GEOPHYSICAL RESEARCH LETTERS, VOL. 23, NO. 9, PAGES 1041-1044, MAY 1, 1996

Fluctuations of the repetition period of Pc1 pearl pulsations

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Abstract. In this paper we investigate fluctuations of the repetition period of geomagnetic Pc1 pearl pulsations. Starting from calculated repetition period we present a general formula for the deviation of the repetition period as a function of wave frequency and stochastic parameters of the medium along the ray trace. We then apply this formula for a dipole magnetic field with a simple plasma distribution, and show that a linear correlation between repetition period and its deviation is predicted. This correlation and the frequency dependence of fluctuations are then compared with experimental values measured from selected Pc1 pearl events observed in Finland.

Introduction

Thirty years ago *Jacobs and Watanabe* [1964] and *Obayashi* [1965] presented the idea that geomagnetic Pc1 pulsations are electromagnetic ion cyclotron waves which propagate in the magnetosphere along geomagnetic field lines. Accordingly, the repetition period of a Pc1 pearl is

$$\tau = \int_0^\ell \frac{dx}{V} \tag{1}$$

where $$V = V_A(1-\omega/\Omega)^{3/2}(1-\omega/2\Omega)^{-1} \tag{2}$$

is the group velocity of ion cyclotron waves with angular frequency ω, V_A is Alfvén velocity, and Ω is ion gyrofrequency. Integral is taken along the field line (dx is the line element) and ℓ equals twice the length of the line between conjugate ionospheres.

By now this idea is widely used as a basis when interpreting observational data and as a starting point when generalizing theory. Note that in all previous publications on this topic the magnetospheric plasma is considered as a regular medium [see e.g. *Troitskaya and Guglielmi*, 1967; *Jacobs*, 1970; *Nishida*, 1978; *Guglielmi*, 1989]. However, in reality the magnetospheric plasma is an irregular medium, i.e. it contains random inhomogeneities of plasma density and other plasma parameters. Therefore the Pc1 repetition period τ also fluctuates randomly even if pulsations remain on the same field line. Fig. 1 depicts a typical sonagram of a Pc1 pearl event with a sequence of wave packets. When analyzed carefully, small fluctuations of repetition period τ are seen. Accordingly, it is important to find the stochastic generalization of the theory of Pc1 propagation and to analyze the observed fluctuations of τ.

In this study we present a theoretical formulation for the stochastic r.m.s. deviation of τ as a function of wave frequency and statistical properties of the medium along the ray trace. Then we apply the theory to a dipole magnetosphere with a simple plasma distribution in order to extract definite numerical estimates. We also compare these estimates with observed fluctuations of Pc1 repetition period using ground-based data from the Finnish magnetometer network.

Theory

Let us regard τ as a stochastic function of the wave packet number and call τ_0 and τ_1 the regular and irregular (stochastic) parts of τ, i.e. $\bar{\tau} = \tau_0$ and $\bar{\tau}_1 = 0$. The line over a symbol means statistical average. We introduce also $\sigma_\tau = \overline{\tau_1^2}^{1/2}$ to denote the deviation of the fluctuations of the repetition period. According to Eqs. (1) and (2), τ depends (via $V_A \propto \rho^{-1/2}$) on mass density ρ and therefore fluctuates due to its random irregularities. Let us now divide ρ to the regular density on the Pc1 ray path $\rho_0(x)$, and the irregular, fast fluctuating $\rho_1(x)$, and assume that $|\rho_1| << \rho_0$. Similarly we divide $V = V_0 + V_1$ with $\bar{V} = V_0$ and $\bar{V}_1 = 0$. The above assumption implies $|V_1| << V_0$, and we can easily find in the leading order of perturbation

$$\sigma_\tau^2 = \frac{1}{4}\int_0^\ell\int_0^\ell \frac{\overline{R(x_1)R(x_2)}}{V_0(x_1)V_0(x_2)}dx_1dx_2, \tag{3}$$

where $R = \rho_1/\rho_0$.

