

STUDY OF THE V, A STRUCTURE OF THE CHARGED LEPTONIC CURRENT

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Although the charged weak interactions of leptons are conventionally described by pure $V-A$ currents, several recent models have mechanisms which admit components of other structures. It is therefore becoming increasingly important to know the experimentally allowed amounts of $V+A$ (as well as $S, P,$ and T) admixture.

In this paper the structure of the charged leptonic weak currents is studied in the framework of two models, the fermion-mirror fermion mixing model connected with many grand unification schemes, and the left-right symmetric $SU(2)_L \times SU(2)_R \times U(1)$ flavor model. Both of these models allow more general V, A structure of the currents. We fit the parameters of these models to experimental data from pseudoscalar meson decay, muon decay, nuclear β -decay, and inverse muon decay. We find that substantial departure from pure $V-A$ is allowed, the best fit values of the parameters being within one standard deviation from their $V-A$ values.

In the mixing model, different mass configurations of the mirror neutrinos are considered. The case with a heavy ($> m_K$) electron mirror neutrino and a light ($\ll m_e$) muon mirror neutrino is disfavored by the fit.

The lower limit of the mass of the right-handed gauge boson W_R in the $SU(2)_L \times SU(2)_R \times U(1)$ model is $m(W_R) > 3.4m(W_L)$ (68.3% c.l.). The left-right symmetric model fits slightly better than the mixing model. The only data explicitly disagreeing with the $V-A$ model is the ξ -parameter in muon decay. It would be of great importance to have this parameter remeasured.

1. Introduction

The conventional description of the charged weak interactions is based on the $V-A$ hypothesis. This corresponds to maximal parity violation in these processes and it is intimately connected with the assumed two-component character of neutrinos. The Glashow-Weinberg-Salam unified electroweak model [1] repeats this structure; there the particle assignment allows only the left-handed fermion currents to couple to the charged intermediate gauge boson W_L .

Experimentally, however, fairly large deviations from the pure $V-A$ theory are still allowed. Accordingly, there is space for alternative theoretical pictures. The purpose of this paper is to investigate two models which deviate from the pure $V-A$ theory and lead to a more general V, A structure of the charged weak

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currents. The most general case with scalar, pseudoscalar and tensor terms in addition to vector and axial vector currents will be analyzed in a later publication* [28].

As the first source for the departure from the pure left-handed coupling we will consider fermion-mirror fermion mixing. Conjugated, or mirror fermions couple to the W_L boson in right-handed currents, but are in other respects similar to the $V-A$ fermions. Mirror fermions appear in many extended schemes, e.g. $SO(n > 10)$ [2], $SU(n > 5)$ [3] and E_8 [4] models, unifying several fermion families in a single representation. For example, in $SO(n > 10)$ models for each ordinary family there exists a corresponding family of mirror fermions. Mirror fermions may, especially when their mass scale is low, mix with the $V-A$ fermions due either to explicit Yukawa terms or dynamically in higher orders [5]. The physical mass eigenstates will then be linear combinations of fermions and mirror fermions with equal charges, and consequently their couplings to the W_L will not have pure $V-A$ structure.

In the first model we assume that there is such mixing of charged leptons and neutrinos with their corresponding mirror particles. We determine the values of the mixing angles by fitting the model to data from the charged current leptonic and semileptonic processes.

The second model we consider is the left-right symmetric electroweak model [6] which is based on the flavor group $SU(2)_L \times SU(2)_R \times U(1)$. The vector boson W_R corresponding to the subgroup $SU(2)_R$ mediates $V+A$ interactions among ordinary fermions (and $V-A$ interactions among mirror fermions). If the mass scale of the parity restoration is not too large, right-handed currents may cause a substantial deviation of the effective couplings from their $V-A$ values. Again we fit the model with the relevant charged current leptonic data, and determine the values of the mass ratio $m(W_1)/m(W_2)$ of the physical charged gauge bosons and the W_L - W_R mixing angle.

In family unification models with mirror fermions, based on the $SO(n)$ or E_8 groups, the gauge symmetry can be broken so that the left-right flavor symmetry $SU(2)_L \times SU(2)_R \times U(1)$ remains at low energies. Then both sources of $V+A$ interactions considered above may be effective. The two models described correspond to the two extreme cases when either of the effects is negligible. The fermion-mirror fermion mixing model is a good approximation if the Higgs couplings and the vacuum expectation values responsible for the mixings are large and the mass scale of parity restoration is high. In the opposite case the effect of the right-handed gauge boson will dominate. The number of parameters is in general so large, that a fit to the available experimental data would not be meaningful. We shall therefore content ourselves with studying the two extreme cases separately.

