Gain scheduling control with Markov transition models

Enso Ikonen
University of Oulu (Finland)
Department of Process and Environmental Engineering
Systems Engineering Laboratory

Outline:

1. Introduction & Adaptive CFMC control
2. Controlled Finite Markov Chains
   MDP, Matlab-toolbox
3. Use of Kullback–Leibler distance
   in adaptive CFMC control
4. Numerical examples
5. Discussion and Conclusions

The Seventh IEEE International Conference on Control and Automation
December 9 - 11, 2009, Christchurch, New Zealand

Introduction

- Model-based control
  - Non-linearities
  - Uncertainties
  - Learning
  - Optimality
- Applications in process engineering
  - Rough models available, Experimention is expensive, On-site tuning required, ...

- In this paper:
  - New adaptive control technique
    - Multiple models and CFMC design
  - New simulation results
    - Feasible domain of applications
  - Promote discussion
    - CFMC techniques in process control

Adaptive CFMC control

- *“Adaptive systems use local (in time and data spaces) approximations around behaviour realizations”* (Kárný et al.)

- Adaptation backed-up by
  - Physico-chemical arguments
  - Prior information
- Avoid (many of the) problems in structure selection and parameter estimation

- Adaptive control based on multiple models:
  - Bank of models
    - *A priori* defined models
    - Physical arguments
  - Off-line control design
    - Model-based design
    - Control specifications
  - On-line selection
    - Choose model/controller to use
    - Based on measured information

Bank of models

- Models based on physical arguments
  - e.g., ordinary differential equations (ode)

$$\frac{d\mathbf{x}}{dt} = f\left(\mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t)\right)$$

  - Bank of models based on unknowns
    - parameter values, reaction paths, etc.

- Conversion to discrete-time format
  - discretization (in time)

$$\mathbf{x}(k) = f\left(\mathbf{x}(k-1), \mathbf{u}(k-1), \mathbf{w}(k-1)\right)$$
$$\mathbf{y}(k) = h\left(\mathbf{x}(k), \mathbf{v}(k)\right)$$

  - states x
  - actions u
  - disturbances w, v

Controlled Finite Markov Chains (CFMC)

- Discrete-time dynamic system

- Finite state model
  - Discretization in space

$$\mathbf{x} \in X_s, s = 1,2,\ldots,S \qquad X = \bigcup_{s=1}^{S} X_s, X_i \cap X_j \neq 0$$

$$\mathbf{u} \in U_a, a = 1,2,\ldots,A \qquad \text{"}$$

$$\mathbf{y} \in Y_m, m = 1,2,\ldots,M \qquad \text{"}$$

$$\mathbf{p}_X(k+1) = \mathbf{P}^{a(k)}\mathbf{p}_X(k)$$

$$p_{s',s} = \int_{X_{s'}} p\left(\mathbf{x}(k) \in X_{s'} \middle| \mathbf{x}(k-1) \in X_s, u(k-1) \in U_a\right) d\mathbf{x}$$

where $\mathbf{P}^{a(k)} = \left[p^a_{s',s}\right]$ is a transition probability matrix

---

- State estimator

$$\mathbf{p}_X^{est}(k) \propto \mathbf{l}_m * \mathbf{P}^{a(k)}\mathbf{p}_X(k-1)$$

$$l_{m,s} = \int_{Y_m} p\left(\mathbf{y}(k) \in Y_{s'} \middle| \mathbf{x}(k) \in X_s\right) d\mathbf{y}$$

where $\mathbf{L} = \left[l_{m,s}\right]$ is a likelihood matrix

Markov Decision Process (MDP)

- Infinite-horizon discounted model

$$J_{s(0)} = \sum_{k=0}^{\infty} g^{k} r_{s(k)}^{p_{s(k)}}$$

$r_s^a = r(s(k), a(k))$ immediate cost

g discount factor

- Bellman's optimality

$$J_s^* = \min\left[r_s^a + g \sum_{s' \in S} p_{s',s}^a J_{s'}^* \right]$$

  - Dynamic programming
  - Value iteration
  - Optimal policy

$$p_s^* = \arg\min_a \left[r_s^a + g \sum_{s' \in S} p_{s',s}^a J_{s'}^* \right]$$

MATLAB Toolbox

cc.oulu.fi/~iko/MGCM

- Comp. load
  - Memory requirements

    $S = \prod n_i$ cells

    SSA transition probabilities

    SA immediate costs
  - Model evaluations

    SA times precision
  - Value iteration

    SA size of Q-matrices
- **P**^{a} is sparse + vectorization

CFMC control design procedure

1. Set model resolution

   X_s, U_a, Y_m, sampling time, ...
2. Set control targets

   r_s^a, g
3. Build & analyse plant model (via simulation)
4. Solve controller (value iteration)
5. Build & analyse closed-loop (stability, comm.classes, basins-of-attraction, transition times, ...)