Let us now make another simplifying assumption and regard $R(x)$ as a statistically homogeneous function. This means that the correlation function $\Gamma_\rho(x_1,x_2) = \overline{R(x_1)R(x_2)}$ only depends on the difference of its arguments: $\Gamma_\rho(x_1,x_2) = \Gamma_\rho(x_1-x_2)$. Then it is natural to change the variables to $\zeta = x_1 - x_2$ and $x = (x_1+x_2)/2$, whence Eq. (3) is transformed to

$$\sigma_\tau^2 = \frac{1}{2}\int_0^\ell \Gamma_\rho(\zeta)d\zeta \int_{\zeta/2}^{\ell-\zeta/2} \frac{dx}{V_0(x+\zeta/2)V_0(x-\zeta/2)}, \tag{4}$$

where we used the symmetry relation $\Gamma_\rho(-\zeta) = \Gamma_\rho(\zeta)$. We ignored here the possible correlation of fluctuations on the back and forth segments of Pc1 trajectory between conjugate points. Had we taken this correlation into account, the right-hand part of Eq. (4) should be multiplied by $\sqrt{2}$. In the case of small-scale irregularities, $\Gamma_\rho(\zeta)$ is non-zero for small values of ζ only. This allows us to neglect the dependence of the x-integral on ζ and to put the upper limit to infinity in the ζ integral. As a result we find

$$\sigma_\tau^2 = \frac{1}{2}\int_0^\ell \frac{dx}{V_0^2(x)}\int_0^\infty \Gamma_\rho(\zeta)d\zeta = \sigma_\rho^2\ell_\rho J_\ell^2(\omega) \tag{5}$$

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Paper number 96GL00965
0094-8534/96/96GL-00965$05.00

where $$\sigma_\rho = [\Gamma_\rho(0)]^{1/2}, \ \ell_\rho = \frac{1}{\sigma_\rho^2}\int_0^\infty \Gamma_\rho(\zeta)d\zeta, \quad (6)$$

and $$J_\ell(\omega) = \left[\int_0^{\ell/2} \frac{dx}{[V_0(x,\omega)]^2}\right]^{1/2}. \quad (7)$$

The physical meaning of these terms is the following: σ_ρ is the deviation of plasma density irregularities and ℓ_ρ is the effective correlation length of these irregularities along the wave packet trace. The function $J_\ell(\omega)$ describes the frequency and field line dependence of σ_τ.

Thus, starting from the expression (1) we have obtained an interesting new result. Frequency dependence of σ_τ is, to first order, determined by the regular distribution $V_0(x,\omega)$ along the ray path. This gives us a possibility to test the model experimentally. It also provides the basis for possible diagnostical applications of the theory.

Results in a dipole field

Let us now make some more quantitative theoretical and numerical estimates by using dipole field and the following simple plasma density distribution along the field line: $\rho(z) = \rho(0)(1-z^2)^{-4}$, where $z = cos\theta$, θ is colatitude (see Guglielmi, 1989). Changing integration variables with the relation $dx = LR_E(1+3z^2)^{1/2}dz$ and using Eqs. (2) and (7), we obtain

$$J_\ell(\omega) = \sqrt{2}\ V_{Ao}^{-1} R_E^{1/2} [L(L-1)]^{1/4}\ [I(\omega)]^{1/2}, \quad (8)$$

where $$I(\omega) = \frac{1}{z_0}\int_0^{z_0} \frac{[1-\omega/2\Omega(z)]^2(1-z^2)^2}{[1-\omega/\Omega(z)]^3(1+3z^2)^{1/2}}dz, \quad (9)$$

$$\Omega(z) = \Omega_0(1+3z^2)^{1/2}/(1-z^2)^3, \quad (10)$$

and $$\Omega_0 = \Omega_E/L^3, \ z_0 = (1-1/L)^{1/2}. \quad (11)$$

Here R_E is Earth radius, V_{Ao} is Alfvén velocity at equator, L is McIlwain's parameter, and $\Omega_E = 3066$ s^{-1} gives the equatorial proton gyrofrequency at L=1. Ω_0 is the corresponding quantity at equatorial distance LR_E, and $\Omega(z)$ gives the gyrofrequency at angle θ and invariant latitude L.

