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Pc 1 induced electromagnetic lift of background plasma in the magnetosphere

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Abstract. We discuss the ponderomotive redistribution of ions along geomagnetic field lines due to the action of Pc 1 pulsations, using a simple diffusion equilibrium model. The field-aligned component of the ponderomotive force is derived for Alfvén waves as well as for ion cyclotron waves propagating in a multicomponent plasma. Our theory gives a possibility for a better understanding of the problem and a convenient way to make rough estimates of the ponderomotive efficiency of Pc 1 waves under concrete magnetospheric conditions. Qualitative analysis of equations and satellite information on the intensity of wave fields show that the ponderomotive forces can have a pronounced effect on plasma. Ground-based observations of Pc 1 pulsations also suggest a ponderomotive redistribution of the background plasma.

1. Introduction

It is reasonable to think that the electromagnetic waves in the Pc 1 frequency range (0.2–5 Hz) may have a pronounced ponderomotive effect on plasma distribution in the magnetosphere. This supposition is supported by both theoretical considerations [*Lundin and Hultqvist*, 1989; *Guglielmi*, 1992; *Guglielmi and Pokhotelov*, 1994, 1996] and satellite observations [*Hultqvist et al.*, 1988; *Gustafsson et al.*, 1990; *Lundin*, 1988; *Kondo et al.*, 1990]. There is also circumstantial evidence deduced from the ground-based observations of Pc 1 pulsations. For example, we have suggested that the enhancement of Pc 1 pearl pulsations after magnetic storms [*Wentworth*, 1964] hastens the refilling of the outer plasmasphere [*Guglielmi et al.*, 1993, 1995; *Pokhotelov et al.*, 1995, 1996]. The possible relation of the substorm-associated pulsations [*Kangas et al.*, 1976] with the rise of O^+ density in the magnetosphere with increasing geomagnetic activity [*Young et al.*, 1982] is another example of this kind (see also *Ashour-Abdalla et al.* [1981], *Lockwood and Titheridge* [1981], *Cahill et al.* [1982], *Fraser and McPherron* [1982], *Kalisher et al.* [1982], *Perraut et al.* [1984], *Krimigis et al.* [1986], *Olsen and Chappell* [1986], and *Fraser et al.* [1992]).

A great deal needs to be done before the ponderomotive redistribution of magnetospheric ions along the geomagnetic field lines under the action of Pc 1 becomes well understood. In the present paper we treat this problem in the framework of a simple diffusion equilibrium model, taking the multicomponent composition of magnetospheric plasma into account when calculating ponderomotive forces. We begin in section 2 with Alfvén waves for the following reasons. First, it is generally believed that Pc 1 pulsations propagate in the magnetosphere to great distances as Alfvén waves. Second, it is commonly known that rays of Alfvén waves coincide with geomagnetic field lines, and hence the problem of field-aligned transfer of wave energy is of particular importance. Third, the problem of self-consistency between the spatial structure of Alfvén waves and spatial distribution of ions is readily solved at least approximately by using the WKB method. This approach provides insight into the topic and offers a clearer view of the ponderomotive redistribution of plasma.

The case of ion cyclotron waves is considerably more complex. (Note that ion cyclotron waves represent Pc 1 propagation in the near-equatorial segment of the trajectory.) Section 3 will be devoted to this subject. We demonstrate the differences between the two cases of Alfvén and ion cyclotron waves. Section 4 contains a number of applications of the theory presented in the preceding sections to the analysis of satellite and ground-based observations of Pc 1 waves.

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2. Redistribution of Ions Under the Action of Alfvén Waves

2.1. Ponderomotive Forces

The ponderomotive forces can be introduced in the Pc 1 theory by using the well-known expression for dielectric permeability of magnetospheric plasma [*Guglielmi et al.*, 1995]. Such a phenomenological approach is convenient in many respects, but it is limited to the hydrodynamic description of the bulk plasma. To remedy this limitation, the single–particle approach will be used here to evaluate the ponderomotive forces.

Miller force. Let B be the background magnetic field and $E_\perp$ the amplitude of Alfvén wave. Then the field–aligned component of the so–called Miller force acting on a charged particle with mass m equals

$$f_M = \frac{m}{4B^2}\partial E_\perp^2/\partial s \qquad (1)$$

where $\partial/\partial s$ is the spatial derivative along the field line. Let us derive (1) by averaging the Lorentz force over a period of Alfvén oscillations in the following manner.

We introduce the local Cartesian coordinates (x, y, z) so that $\mathbf{B} = (0, 0, B)$ and $\mathbf{E} = (E, 0, 0)$ at the origin. Then the magnetic field of the monochromatic ($E \propto \exp(-i\omega t)$) linearly polarized Alfvén wave has the components $\mathbf{b} = (0, b, 0)$, where

$$b = (-i/\omega)\partial E/\partial s. \qquad (2)$$

The field–aligned component of the Lorentz force equals evb, where e is the particle charge and v is the x component of the particle velocity due to the action of $\mathbf{E}$. Averaging over the oscillation period is obtained by the rule [*Landau and Lifshitz*, 1988]

$$\overline{vb} = \frac{1}{2}\mathrm{Re}(vb^*) \qquad (3)$$

$$v = -i\omega E/\Omega B \qquad (4)$$

where $\Omega = eB/m \gg \omega$. Now, substituting (2) and (4) into (3) gives (1).