Kullback – Leibler distance

- Comparison of distributions
  - K–L distance

$$D(R\|S) = \sum_{s\in S} R(s)\log\frac{R(s)}{S(s)}$$

  - K–L distance between conditional distributions

$$\overline{D}(R\|S_q) = \sum_{(s,a)\in S\times A} R(s,a) D\left(R^{sa}\|S_q^{sa}\right)$$

$S(s) = \Pr(s)$ model

$R(s) = \dfrac{N(s)}{k}$ data

$N(s)$ count of occurances

k data length

pmf's:

$S_q^{sa} = S_q(s'|s,a)$ model q

$R^{sa} = R(s'|s,a)$ data

$R(s,a) = \dfrac{N(s,a)}{k}$

Model selection

- Empirical distribution (*R*)
  - Count from a sequence of plant operating data

$$R^{sa}\left(s'\right)=\frac{N\left(s'\middle|s,a\right)}{k}$$

- Model-based distribution (*S*)
  - Corresponding prediction using model q

$$S_q^{sa}\left(s'\right)=p_{s',s}^{a} \quad \text{model } q$$

$$q^{*}=\arg\min_{q}\sum_{(s,a)}D\left(R^{sa}\middle\|S_q^{sa}\right)$$

Adaptive control algorithm

- Procedure

  1. Select plant descriptions
  2. Build CFMC maps
  3. Set up and solve control problems
  4. Select model / controller to use

- Design problems: how to select...
  - ...plant descriptions? (use prior info)
  - ...discretization?
  - ...tuning parameters?
  - ...when to switch?

Numerical examples van der Vusse CSTR

A. MTM/MDP CSTR control
B. GS of ideal CSTR
C. GS of non-ideal CSTR

- Continuos stirred tank reactor
- Convert substance A to product B

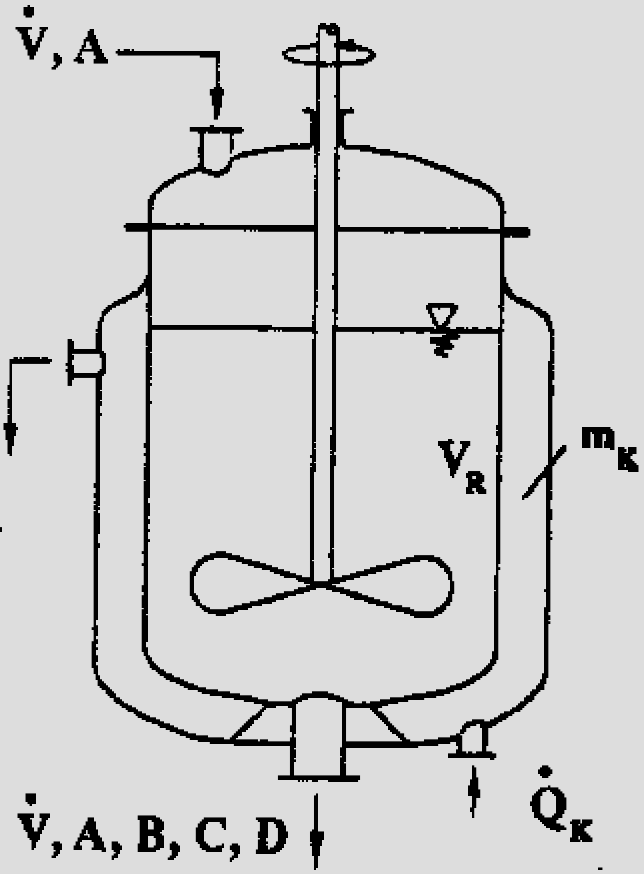
$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

$$2A \xrightarrow{k_3} D$$

$$\dot{c}_A = \frac{\dot{V}}{V_\mathrm{R}}\left(c_{A0} - c_A\right) - k_1 c_A - k_3 c_A^2$$

$$\dot{c}_B = -\frac{\dot{V}}{V_\mathrm{R}} c_B + k_1 c_A - k_2 c_B$$

- Nonlinear, non-minimum phase



Control of CSTR using MTM/MDP

- discretization
  - 861+1 state cells, c_A x c_B
  - 17 actions, V'/V_R
- model building
  - 100 evaluations / cell-action pair
  - ~2h on PC
- control design
  - for five temperatures 105,107.5,110,112.5,115 °C
    - 5 set points c_B^{ref} each
  - Table I for 110°C
    - # communicating classes
    - size of basin-of-attraction

$$c_A = \{1.00, 1.05, \ldots, 3.00\} \ (41)$$

$$c_B = \{0.50, 0.55, \ldots, 1.50\} \ (21)$$

$$\frac{\dot{V}}{V_R} = \begin{Bmatrix} 2,6,10,11, \\ 12,12.5,13.0,\ldots,16, \\ 17,18,20,26 \end{Bmatrix} \ (17)$$