The repetition period $\tau_0(\omega)$ in the dipole magnetosphere

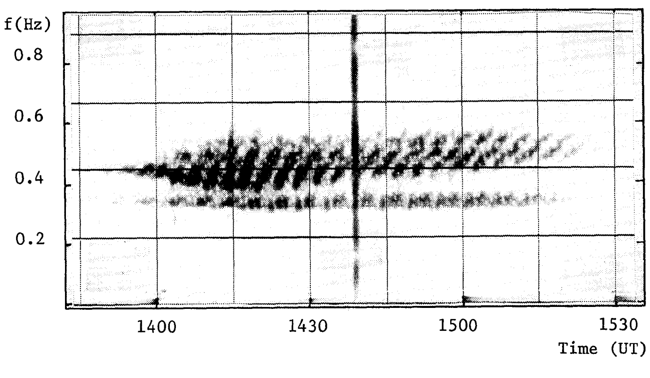

Fig. 1. Sonagram for one Pc1 pearl event observed at Sodankylä, Finland, on September 30, 1990. The upper band was used in the analysis. (The narrow vertical line is due to an external disturbance, probably a lightning.)

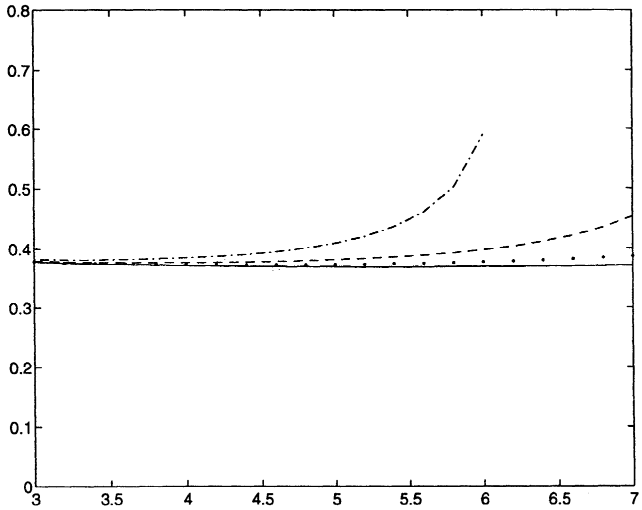

Fig. 2. The proportionality factor $A(L,\omega/\Omega_0)$ as a function of L (calculated in steps of 0.2) for a few relevant frequencies. The solid, dotted, dashed and dash-dotted lines correspond to the frequencies of 0.2 Hz, 0.5 Hz, 1 Hz and 2 Hz, respectively.

is given by the following well known formula (see e.g. *Nishida*, 1978; *Guglielmi*, 1989):

$$\tau_0(\omega) = 4R_E V_{Ao}^{-1}[L(L-1)]^{1/2}K(\omega), \quad (12)$$

where $$K(\omega) = \frac{1}{z_0}\int_0^{z_0} \frac{[1-\omega/2\Omega(z)]}{[1-\omega/\Omega(z)]^{3/2}}(1-z^2)dz. \quad (13)$$

We can now easily find the following relation:

$$\sigma_\tau = \sigma_\rho(\ell_\rho/R_E L)^{1/2}\tau_0 \cdot A(L,\omega/\Omega_0). \quad (14)$$

where $$A(L,\omega/\Omega_0) = \frac{\sqrt{2}}{4}\frac{I^{1/2}(\omega)}{K(\omega)}z_0^{-1/2}. \quad (15)$$

It is interesting to note that the proportionality factor $A(L,\omega/\Omega_0)$ is fairly constant (about 0.4) for physically relevant L and ω values, as shown in Fig. 2. Therefore, Eq. (14) expresses a nearly constant linear correlation between σ_τ and τ_0 with σ_ρ, $\ell_\rho^{1/2}$ and trivial scaling factors as coefficients. As is seen in Fig. 2, largest deviations from constancy are found for high values of frequency and L. However, since the high-frequency pulsations are produced at fairly low L shells (*Erlandson et al.*, 1990), the physical value of A is approximately 0.4.

