Lecture 1

Jussi Klemelä

September 15, 2009

1 Topics

1. Finanssi-instrumentit
2. Jakaumien estimointi
   (a) Yksiulotteisten jakaumien estimointi
   (b) Moniulotteisten jakaumien estimointi
3. Mallintaminen diskreeteillä ja jatkuvilla aikasarjoilla
4. Portfolion valinta
5. Hinnoittelun perusteet
   (a) Black-Scholes hinnoittelu
   (b) Hinnoittelu epätäydellisissä malleissa
6. Volatiliteetin mallintaminen

2 Instruments

We use the term "instrument" or "asset" to mean a financial instrument which can be bought and sold, like a stock, bond, gold, or a currency. We shall give an overview of basic assets that are available in financial markets. After that, we shall give an overview of derivative assets like futures and options, whose price is derived from the price of basic assets.

2.1 Stock Indeces

2.1.1 Definition of a Stock Index

Stock indices are usually averages of stock prices, weighted by market capitalization: the value of a stock index of N stocks at time t is calculated by formula

$$I_t = C \sum_{i=1}^{N} n_i S_t^i, \tag{1}$$

where C is a constant, n_i is the number of shares of stock i, and S_t^i is a suitably adjusted price of stock i at time t. The definition of the stock index depends on three parameters: the constant C, the numbers n_i, and the numbers S_t^i.

1. The constant C can be chosen, for example, to make the value of the index equal to 100 at a given past day. When the constitution of the index is changed, then the constant C is changed, to keep the index equal to 100 at the chosen day.

2. The value of n_i can be the total number of shares of stock i, but it can also be the number of freely floating stocks. Float market capitalization excludes stocks which are not freely floating (cannot be bought in the open market).

3. The value S_t^i is calculated differently depending on whether the index is a *pure price index* or a *total return index*.

   (a) Pure price indeces adjust the price S_t^i to stock splits, spin-offs, and exceptional dividends.

   (b) Total return indeces take into account also the regular dividends.

   For statistical analysis of the price fluctuations one has to use total return indeces, or adjust the pure price indeces. The *adjusted close* of a stock is the daily closing price of a stock which is adjusted to cash dividends, stock dividends, stock splits, and also to more complex corporate actions, such as rights offerings. The calculation of adjusted closing price is often made by data providers.

2.1.2 Examples of Stock Indeces

U.S. Indeces We mention Dow Jones Industrial Average, S&P 500, and Nasdaq-100 stock indeces.

- Dow Jones Industrial average is an exception of the rule (1), because in its calculation the components are not weighted by the number of shares.

- S&P 500 was created at 4 March 1957. It was calculated back until 1928 and the basis value was taken to be 10 from 1941 until 1943.

  The S&P 500 is a price return index, but there exists also total return versions (dividends are invested back) and net total return versions (dividends minus taxes are invested back) of the S&P 500 index.

  The S&P 500 is a market value weighted index: prices of stocks are weighted according to the market capitalizations of the companies. Since 2005 the index is float weighted, so that the market capitalization is calculated using only stocks that are available for public trading.

- NASDAQ-100 is calculated since 31 Januar 1985. The basis value was at that day 250.

  NASDAQ-100 is a price index, so that the dividends are not included in the value of the index.

  NASDAQ-100 is a different index than Nasdaq Composite, which is based on 3000 companies. NASDAQ-100 is calculated suing the 100 largest companies in Nasdaq composite. NASDAQ-100 is a market value weighted index, but the influence of the largest companies is capped (the weight of any single company is not allowed to be larger than 24%).

  NASDAQ opening hours are from 9:30 until 16:00 (15:30 until 22:00 CET).

European Indeces We mention DAX 30 stock index.

- DAX 30 (Deutscher AktienindeX) was created at 1. July 1988. The basis value is 1000 at 31 December 1987.

  DAX 30 is a performance index (dividends are reinvested in calculating the value of the index).

  DAX 30 stock index is a market value weighted index of 30 largest German companies. Market value is calculated using only free floating stocks (stocks that are not owned by an owner which has more than 5% of stocks). Largeness is measured by taking into account both the free floating market value and the transaction volume (total value of

the stocks that are exchanged in a given time period). The weight of any single company is not allowed to be larger than 10%.

DAX is calculated using prices in XETRA. The opening hours of XETRA are 9:00-17:30 CET. Note that L-DAX (late DAX) is calculated also during 8:00-9:00 and 17:45-20:00 CET using the floor prices of Frankfurt stock exchange and X-DAX is calculated using DAX future prices which are available from derivative exchange Eurex during 8:00-22:00 CET.

3 Fixed Income Instruments

To put it simply, fixed income research studies how much one should pay today, in order to receive a cash payment at a future day. One dollar today is better than one dollar tomorrow, and fixed income research tries to quantify this phenomenon.

3.1 Zero-Coupon Bonds

Zero-coupon bond or pure discount bond is a certificate which gives the owner a nominal amount P at the future maturity time T.

3.2 Coupon Bearing Bonds

Most bonds make regular payments (coupons) before the final payment at maturity. A coupon bond can be defined as a series of payments $P_1, \ldots, P_N$ at times $T_1, \ldots, T_N$. The terminal payment contains the principal and the final coupon payment.

For example, a five year 4% semi-annual coupon bond with 1000\$ face value makes ten 20\$ payments every six months and the final payment of 1000\$. Thus $P_i = 20\$$ for $i = 1, \ldots, N-1$ and the last payment is $P_N = 1020\$$, where $N = 10$.