It is interesting to compare (1) with the original equation,

$$f_M = -\frac{e^2}{4m\omega^2}\partial E_\perp^2/\partial s \qquad (5)$$

put forward by *Miller* [1958] in his pioneering work for high–frequency ($\omega \gg \Omega$) electromagnetic fields. We call special attention to the different dependence on mass and charge. One can see that the Miller force in the low–frequency limit ($\omega \ll \Omega$) is independent of e but, similar to inertia, proportional to m.

Lundin–Hultqvist force. In addition to (1), the "magnetic pumping" force

$$f_{LH} = -\frac{m}{2B^2}E_\perp^2\partial \ln B/\partial s \qquad (6)$$

exists if the background magnetic field is inhomogeneous [*Lundin and Hultqvist*, 1989]. The origin of this force and the form of (6) are made clearer in considering the effective magnetic moment of the charged particle in motion under the action of the electric field of Alfvén wave. Note that f_{LH}, like f_M in the low–frequency limit, is independent of the particle charge but proportional to m.

Total force. Let us denote $f = f_M + f_{LH}$. With a little manipulation this total force can be rearranged to the form

$$f = -ma \qquad (7)$$

where the field–aligned acceleration is equal to

$$a = (b^2/8\mu_0\rho)\partial \ln \rho/\partial s. \qquad (8)$$

Here we have made use of the known relation

$$E_\perp = c_A \cdot b \propto B/\rho^{1/4} \qquad (9)$$

where $c_A = B/(\mu_0\rho)^{1/2}$ is the Alfvén speed, ρ is the plasma density, and μ_0 is the permeability of free space [*Alfvén and Fälthammar*, 1963]. We can see that f is independent of B and directed toward decreasing ρ. It should be particularly emphasized that these properties are not valid in the case of ion cyclotron waves (see section 3).

2.2. Diffusion Equilibrium

We study now the diffusion equilibrium of electrons (e) and ions (i) when is taken into f into account. The equations of force balance have the form

$$T\partial \ln N_e/\partial s = m_e g - eE_\parallel + f_e \qquad (10)$$

$$T\partial \ln N_i/\partial s = m_i g + eE_\parallel + f_i \qquad (11)$$

where $N_{e,i}$ stands for the concentration of particles and g is the gravitational acceleration. For simplicity we have assumed that ions are singly charged and plasma is considered to be isothermal ($T_e = T_i = T$). Using (10) and (11) and the quasi–neutrality condition, we obtain the ambipolar electric field

$$E_\parallel = -\frac{m_+}{2e}C \qquad (12)$$

where

$$C = g - a \qquad (13)$$

and $m_+ = \rho/N$ is the mean ion mass ($N = \sum N_i$ is the total ion density).

Substituting (12) into (11), we obtain

$$T\partial \ln N_i/\partial s = (m_i - \frac{m_+}{2})C. \qquad (14)$$

Equation (14) differs from the common form only in that g is replaced by C, the difference between the gravitational and ponderomotive accelerations. Reducing the gravitational acceleration due to the action of ponderomotive forces leads to interesting consequences. Let us assume first that all ions have equal mass. Then $m_+ = m_i$, and (14) may be rewritten as

$$\left(c_s^2 + \frac{\alpha}{\sqrt{\rho}}\right)\partial\rho/\partial s = g\rho \qquad (15)$$

where $c_s = (2T/m_i)^{1/2}$ and $\alpha = b_0^2/8\mu_0\sqrt{\rho_0}$; $b_0 = b(r_0)$ and $\rho_0 = \rho(r_0)$ are values at a certain point r_0 on a given field line of the geomagnetic field.

At high latitudes the magnetic field lines are almost radial, and therefore (15) can be replaced by

$$\frac{1}{\rho}\left(c_s^2 + \frac{\alpha}{\sqrt{\rho}}\right)\frac{d\rho}{dr} = -\frac{\kappa M}{r^2} \qquad (16)$$

where r is the geocentric distance, M is Earth's mass, and κ is the gravitational constant. After integration we obtain

$$\frac{r_0}{r} = 1 + \beta \left\{ \ln\left(\frac{\rho(r)}{\rho_0}\right) + \gamma\left[1 - \sqrt{\rho_0/\rho(r)}\right] \right\} \qquad (17)$$

where $\beta = c_s^2 r_0/\kappa M$ and $\gamma = b_0^2/8\mu_0 N_0 T$. The efficiency of the ponderomotive redistribution of plasma is characterized by the value of the dimensionless parameter γ. When passing from $\gamma \ll 1$ to $\gamma \gg 1$, the exponential density profile $\rho(r)$ changes to power form at the distance $r \sim r_0$. If $\beta \ll 1$, then a strong modification of plasma can take place, say, at a distance $r \sim 2r_0$ if

$$\gamma > \exp\left(-\frac{1}{2\beta}\right) \qquad (18)$$

We now analyze multicomponent plasma, using the system of quasi-linear equations (14). We will argue that the acceleration G is not equal to zero anywhere but rather downward everywhere. This finding is evident in the case of a single-component plasma, but we will prove it in the general case. As a consequence, the ponderomotive acceleration a is pointing upward; i.e. we are dealing here with "electromagnetic lift" of the background plasma. However, a is strictly less in magnitude than the gravitational acceleration.