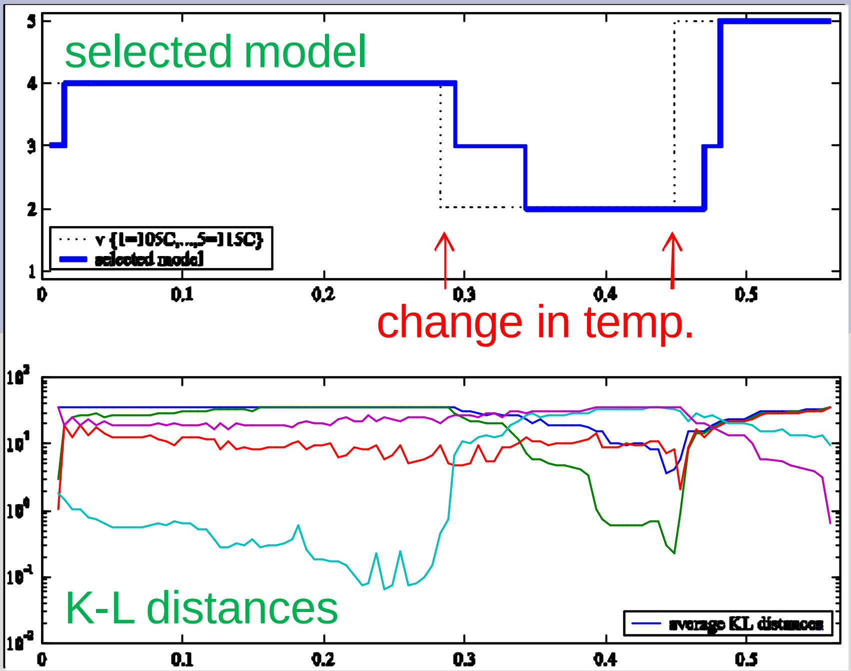
TABLE I

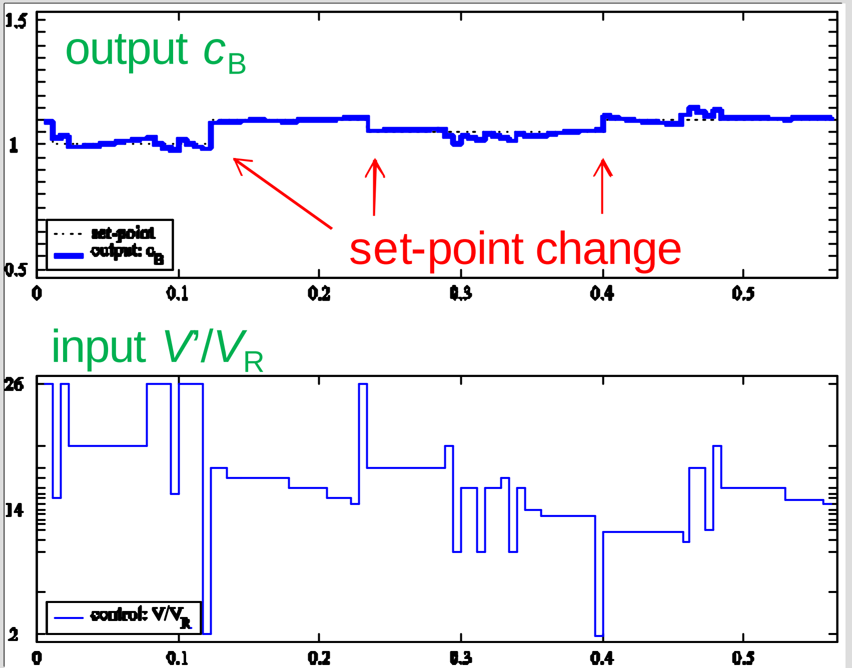
ANALYSIS OF CLOSED-LOOP PERFORMANCE FOR PLANT IN 110C.

c_B^{sp}	incl: #cc	#ba	only: #cc	#ba	τ_{ave}	τ_{max}
0.8	1	515.4	0	0	15.6	35.9
1.0	2	861	2	861	43.0	75.0
1.05	5	861	5	861	159.5	317.4
1.1	2	861	2	861	25.6	70.7
1.15	1	858.5	0	0	19.4	63.2

Gain scheduling of an ideal CSTR

- five models
  - for five temperatures
    105,107.5,110,112.5,115 °C
- – K–L distance to select between models
  - empirical distribution *R* from a moving window of *w* samples (*w*=20)