4 Currencies

The price of a currency is determined only in relation to some other currency. For example, we can trade the relation of Euro to U.S. dollar or the relation of Euro to Franc of Schwizerland.

5 Derivatives

5.1 Forwards and Futures

5.1.1 Forward

A forward is a contract written at time t_0, with a commitment to accept delivery of (or to deliver) the specified number of units of the underlying asset at a future date t_1, at forward price F_{t_0}. The current price of the underlying is called the spot price. At time t_0 nothing changes hands, all exchanges will take place at time t_1. A long position is a commitment to accept delivery at time t_0. A short position is a commitment to deliver the contracted amount.

5.1.2 Futures contract

The instrument is called futures contract if the forward commitment is made through a homogenized contract (the size, expiration date are preset), if the trading is done in a formal exchange, and if there is formal mark-to-market. Futures come with daily marking to market. Forward contracts may not require any marking to market until the expiration day.

5.2 Options

5.2.1 Calls and Puts

An European call option C_t gives the right to buy an asset at the given expiration time T at the given strike price K. An European put option P_t gives the right to sell an asset at the given expiration time T at the given strike price K. Thus the value of the European call option is at the expiration time T equal to

$$C_T = \max\{S_T - K, 0\}$$

and the value of the European put option is

$$P_T = \max\{K - S_T, 0\}.$$

5.2.2 Exercise Time

The buyer of a call or a put option receives the right to buy or sell the underlying instrument. There exists three basic modes concerning the right to exercise the option:

1. An European option can be exercised only at the expiration date.

2. An American option can be exercised at any time before the expiration date. Thus an American option is more expensive than the corresponding European option.

3. A Bermudan option can be exercised at more than one time before the expiration.

We shall mostly discuss European options, and below the term "option" without a further qualification refers to an European option.

5.2.3 Uses of Options

Options can serve many different purposes. We mention the following uses of options.

1. Aristotle tells about philosopher Thales of Miletus (625-547 B.C.) who bought options on olive presses (a right to use olive presses). A large crop of olive followed, and he was able to sell with profit the right to use presses. Shorting the price of olives would have involved an unlimeted downside risk, unlike the strategy of buying options (in effect, buying put options on the price of olives).

2. A call option can be used to give a compensation to managers.

3. Options provide insurance: buying a put option gives an insurance in the case one has to sell in a future time an asset one possesses and buying a call option gives an insurance to the case when one has to buy in a future time an asset one does not possess.

    - Buying a put or a call option on a stock gives an insurance policy for an investor.
    - Buying a put option on an exchange rate gives an insurance policy for a company receiving payments on a foreign currency.

4. Options may be used in speculation because they allow the use of leveraging, speculating on falling prices and profit on various other schemes.

5.2.4 Exotic options

We say that an option is exotic if it is not an European or American call or put option. We have already mentioned Bermudean options, which we consider to be exotic. Other exotic options include Asian options and Barrier options.

1. The value of an Asian call option at the expiration is

$$C_T = \max\{0, M_T - K\},$$

where

$$M_T = \frac{1}{N}\sum_{i=1}^{N} S_{t_i}$$

with $t_1 < \cdots < t_N = T$ a grid of time points.

2. Barrier options disappear if the underlying either exceeds, or goes under the barrier. Alternatively, barrier option could have value only if it has exceeded, or went under the barrier. Knock-in options come into existence if some barrier is hit and knock-out options cease to exist if some barrier is hit. One speaks of up-and-out, down-and-out, up-and-in, down-and-in options. We have

vanilla call, strike K
= knock-in, K-call, barrier H + knock-out, K-call, barrier H.

5.2.5 Interest Rate Options

Caps, floors and swaptions are the basic interest rate derivatives.

Caps and Floors An interest rate cap is a derivative in which the buyer receives payments at the end of each period in which the interest rate exceeds the agreed strike price. A cap can be expressed as a series of caplets. The payoff of a caplet with notional value N, day count fraction τ, strike price at K, is

$$N \cdot \tau \cdot \max\{L - K, 0\},$$

where L is the underlying interest rate. For example, suppose that the six month USD LIBOR rate sets 3% at 1st of March. Then a caplet on the six month USD LIBOR rate, with the expiry at 1st of March, struck at 2%, with the notional of 1000 dollars, pays $1000\$ \times 0.5 \times (0.03 - 0.02)$.

A floor is a series of floorlets. The payoff of a floorlet is

$$N \cdot \tau \cdot \max\{K - L, 0\},$$

where L is the underlying interest rate. K is the strike price, N is the notional value, and τ is the day count fraction.

Swaption A *swap* is a commitment to exchange the payments originating from a fixed leg and a floating leg. The buyer of a *receiver swap* obtains payments with a fixed interest rate and pays payments with a variable interest rate. The buyer of a *payer swap* obtains payments with a variable interest rate and pays payments with a fixed interest rate.

A *swaption* is an option on a swap. The buyer of a swaption pays a premium and gets the right to enter an interest rate swap at the given date. A receiver swaption is a call on a receiver swap. It is an insurance against falling interest rates. A payer swaption is a call on a payer swap. It is an insurance against raising interest rates. The fixed rate of the underlying swap is called the strike of the swaption. In additon, we have to specify the expiration date of the option and the tenor (time to maturity at exercise of the option) of the swap.

6 Examination

Possible questions in the examination:

1) Explain (a) forward contract and (b) futures contract.

2) Define an European call option and an European put option.