Now, assuming that $G = 0$ somewhere, we can deduce from (14) that $\partial\rho/\partial s = 0$, and therefore $a = 0$ (see equation (8)). From this deduction it follows that $G = g$, coming into conflict with the initial assumption. Since $a < g$ at sufficiently small distances with certainty, and G is continuous and nonzero at any point, our claim is proven. In particular, this result implies that the plasma density decreases monotonically with distance from the Earth, even though the Alfvén wave is of high amplitude.

The last conclusion calls for two remarks. First, the possible interference structure of Alfvén waves is disregarded here. Therefore we do not consider a nonmonotonic behavior of plasma density in the antinode of a standing Alfvén wave [*Allan*, 1992, 1993a, b). Second, to avoid confusion, it should be particularly emphasized that the above conclusion about the monotonous decrease is equally valid for ion cyclotron waves but in the low-frequency limit only (see section 3.2).

Thus, although the dependence of acceleration G upon concentrations N_i is rather complicated, we know that the sign of G coincides with the sign of g. It follows that qualitatively the ion species are distributed along the geomagnetic field lines in the same way as they would be without ponderomotive forces. In particular, a mixture of light ($i = 1$) and heavy ($i = 2$) ions is distributed, so that N_2 decreases monotonically with distance from the Earth and N_1 has a maximum at some distance.

Parameter γ has been introduced so that $\gamma + 1$ indicates how many times the local scale height at the point r_0 is increased under the action of ponderomotive forces on the plasma with one ion type. In the case of binary mixture of ions, let us coincide r_0 with the maximum of light ions. Then it follows from (8) and (14) that the local scale height at this point is increased $\gamma' + 1$ times, where

$$\frac{\gamma'}{\gamma} = \frac{2m_2}{m_1}\left(\frac{m_2 - 2m_1}{m_2 - m_1}\right)^2 \qquad (19)$$

For example, in the mixture of O^+ and H^+ ions the parameter γ' is larger than γ by a factor of 28. In this regard it can be said that the effectiveness of ponderomotive forces increases if the plasma contains ions of more than one type.

3. Redistribution of Ions Under the Action of Ion Cyclotron Waves

Recall that the Alfvén and ion cyclotron waves are assigned to the same branch of the dispersion curve; Alfvén waves correspond to the quasi-transverse propagation, and ion cyclotron waves correspond to the quasi-longitudinal propagation [e.g., *Stix*, 1962]. It is not surprising, then, that the Alfvén wave may be altered to the ion cyclotron wave and vice versa as the wave propagates in the magnetosphere. Thus it is necessary to obtain equations for ion cyclotron waves similar to those derived above for Alfvén waves. We shall restrict our consideration to the special case of strictly field-aligned propagation for simplicity.

3.1. Basic Equations

A transverse electromagnetic ion cyclotron wave propagating along the background magnetic field $\mathbf{B}$ has the left hand circular polarization with electric components

$$E_x = E_\perp e^{-i\omega t}, \qquad E_y = -iE_x \qquad (20)$$

and magnetic components

$$b_x = \frac{1}{\omega} \cdot \frac{\partial E_\perp}{\partial s} e^{-i\omega t}, \qquad b_y = -ib_x \qquad (21)$$

where x and y are the Cartesian coordinates perpendicular to $\mathbf{B}$ The Miller force f_M results from averaging the Lorentz force over the oscillation period. Since f_M is quadratic in $E_\perp$, the linearized equation of motion will suffice for our purpose:

$$v_x = \frac{ieE_\perp}{m(\omega - \Omega)}, \qquad v_y = -iv_x. \qquad (22)$$

In deriving (22) we have neglected the unperturbed motion of the charged particle.

Combining (21) and (22) and averaging, we obtain

$$f_M = -\frac{e^2 \partial E_\perp^2/\partial s}{2m\omega(\omega - \Omega)}. \qquad (23)$$

Similarly,

$$f_{LH} = -\frac{e^2\Omega}{2m\omega(\omega - \Omega)^2} E_\perp^2 \partial \ln B/\partial s. \qquad (24)$$

Then, by adding the forces (23) and (24) we obtain (see also *Shukla and Stenflo* [1985]

$$f = \frac{e^2}{2m\omega\,(\Omega-\omega)}\left[\frac{\partial E_\perp^2}{\partial s} - \frac{\Omega}{\Omega-\omega}E_\perp^2\frac{\partial \ln B}{\partial s}\right]. \quad (25)$$

Let us compare (23)–(25) with the similar equations (1) and (6)–(8) for the Alfvén wave. These two sets of equations deviate considerably. Furthermore, (25) does not yield (7) in the limit $\omega \to 0$, as might be hoped for, but tends to infinity. As we will see below, the inclusion of the ambipolar electric field eliminates infinity at $\omega \to 0$ and allows us to obtain the ponderomotive force of ion cyclotron waves in the low-frequency limit.