Comparing theory with observations

In order to compare the above theory with experimental data we studied Pc1 pearl events observed at Sodankylä (L = 5.1), Oulu (L = 4.3) and Nurmijärvi (L = 3.3) from 1975 to 1990. We selected 20 events passing the following criteria: Average Pc1 frequency was almost constant; Pc1 band was broad enough; the chain of oscillations contained at least 12 wave packets; pulsation amplitude did not suffer from abrupt changes. One of the selected events is shown in Fig. 1. Most selected events occurred in the morning or day hours during quiet or moderate magnetic activity.

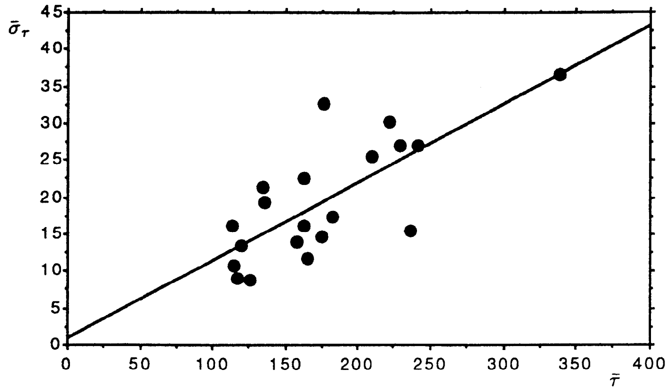

Fig. 3. Scatterplot of the 20 Pc1 events in the $\bar{\tau} - \bar{\sigma}_\tau$ plane. Regression line is also shown.

For each of the 20 Pc1 pearl events we calculated the average repetition time τ and its deviation σ_τ at two frequencies of the same band. (The read-out error of times from the sonagram was estimated to be about 5s). The values obtained at the lower (higher, resp.) frequency are denoted by index 1 (2). A typical difference between the two frequencies was 0.1–0.15 Hz. Furthermore, average values of quantities measured at the two frequencies of one band were calculated. These are denoted by bars. The average frequency varied from 0.3 Hz to 1.5 Hz.

In Fig. 3 we have depicted the scatterplot of all 20 events in $\bar{\tau} - \bar{\sigma}_\tau$ plane. A fair correlation ($r^2 = 0.55$) between the two quantities is found with a linear regression equation of $\bar{\sigma}_\tau = 0.99 + 0.11 * \bar{\tau}$. Thus Fig. 3 provides observational evidence for the above idea of a linear relation between σ_τ and τ, as expressed in Eq. (14).

The observed properties of events also reproduce the well known inverse relation between repetition period and frequency of Pc1 pearls (see e.g. *Troitskaya and Guglielmi*, 1967). Using average values for events observed at Sodankylä we find $\bar{f} = 103 * \bar{\tau}^{-1} (r^2 = 0.63)$. From this relation and the above mentioned correlation between $\bar{\tau}$ and $\bar{\sigma}_\tau$, it is clear that $\bar{\sigma}_\tau$ was also found to be inversely correlated with the average frequency $\bar{f}$. Thus, on an average, larger values of $\bar{\tau}$ and $\bar{\sigma}_\tau$ were observed at low frequencies.

However, while this relation is true for the statistical ensemble of events, a different frequency dependence was found when each event was studied separately. Let us first note that the repetition period for the upper part of the band (τ_2) was slightly larger than that of the lower part (τ_1) for most events. The two periods were found to be very well correlated ($r^2 = 0.96$) with $\tau_2 = 9.9 + 0.99 * \tau_1$. The non-zero intercept of this regression is due to the changing slope of pearls during the event. This intercept gives the statistically averaged time delay between the upper and lower frequencies of about 10 s. Thus the higher frequency part of a pearl propagates slower and gets delayed with respect to the lower frequencies. Similarly, the deviations calculated for the upper (σ_2) and lower (σ_1) frequency parts of a pulsation band were seen to be fairly well correlated ($r^2 = 0.67$) with $\sigma_2 = 3.3 + 1.1 * \sigma_1$. Out of 20 events, 17 had σ_2 larger than σ_1. A typical value of the difference $\sigma_2 - \sigma_1$ is 4-5 s while that for σ_2 (σ_1) is 17-18 s (12-13 s). Accordingly, the upper frequencies of a pulsation band suffered a larger deviation than the lower frequencies. This is in an interesting difference to the above discussed inverse relation between $\bar{f}$ and $\bar{\sigma}_\tau$ based on statistical averages.

