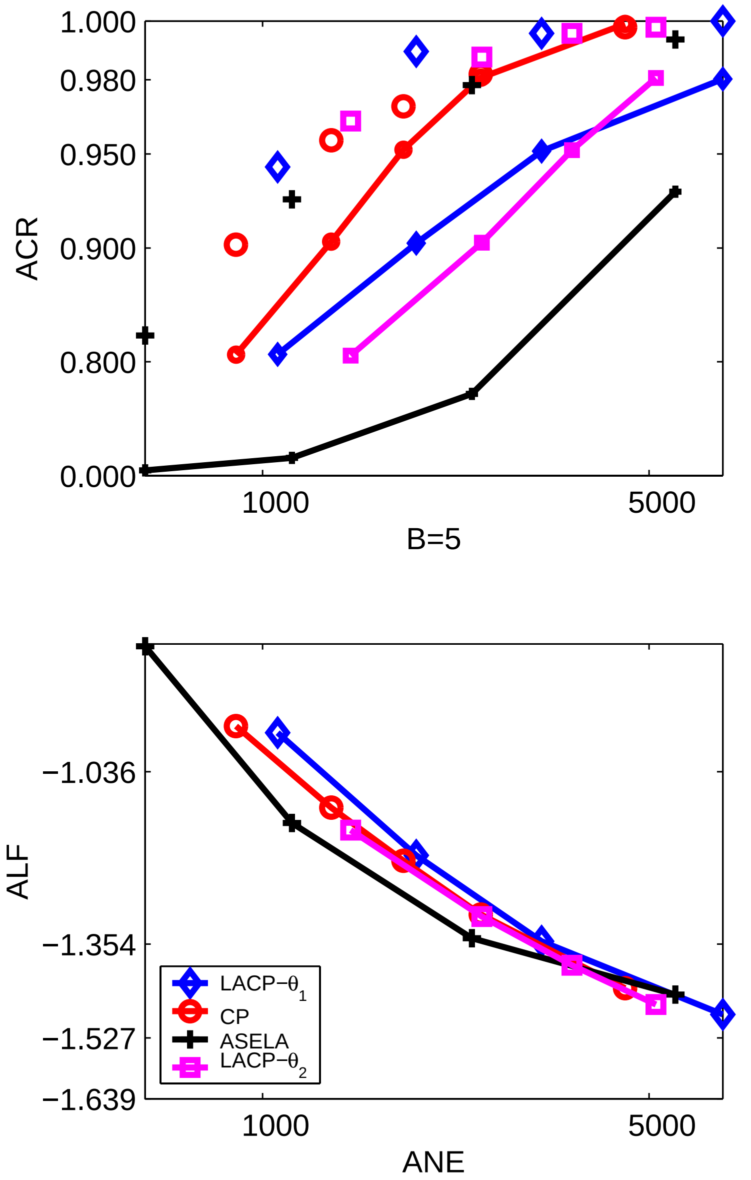


Sample realizations of the function to be minimized, I={4, 32}.

Averages on 750 test runs (I=32)

- lower estimates $\underline{CP}_n\left(i_n^*\right)$
  - ASELA: $N^{ASELA}=\{5I,20I,80I,320I\}$, $\gamma^{ASELA}=0.25$ +——+
  - Maximization of CP: $CP^{stop}=\{0.8,0.9,0.95,0.98\}$ o——o
  - LACP $\theta_1=0.5$, $\theta_2=0$: $N^{ASELA}=5/I$ -"- ◊——◊
  - LACP $\theta_1=0$, $\theta_2=0.1$: $N^{ASELA}=5/I$ -"- □——□

    $$p_n^{**}=\theta_1\left(1-\underline{CP}_n\left(i_n^*\right)\right)+\theta_2$$
- correct responses (ACR):
  - ASELA +
  - Maximization of CP o
  - LACP $\theta_1=0.5$, $\theta_2=0$ ◊
  - LACP $\theta_1=0$, $\theta_2=0.1$ □
- losses (ALF): $\frac{1}{n}\sum_{k=1}^{n}\xi_k$
  - ASELA +——+
  - Maximization of CP o——o
  - LACP $\theta_1=0.5$, $\theta_2=0$ ◊——◊
  - LACP $\theta_1=0$, $\theta_2=0.1$ □——□
- Comparable parameterization ?