In the case of ion cyclotron waves we have

$$eE_\parallel = -\frac{m_+}{2}g + F \quad (26)$$

$$T\partial \ln N_i/\partial s = \left(m_i - \frac{m_+}{2}\right)g + F_i \quad (27)$$

where

$$F_i = f_i + F \quad (28)$$

$$F = \frac{1}{2}\left(f_e - \sum \eta_i f_i\right) \quad (29)$$

Here, $\eta_i = N_i/N$ is the relative concentration of ions, and f_e and f_i are defined by (25) with corresponding values of charge and mass.

Equation (28) may be rearranged to give

$$F_i = \frac{\varepsilon_0 E_\perp^2}{2N}\langle\left[\frac{\omega_{0e}^2\,(1+\mu_i)}{(\Omega_e+\omega)\,(\Omega_i-\omega)} - \frac{n^2-1}{2}\right]\frac{\partial \ln E_\perp^2}{\partial s} - \left[\frac{\omega_{0e}^2\,[\Omega_e(2\Omega_i-\omega)+\mu_i\Omega_i(2\Omega_e+\omega)]}{(\Omega_e+\omega)^2(\Omega_i-\omega)^2} + \frac{B}{2}\frac{\partial n^2}{\partial B}\right]\frac{\partial \ln B}{\partial s}\rangle \quad (30)$$

where $\omega_{0\alpha} = (e^2N_\alpha/\varepsilon_0 m_\alpha)^{1/2}$, $\alpha = e, i$, $\mu_i = m_e/m_i$, ε_0 is the permittivity of free space and

$$n^2 = \sum \frac{\omega_{0\alpha}^2}{\omega\,(\Omega_\alpha-\omega)} + 1. \quad (31)$$

Now it is easy to check that (27) takes the form (14) for $\omega \to 0$; in this case the acceleration (8) is replaced by

$$a' = -(1/4)\partial(E_\perp/B)^2/\partial s. \quad (32)$$

It remains to be described how the spatial structure of the wave field is related to the distribution of ions. This is a challenging task, since the distribution of ions in turn is determined by the wave structure. For the purpose of finding the spatial distribution of ions in an explicit form it is convenient to use the following straightforward relation:

$$E_\perp \propto (B/n)^{1/2} \quad (33)$$

Probably there is no other simple way to solve the problem of ponderomotive redistribution of ions without numerical integration of the nonlinear wave equation. One argument in favor of (33) is that it offers a natural extension of (8) to higher-frequency oscillations. For $\omega \ll \Omega$ and $n^2 \gg 1$, (32) and (33) coincide with (8) and (9), respectively. The second argument relies on the assumption that the excitation and propagation of Pc 1 ion cyclotron waves in the magnetosphere occur most commonly in waveguides stretched along geomagnetic field lines (see *Guglielmi* [1989] for a more detailed discussion). Considering that the effective cross section of the waveguide changes along a geomagnetic field line as B^{-1}, relation (33) can be obtained in the framework of ray theory.

Substituting relation (33) into equation (30) gives

$$F_i = -\frac{\varepsilon_0 E_\perp^2}{2N}\langle\left[\frac{\omega_{0e}^2(\Omega_e\Omega_i+\omega^2)(1+\mu_i)}{(\Omega_i-\omega)^2(\Omega_e+\omega)^2} + \frac{n^2-1}{2} + \frac{B}{2n^2}\frac{\partial n^2}{\partial B}\left(\frac{\omega_{0e}^2(1+\mu_i)}{(\Omega_e-\omega)(\Omega_e+\omega)} + \frac{n^2+1}{2}\right)\right]\frac{\partial \ln B}{\partial s} + \frac{1}{2n^2}\left[\frac{\omega_{0e}^2(1+\mu_i)}{(\Omega_i-\omega)(\Omega_e+\omega)} - \frac{n^2-1}{2}\right]\sum_j \frac{\partial \ln n^2}{\partial N_j}\frac{\partial N_j}{\partial s}\rangle \quad (34)$$

3.2. Diffusion Equilibrium

Although equations (27), (31), (33), and (34), describing the diffusion equilibrium of multicomponent magnetospheric plasma with Pc 1 waves, are fairly cumbersome, some properties of the solutions of these equations are understood by analyzing some special and limiting cases.