The experimental results we use in our analysis come from the following processes:

- (1) pseudoscalar ℓ 2-decay;

* Also many subquark models lead naturally to a more general structure of the charged current. However, we do not explicitly consider them in this paper.

- (2) muon β decay;
- (3) nuclear β decay;
- (4) inverse muon decay.

Our study extends and generalizes earlier work by us [7] and by Nandi et al. [8] on the fermion–mirror fermion mixing model, and the analysis of Beg et al. [9] on the left–right symmetric model.

2. Fermion–mirror fermion mixing model

Consider a family unifying group with fermion and mirror fermion representations. In the weak $SU(2)_L \times U(1)$ subgroup one then has the following leptonic weak eigenstates:

$$\begin{pmatrix} \nu'_\ell \\ \ell' \end{pmatrix}_L, \quad \begin{pmatrix} N'_\ell \\ L'_\ell \end{pmatrix}_R, \quad \nu'_{\ell R}, \quad \ell'_R, \quad N'_{\ell L}, \quad L'_{\ell L}, \quad (1)$$

where $\ell = e, \mu, \tau$ labels the extended lepton family consisting of an ordinary charged lepton and its neutrino and the corresponding mirror particles. The charged weak lagrangian for each extended generation has the form

$$\begin{aligned} \mathcal{L}^{cc} &= -\frac{g}{2\sqrt{2}} W_\mu \{ \bar{l}' \gamma^\mu (1 - \gamma_5) \nu'_l + \bar{L}'_l \gamma^\mu (1 + \gamma_5) N'_l \} + \text{h.c.} \\ &\equiv -\frac{g}{2\sqrt{2}} W_\mu J^\mu + \text{h.c.} . \end{aligned} \quad (2)$$

The mass eigenstates (unprimed) are obtained by the following rotations:

$$\begin{aligned} \begin{pmatrix} \ell \\ L_\ell \end{pmatrix} &= \begin{pmatrix} \cos \theta_\ell & -\sin \theta_\ell \\ \sin \theta_\ell & \cos \theta_\ell \end{pmatrix} \begin{pmatrix} \ell' \\ L'_\ell \end{pmatrix} \equiv R(\theta_\ell) \begin{pmatrix} \ell' \\ L'_\ell \end{pmatrix}, \\ \begin{pmatrix} \nu_\ell \\ N_\ell \end{pmatrix} &= \begin{pmatrix} \cos \phi_\ell & -\sin \phi_\ell \\ \sin \phi_\ell & \cos \phi_\ell \end{pmatrix} \begin{pmatrix} \nu'_\ell \\ N'_\ell \end{pmatrix} \equiv R(\phi_\ell) \begin{pmatrix} \nu'_\ell \\ N'_\ell \end{pmatrix}. \end{aligned} \quad (3)$$

The current J^μ then has the form

$$\begin{aligned} J^\mu &= (\bar{\ell}, \bar{L}_\ell) \gamma^\mu [R(\theta_\ell) R^{-1}(\phi_\ell) - R(\theta_\ell) R(\phi_\ell) \tau_3 \gamma_5] \begin{pmatrix} \nu_\ell \\ N_\ell \end{pmatrix} \\ &\equiv J^\mu(\ell \nu_\ell) + J^\mu(\ell N_\ell) + J^\mu(L_\ell \nu_\ell) + J^\mu(L_\ell N_\ell), \end{aligned} \quad (4)$$

where

$$\begin{aligned} R(\theta_\ell) R^{-1}(\phi_\ell) &= \begin{pmatrix} V_\ell & -\tilde{V}_\ell \\ \tilde{V}_\ell & V_\ell \end{pmatrix}, \\ R(\theta_\ell) R(\phi_\ell) &= \begin{pmatrix} A_\ell & -\tilde{A}_\ell \\ \tilde{A}_\ell & A_\ell \end{pmatrix}. \end{aligned} \quad (5)$$

We have used the following short-hand notation for the mixing angles:

$$\begin{aligned}
 \cos(\theta_\ell - \phi_\ell) &= V_\ell, \\
 \sin(\theta_\ell - \phi_\ell) &= \tilde{V}_\ell, \\
 \cos(\theta_\ell + \phi_\ell) &= A_\ell, \\
 \sin(\theta_\ell + \phi_\ell) &= \tilde{A}_\ell.
 \end{aligned}
 \tag{6}$$

The V_ℓ and A_ℓ parametrize the diagonal currents $J(\ell\nu_\ell)$ and $J(L_\ell N_\ell)$, whereas \tilde{V}_ℓ and \tilde{A}_ℓ appear in the non-diagonal currents $J(\ell N_\ell)$ and $J(L_\ell \nu_\ell)$.