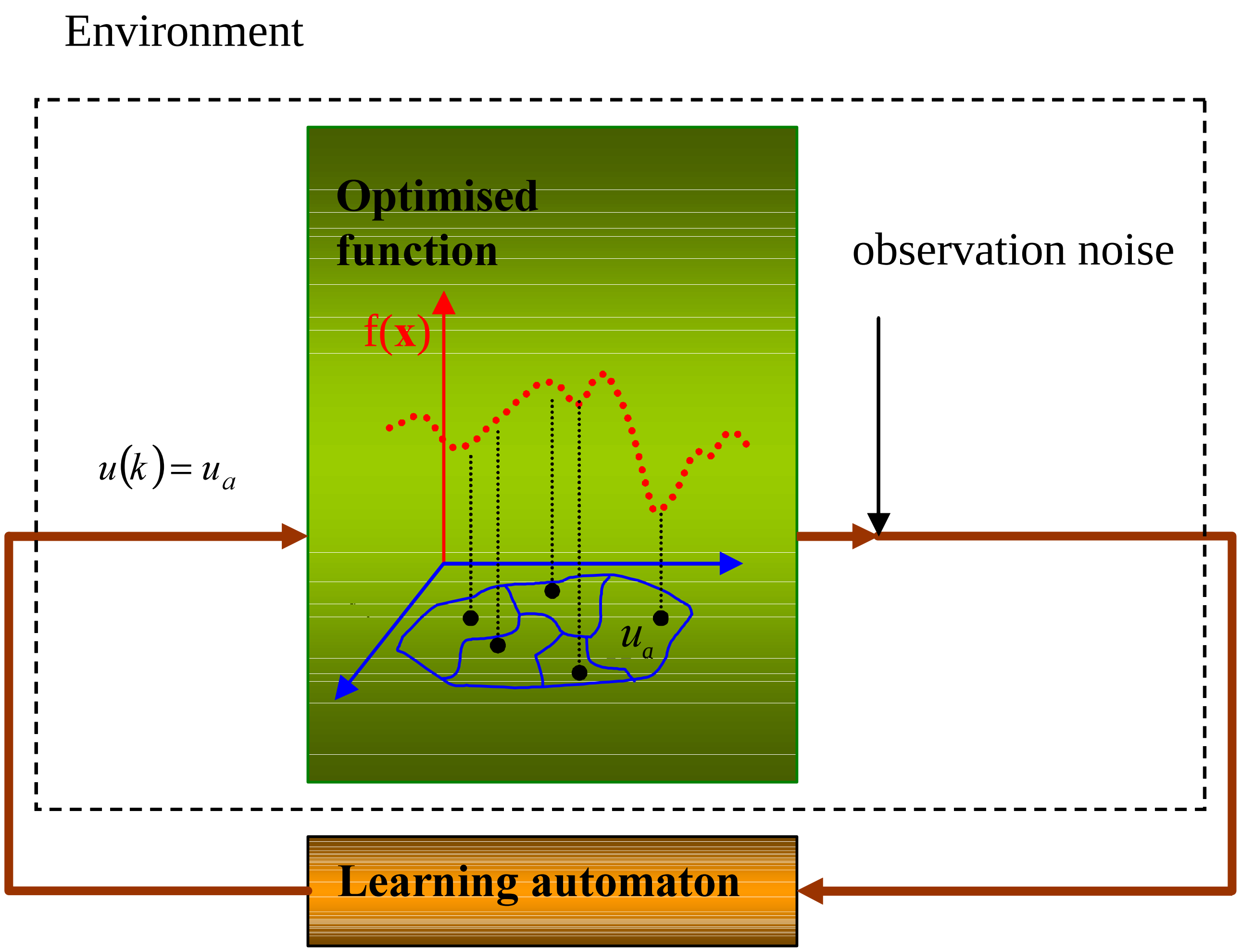
Conclusions

- Combine learning automata & confidence probabilistics
- Detailed algorithm & simulation example