In the low-frequency limit, (27), (31), (33), and (34) transform to (8), (9), (13), and (14), as discussed in the preceding section. This brings up the question: How low a frequency is low in this respect? The answer depends on the wave amplitude. If $\gamma \ll 1$, the condition $\omega \ll \min\{\Omega_i\}$ is needed to recover the MHD description. Here, $\min\{\Omega_i\}$ is the gyrofrequency of the heaviest ion. If $\gamma \gg 1$, the condition is $\gamma\omega \ll \min\{\Omega_i\}$.

In case the plasma contains only one ion type, the set of basic equations reduces to

$$\left[\rho^{\frac{1}{2}} + \frac{c^2E_{\perp 0}^2\sqrt{\rho_0}}{4c_s^2B_0^2(1-\nu)^{\frac{1}{2}}(1-\nu\Omega_0/\Omega)^{\frac{1}{2}}}\right]\nabla_\parallel\rho = \frac{g_\parallel}{c_s^2}\rho^{\frac{3}{2}} - \frac{E_{\perp 0}^2\sqrt{\rho_0}\,\nu(\Omega_0/\Omega)}{4c_s^2B_0^2(1-\nu)^{1/2}(1-\nu\Omega_0/\Omega)^{3/2}}\left(\frac{\rho}{B}\right)\nabla_\parallel B, \quad (35)$$

where $\nu = \omega/\Omega_0$, Ω_0 is the ion gyrofrequency at a certain point on the field line. This equation can be written as

$$\frac{dy}{dx} = \frac{Py^{3/2} - Qy}{y^{1/2} + R}, \quad (36)$$

where, for a dipole magnetic field

$$P(x) = \frac{2R_E g_E}{c_s^2 L}\left[\frac{x}{(1-x^2)^2}\right]$$

$$Q(x) = \frac{E_{\perp 0}^2}{4c_s^2B_0^2}\frac{\nu}{\sqrt{1-\nu}}\frac{3x(5x^2+3)(1-x^2)^2}{(1+3x^2)^{3/2}\left[1-\nu\frac{(1-x^2)^3}{\sqrt{1+3x^2}}\right]^{3/2}}$$

$$R(x) = \frac{E_{\perp 0}^2}{4c_s^2B_0^2}\frac{1}{\sqrt{1-\nu}}\left[1-\nu\frac{(1-x^2)^3}{\sqrt{1+3x^2}}\right]^{1/2}$$

and $y = \rho/\rho_0$, $B_0 = B_E/L^3$, $\Omega_0 = \Omega_E/L^3$, $L = (1 - x_0^2)^{-1}$, $R_E = 6371$ km, $g_E = 9.8$ m/s^2, $B_E = 3.1 \times 10^{-5}$ T, $\Omega_E = 3 \times 10^3$ s^{-1}, $x = \cos\theta$, $x_0 = \cos\theta_0$ (θ_0 is the colatitude of intersection of a dipole field line with the Earth's surface), and ρ_0 is the plasma density at the equator of a field line.

Equation (36) demonstrates a new property of the plasma distribution $\rho(x)$ which was absent in the case of Alfvén waves (see section 2.2), namely, it follows from (36) that a nonmonotonic distribution occurs and a maximum is formed at the equator ($x = 0$), if $E_{\perp 0}$ is greater than [*Guglielmi*, 1992]

$$E_* = \frac{2\sqrt{2}}{3}\left(\frac{R_E g_E}{L}\right)^{\frac{1}{2}}\left[\left(\frac{\Omega_0}{\omega}\right)^{\frac{1}{2}} - \left(\frac{\omega}{\Omega_0}\right)^{\frac{1}{2}}\right] B_0 \quad (37)$$

The critical field E_* [mV/m] $= 231.46$ $[(1-\nu)/\sqrt{\nu}]$ $L^{-3.5}$ is smaller the greater the McIllwain parameter L and the normalized frequency $\nu = \omega/\Omega_0$ are (Figure 1).

Let us study some partial solutions of (36). The boundary condition $y = 1$ at $x = 0$ follows from the definition of y. The solution $\rho(x)$ is controlled by three dimensionless parameters, $\epsilon = E_{\perp 0}/E_*$, $\nu = \omega/\Omega_0$, and $\sigma = c_g/c_s$, where $c_g = (2R_E g_E/L)^{1/2}$. Some of the solutions obtained by numerical integration are plotted in Figure 2. We can see the qualitative redistribution of plasma when $E_{\perp 0}$ increases above the critical value $E_* = 0.58$ mV/m at $L = 5$ and $\nu = 0.5$.

4. Analysis of Observations

The simple theory outlined above applies to the equilibrium distribution of plasma along geomagnetic field lines. It hardly needs saying that in actual conditions the equilibrium can never be attained. Nevertheless, the general features as well as the special cases of the above theory may be used as a means to analyze satellite and ground-based measurements.