In order to further study the frequency dependence of σ_τ, let us introduce the dimensionless parameter (actually the logarithmic derivative of the deviation) $F = d\ell n\sigma_\tau / d\ell n\omega$. The theoretical value of F can be calculated from Eq. (8). Instead of showing the lengthy formula we have plotted in Fig. 4 the frequency ratio ω/Ω_0 versus the value of F for $L = 5$. Numerical calculations show that this curve changes only little if plasma density distribution or L are varied, once $L > 3$. As seen in Fig. 4 fluctuations of the Pc1 repetition period increase monotonously (but nonlinearly) with frequency, in agreement with the above discussed observation of σ_2 being greater than σ_1. The distribution of observed values of F is presented in Fig. 5. It is characterized by a maximum at $F = 1 - 1.5$. As seen from Fig. 4, this range of F corresponds to ω/Ω_0 of about 0.55–0.65. Accordingly, for most selected Pc1 events our method finds nearly the same frequency in a physically relevant range of values.

Discussion

Let us first comment on the restrictions made in the derivation of Eq. (5). The perturbative treatment is only allowed if density fluctuations are small, i.e. $|\rho_1| << \rho_0$. This condition is expected to be valid for quiet and moderate geomagnetic times whence Pc1

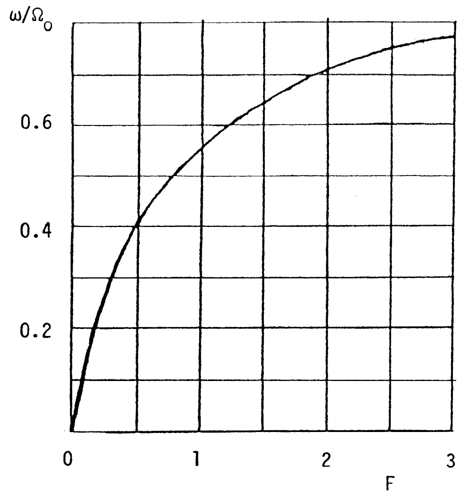

Fig. 4. Frequency ratio ω/Ω_0 versus parameter F calculated for L = 4.5.

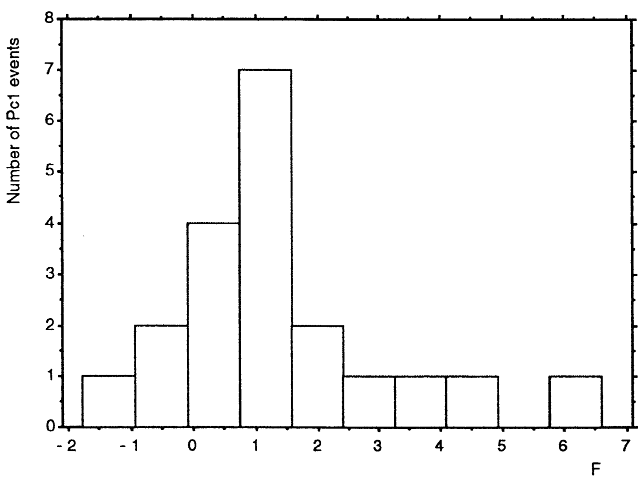

Fig. 5. Frequency distribution of the observed values of F for 20 Pc1 events.

pearls prefer to occur. During such times the conditions on small-scale irregularities and the homogeneity of the correlation function are also best guaranteed.

Furthermore, the dipole approximation used for numerical comparisons should apply quite well since most Pc1 pearls occur at fairly low L-values (L = 3–6) in the morning sector where the field lines are much less perturbed than e.g. in the night sector.