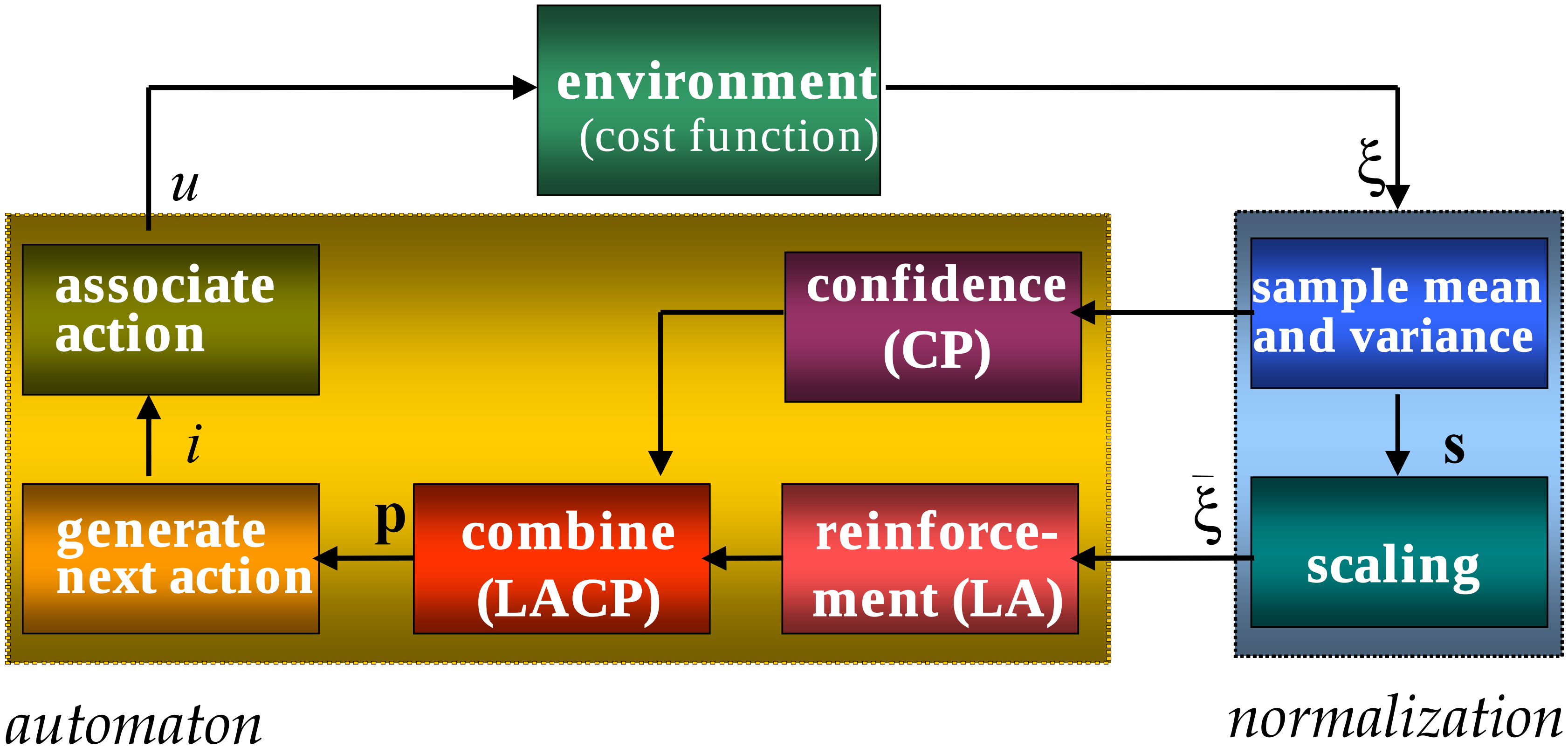
- **Improved feasibility** in process engineering applications:
  - one plant in **on-line** operation
  - slow processes
  - expensive to experiment
  - noisy signals
  - non-linear problems

  - a measure for **goodness** of solution
  - less sensitive in the selection of learning algorithm **parameters**
  - less samples & less **time** *vs*. increased costs / time => **viability**

  - random search *vs*. gradient-based techniques
    - **discretization** of search space *vs*. continuous space
    - theoretically justified *vs*. **efficient** in practice



Environment
Optimised function
f(x)
observation noise
u(k) = u_a
u_a
Learning automaton



environment
(cost function)
u
ξ
associate action
confidence (CP)
sample mean and variance
i
s
ξ̄
generate next action
p
combine (LACP)
reinforce-ment (LA)
scaling
automaton
normalization