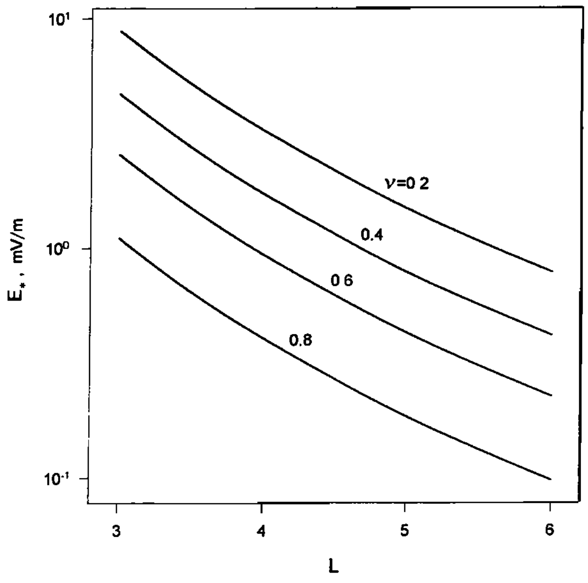

Figure 1. Dependence of the critical field E_* on the McIllwain parameter L for a few values of the normalized frequencies $\nu = \omega/\Omega_0$.

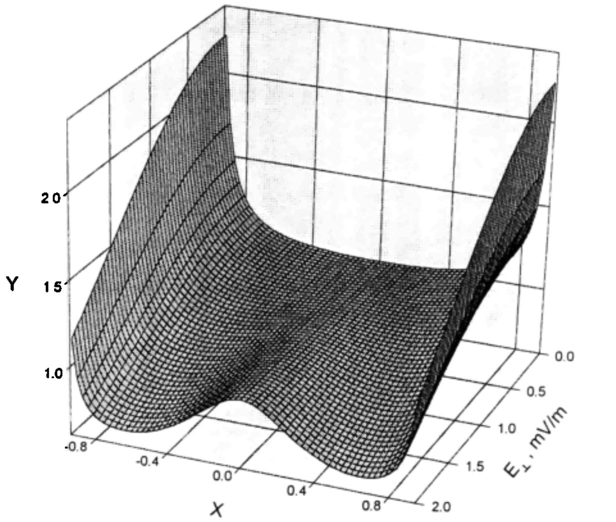

Figure 2. Distribution of normalized plasma density $y = \rho/\rho_0$ as a function of $x = \cos\theta$ and wave amplitude $E_{\perp 0}$ ($\nu = \omega/\Omega_0$ was used).

4.1. Satellite Observations

Measuring the wave frequency $f = \omega/2\pi$ and amplitude $E_{\perp 0}$ at the top of the field line would suffice to estimate the dimensionless parameter $\epsilon = E_{\perp 0}/E_*$ which characterizes the efficiency of ponderomotive forces (see equation (37)). Instead of $E_{\perp 0}$, the amplitude $b_0 = nE_{\perp 0}$ of magnetic field oscillations can be used in the ratio $\epsilon = b_0/b_*$, where

$$b_*(\text{nT}) = 0.342\left[\frac{N_0}{L}\left(\frac{\Omega_0}{\omega} - 1\right)\right]^{1/2} \quad (38)$$

In such a case one has to measure not only f and b_0 but also N_0. Let us consider some illustrative examples.

Bossen et al. [1976] detected Pc 1 magnetic pulsations by the geosynchronous ATS 1 satellite. Figure 1 of their work shows a transverse wave with $f \simeq 0.25$ Hz and $b_0 \simeq 3.5$ nT. In a dipole magnetic field, $\Omega_0 = 10.43$ s^{-1} at $L = 6.6$, whence $\nu \simeq 0.15$. We need the cold background plasma density in addition to these parameters. Lacking precise information, we use the characteristic value $N_0 \simeq 10$–30 cm^{-3}. Then $b_* \simeq 1$–1.7 nT and $\epsilon \simeq 2$–3.5, i.e., we have now a severe redistribution of plasma under the action of ponderomotive forces.

Fraser et al. [1992] observed the ion cyclotron waves at $f \simeq 0.1$ Hz by ISEE 1 and 2 satellites on the inbound pass near equatorial dusk on August 22, 1978. On an average, $E_{\perp 0} \simeq 0.5$ mV/m, $b_0 \simeq 0.65$ nT, and $N_0 \simeq 10$ cm^{-3} at $L \simeq 7.5$ according to Table 1 and Figure 11 of their paper. It follows that $\nu \simeq 0.1$, $E_* \simeq 0.57$ mV/m and hence $\epsilon \simeq 0.87$. The value of the critical magnetic field $b_* \simeq 1.2$ nT leads to a somewhat different estimation of $\epsilon \simeq 0.55$. In any case the ponderomotive redistribution of plasma may be sizable if not as large as in the preceding example.