The clear concentration of observed pearl events to only a small range of F gives good evidence for the theory to explain the observed fluctuations of repetition period. A mere scaling argument or any other known mechanism could not cause such a concentration.

The fact that the higher frequencies of a Pc1 pearl band experience larger fluctuations than the lower frequencies may, in addition to the frequency dependence given by the theory, be slightly further enhanced due to the higher frequencies spending a longer time (about 10 s more, as presented earlier) in the magnetosphere, thus getting more vulnerable for random plasma fluctuations. However, this effect alone can not be the dominant reason since it does not produce a constant value for F.

Most measured values of F being positive and corresponding to physically relevant frequencies gives additional support for the presented theory. However, for three events, a negative value of F was obtained (see Fig. 5). These may, of course, be ascribed to experimental errors. Another explanation is also possible. In the above treatment we have discussed left-handed waves, but Pc1 waves may also occasionally propagate in the right-handed (R) magnetosonic mode. The group velocity of R-mode waves is obtained from Eq. (2) by substituting $\omega \rightarrow -\omega$. Using this substitution the frequency dependence of σ_τ can be calculated for the R-mode from Eq. (9), resulting in a negative value of F.

Let us also briefly mention some diagnostical possibilities of the theory. The correlation coefficient between $\bar{\tau}$ and $\bar{\sigma}_\tau$ can be used to estimate the parameter $Q = \sigma_\rho(\ell_\rho/R_E)^{1/2}$ which characterizes the flatness of the magnetospheric plasma along the Pc1 trace. Using the theoretical value of $A \simeq 0.4$, the expected $L \simeq 5$ and the observed coefficient of proportionality 0.11, we find $Q \simeq 0.6$.

We have neglected the effect of heavy ions in Eq. (2). This omission affects the absolute frequency scale but does not invalidate the above comparison of the observed and theoretical frequency dependences of σ_τ. In particular, the frequency obtained from observed values of F was about half the ion (proton) gyrofrequency while Pc1 frequency observed on the ground was below equatorial He^+ gyrofrequency. Thus, a more correct absolute frequency scale is obtained if He^+ rather than proton gyrofrequency was used in Eq. (11) and, consequently, if Ω_E had one fourth of its value given below Eq. (11). (With this substitution, the frequency range would also correspond to the frequency of the maximal wave growth; see e.g. *Kozyra et al.*, 1984). However, the frequency dependence of σ_τ as tested by F is, because of its definition as a logarithmic derivative, determined by the form (slope) rather than the absolute value of the dispersion relation. Taking He^+ ions into account, the dispersion relation of left-handed waves below the He^+ gyrofrequency (class I waves in the terminology of *Rauch and Roux*, 1982) greatly resembles the form of the dispersion relation of waves below the proton gyrofrequency when heavy ions are neglected. Thus we are confident that the results obtained above for the frequency dependence remain valid even if heavy ions were taken into account.

In order to briefly mention some of the diagnostical possibilities of the method, we can e.g. obtain an estimate of the plasma density at the equator in the following way. Using the observed value of F we will first determine from Fig. 4 the corresponding ratio ω/Ω_0 and, knowing the observed ω, we can find Ω_0 and then L. The plasma density at the equator then is found by the inversion of Eq. (1) using the measured values of τ, ω and L.

Conclusion

We have studied fluctuations of the repetition period of Pc1 pearls in a theoretical framework, using a few physically motivated simplifying assumptions. We found out that the fluctuations depend on the effective correlation length of plasma irregularities and a frequency dependent part which is determined by the regular plasma density distribution along the Pc1 ray path.

Using the dipole field and a simple plasma distribution we derived explicit formulas for fluctuations and demonstrated that an approximately linear relation between the repetition period and its fluctuations follow. This prediction was verified by measured data from a number of pearl events. In addition, the frequency dependence of fluctuations was shown to be in accordance with theoretical expectations. We discussed the limitations of the model and outlined some possibilities for diagnostic applications.

Acknowledgements. The authors acknowledge the financial support of the Academy of Finland.

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(Received 14 June, 1994; revised 3 September, 1995; accepted 25 September, 1995)