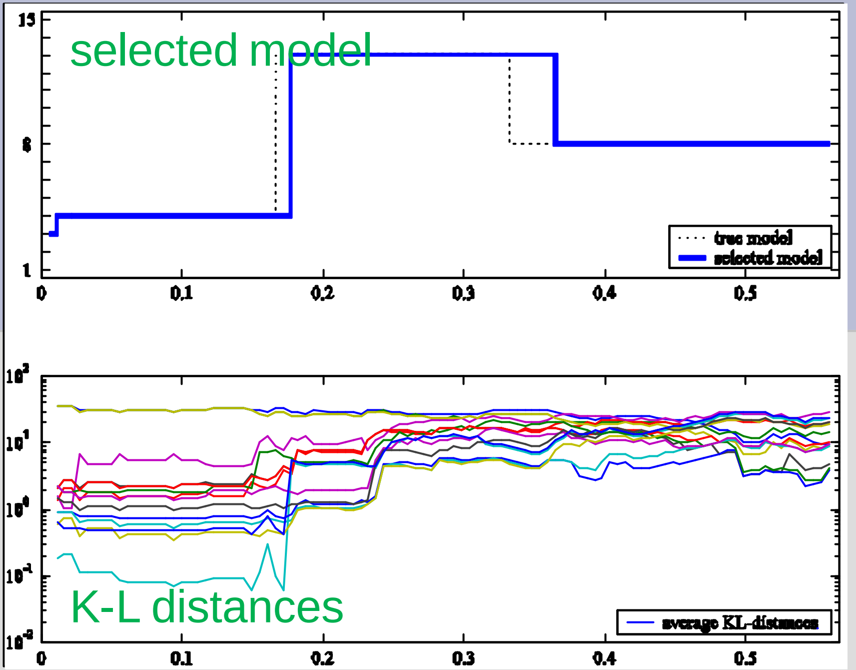


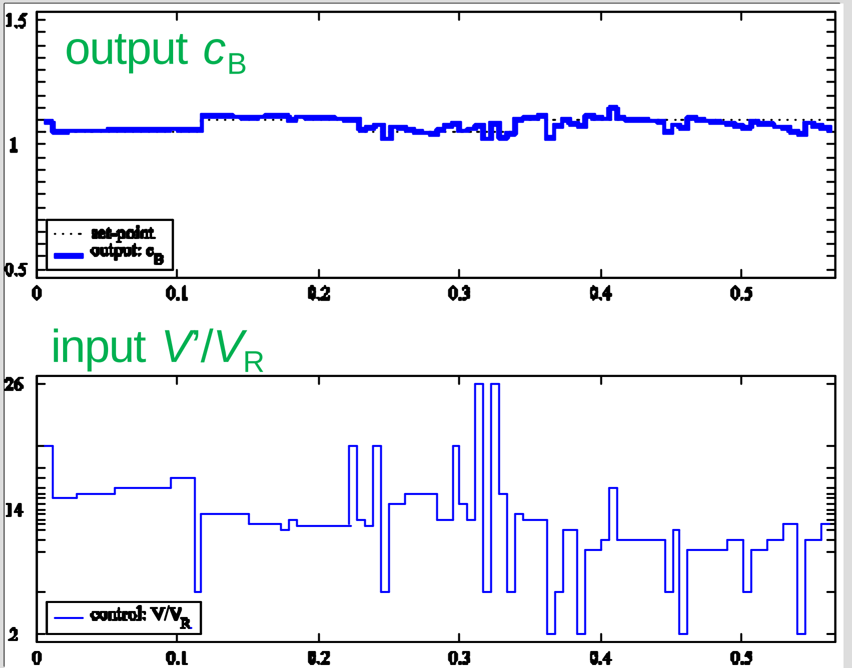
Gain scheduling of non-ideal CSTR

- two separate CSTR in different temperatures
- one measurement
  - random stream (model of channeling)
- 15 models:
  - 100% models (5)

    105,107.5,110,112.5,115 ℃
  - 50% – 50% models (10)

    105 +107.5 107.5+110 110+112.5 112.5+115
    105+110 107.5+112.5 110+115
    105+112.5 107.5+115
    105+115





Discussion & conclusions

- Aims:
  - Examine possibilities of MTM/MDP in control of industrial processes
  - Emphasis on physical models => adaptive control based on multiple models

- Results
  - K-L distance for comparing cond. distributions

- Future
  - State estimation
  - Adaptation of MTM's (machine learning) *vs*. approximate DP (e.g., smooth models) *vs.* prior models (process identification)

Discussion & conclusions

- Aims:
  - Examine possibilities of MTM/MDP in

THANK YOU

physical models → adaptive control based on multiple models

- Results
  - K-L distance for comparing cond.
  - State estimation
  - Adaptation of MTM's (machine learning) *vs*. approximate DP (e.g., smooth models) *vs.* prior models (process identification)