Mursula et al. [1994] used the electric field measurements by the Freja satellite to study Pc 1 pulsations at ionospheric heights. The wave packet with $f \simeq 1$

Hz, $E_\perp \simeq 4$ mV/m was registered near 645 km altitude at 62° geomagnetic latitude on November 18, 1992, during very quiet geomagnetic conditions. Using the dipole approximation of geomagnetic field, we obtain $L \simeq 5$, so $\nu \simeq 0.26$ and $E_* \simeq 1.2$ mV/m. To estimate $E_{\perp 0}$ we use the relations (31) and (33) in the form $E_{\perp 0} \simeq E_\perp (B_0/B)[(1-\nu)(\rho/\rho_0)]^{1/4}$. The B ratio and the ν factor attain the values 6×10^{-3} and 0.9, respectively. Let us assume $N \simeq 10^5$ cm^{-3}, $m_i = m_{O^+}$, and $N_0 \simeq$ 10–100 cm^{-3}, $m_i = m_{H^+}$. Then $(\rho/\rho_0)^{1/4} \simeq$ 10–20. In total, $E_{\perp 0} \simeq$ 0.2–0.4 mV/m, and therefore $\epsilon \simeq$ 0.16–0.33.

The dimensionless parameters γ and γ' introduced in section 2.2 are also useful in analyzing satellite observations. As an example, take $E_\perp \simeq 10^2$ mV/m for an Alfvén wave at a height of 10^4 km in the auroral zone. This value corresponds to the Viking satellite measurements [*Lundin and Hultqvist*, 1989]. Parameter γ can be rearranged to give

$$\gamma = (m_i/8T)(E_\perp/B)^2. \tag{39}$$

Now assume that $T \simeq 10^4$ K and m_i is the proton mass. Then $\gamma \simeq 1.7$; i.e., a strong plasma density change is expected.

4.2. Ground-Based Observations

According to the theory of Pc 1 pulsations [*Cornwall*, 1965], the greater the plasma density N_0 is, the smaller the carrier frequency f is. Consequently, if Pc 1 induced magnetic lift (or magnetic pumping) indeed occurs, the frequency will gradually decrease in the course of the event. Ground-based observations show that pearl Pc 1 wave series decreasing with midfrequency, $df/dt < 0$, are abundant [*Offen*, 1972; *Matveeva et al.*, 1972; *Dovbnya et al.*, 1974; *Guglielmi*, 1974]. Figure 3 shows an example of such a series.

On the other hand, several authors [*Pope, 1965*; *Kenney and Knaflich*, 1967] have indicated statistically that the repetition period τ of Pc 1 wave packets increases as the carrier frequency decreases. Hence one might expect that not only f but also τ is subject to ponderomotive modulation. Let us introduce the dimensionless parameter

$$\Lambda = \frac{d \ln \tau/dt}{d \ln f/dt} \tag{40}$$

as a measure of the combined variation of $\tau(t)$ and $f(t)$. An important point is that Λ can be estimated theoretically but can also be obtained from ground-based observations.

The following relation was recently presented by A. Guglielmi et al. (Range finding of Alfvén oscillations and the remote diagnostics of magnetospheric plasma density by using the ground-based MHD ranger, submitted to *J. Geophys. Res.*, (1996):

$$\tau f \simeq \frac{R_E}{2\pi V_{sw}} \sqrt{\frac{\Omega_E \Omega_m}{L}} \tag{41}$$

where V_{sw} is the solar wind velocity, Ω_m is the gyrofrequency of protons at the magnetopause, and L is the McIllwain parameter of the waveguide along which the Pc 1 wave packet propagates in the magnetosphere. One can imagine two different possibilities. First, L remains constant in the course of a plasma redistribution. In that case, $\Lambda = -1$. Actually, (41) was derived without considering dispersion effects. It can be shown that including wave dispersion would lead to the inequality $\Lambda < -1$ in the case of constant L.

The second possibility is that L increases with time, $dL/dt > 0$. In such a case, $\Lambda > -1$ in order to fit (41). (It can also be shown that including dispersion further enhances the inequality $\Lambda > -1$.) The case

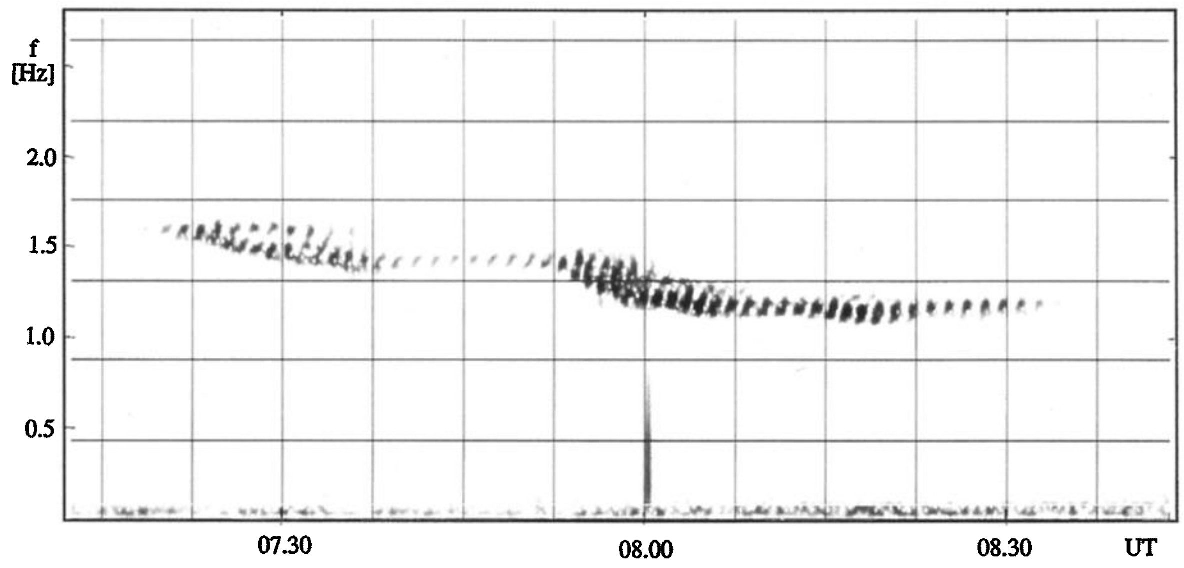

Figure 3. Sonogram of pearl Pc 1 event with decreasing frequency observed at Oulu, Finland, on November, 24, 1993, at about 0800 UT. The vertical bar is due to time signal.

of $dL/dt > 0$ corresponds to the commonly accepted model of the plasmasphere refilling after a magnetic storm and the excitation of ion cyclotron waves in the vicinity of the expanding plasmapause [e.g., *Thorne*, 1974]. The results presented above suggest that the Pc 1 waves play an active role in the process of plasmasphere refilling, not just passively moving with the plasmapause away from the Earth. In conclusion, Λ is predicted to be smaller (larger) than -1 according to the first (second) possibility.

In order to estimate the experimental value of Λ we studied Pc 1 events observed at the two Finnish observatories of Oulu and Sodankylä. We selected 32 series of Pc 1 waves with $\dot{\omega} < 0$ occurring in August–November 1993. The mean frequency and repetition period are 0.92 ± 0.06 Hz and 131 ± 8 s, respectively. The Kp index during all events was smaller than 3+.

Figure 4 shows the distribution of the value of Λ for the selected events. Parameter Λ varies between -2.1 and -0.1 with the mean value -0.68 ± 0.08 and rms 0.47, suggesting that the second possibility is more favored, although the first one may also be realized sometimes. Since it is possible that the diurnal variation of the plasmapause position may also affect the change of wave parameters, we have studied the day-to-day variation of τ and f in a more or less fixed local time sector. Among the wave events selected there were a few cases satisfying these constraints. For all but one event the value of Λ so calculated was found to be larger than -1, typically -0.4.

An effort was made to eliminate the L parameter from (41), thereby improving the theoretical estimate of Λ in the framework of the second possibility. We use the theoretical relation $\tau \propto L^4\sqrt{N_0}$ [*Guglielmi*, 1989] and the empirical result

$$N_0(\mathrm{cm}^{-3}) = (1.0 \pm 0.7) \times 10^5 L^{-3.98}$$

obtained by *Farrugia et al.* [1989]. Then

$$\tau^{1.25} f = \mathrm{const} \qquad (42)$$

which gives $\Lambda = -0.8$. This value is in rather good agreement with the empirical mean value $\Lambda = -0.68$.

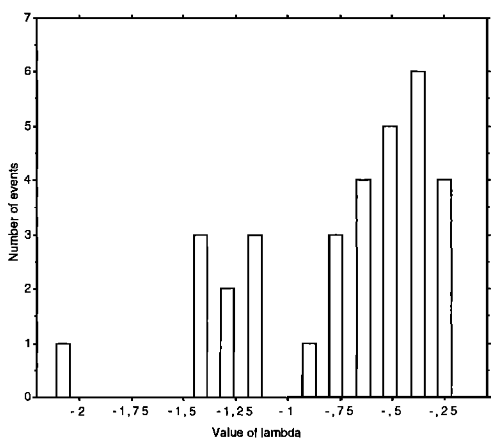

Figure 4. The distribution of the Λ parameter calculated for 32 selected Pc 1 events observed on the ground.

5. Conclusion

We studied the ponderomotive redistribution of ions along the geomagnetic field lines, using a simple diffusion equilibrium model. Naturally, the equilibrium assumption is an idealization, especially at high latitudes. Plasma outflow persists at high latitudes, as proven by theoretical calculations [*Banks and Holzer*, 1968; *Lundin and Hultqvist*, 1989] as well as by satellite measurements [*Hultqvist et al.*, 1988; *Lundin*, 1988; *Kondo et al.*, 1990]. At low latitudes an equilibrium, or more precisely, quasi-equilibrium, seems possible. Our theory predicts the formation of a maximum of plasma density at the equator if an intense Pc 1 wave action persists long enough (see also *Allan* [1992]).

We think that the present theory offers an exciting analytical potential to better understand and estimate the ponderomotive efficiency of Alfvén and ion cyclotron waves under concrete magnetospheric conditions by using simple parameters like γ, γ', ϵ, Λ, and the given relations. We conclude from our estimations that a ponderomotive redistribution of plasma in the magnetosphere under the action of Pc 1 waves is considerable.

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