The following definitions will be useful later:

$$\begin{aligned}
 \alpha_\ell &\equiv \frac{2 \cos(\theta_\ell + \phi_\ell) \cos(\theta_\ell - \phi_\ell)}{\cos^2(\theta_\ell + \phi_\ell) + \cos^2(\theta_\ell - \phi_\ell)} = \frac{2V_\ell A_\ell}{V_\ell^2 + A_\ell^2}, \\
 \tilde{\alpha}_\ell &\equiv -\frac{2 \sin(\theta_\ell + \phi_\ell) \sin(\theta_\ell - \phi_\ell)}{\sin^2(\theta_\ell + \phi_\ell) + \sin^2(\theta_\ell - \phi_\ell)} = -\frac{2\tilde{V}_\ell \tilde{A}_\ell}{\tilde{V}_\ell^2 + \tilde{A}_\ell^2}, \\
 \beta_\ell &\equiv \cos 2\theta_\ell = V_\ell A_\ell - \tilde{V}_\ell \tilde{A}_\ell.
 \end{aligned}
 \tag{7}$$

Charged mirror fermions must have a mass heavier than $20 \text{ GeV}/c^2$, since they have not been observed in e^+e^- collisions. Mirror neutrinos, in turn, may in principle be light, and will thus open new channels in weak processes. To be as general as possible we shall consider the following four separate cases:

- model a: $m_{N_e}, m_{N_\mu} \ll m_e$;
- model b: $m_{N_e} \ll m_e, m_{N_\mu} > m_K$;
- model c: $m_{N_e}, m_{N_\mu} > m_K$;
- model d: $m_{N_\mu} \ll m_e, m_{N_e} > m_K$.

According to the measurement of the muon range spectrum in the $K^+(\pi^+)$ decay, the existence of a massive neutrino in the mass interval $160\text{--}230$ ($5\text{--}30$) MeV/c^2 and its coupling to the muon are severely restricted [10]. This partly supports our classification with no neutrinos in the intermediate mass range. We shall neglect all light ($\ll m_e$) neutrino masses. Furthermore, all neutrinos are assumed to be Dirac particles. As far as we know, there are no experimental facts contradicting this assumption. In many general unified models, however, the small mass of the left-handed neutrinos can be explained by conjecturing that the neutrinos are Majorana particles with the left-handed component being suppressed by (super)heavy mass scales and the right-handed component remaining (super)heavy.

Case a is unphysical if one rigorously believes in the cosmological limit for the number of the light neutrinos ($N_\nu < 3\text{--}4$) [11]. This limit is already saturated by the ordinary neutrinos. Cases b and d might seem unnatural because of their obvious $e\text{--}\mu$ asymmetry. The most natural is perhaps model c. In our previous analysis [7]

we restricted ourselves to case c. Nandi et al. [8] considered cases a, b and c. In what follows we will investigate all four cases.

2.1. $\pi_{\ell 2}$ AND $K_{\ell 2}$ DECAYS

The partial widths for $\ell 2$ decays of the pseudoscalars π and K are given by

$$\Gamma(P \rightarrow \ell \bar{\nu}_\ell) = \frac{\tilde{G}^2}{8\pi} f_P^2 c_P^2 \frac{m_\ell^2}{m_P^3} (m_P^2 - m_\ell^2)^2 \frac{V_\ell^2 + A_\ell^2}{2}, \tag{8}$$

where f_P , $P = \pi, K$ is the decay constant and c_P stands for the Cabibbo factor. The coupling constant \tilde{G} is defined by $\sqrt{\frac{1}{2}} \tilde{G} = g^2 / 8M_{W_L}^2$. In models a, b and d also the non-diagonal decay $P \rightarrow \ell \bar{N}_\ell$ is allowed. The width for this channel is given by eq. (8) with V_ℓ and A_ℓ replaced by $-\tilde{V}_\ell$ and \tilde{A}_ℓ , respectively.

We use in our analysis the ratios of the total electronic and muonic $\ell 2$ widths

$$R_P = \frac{\Gamma_{\text{total}}(P_{e2})}{\Gamma_{\text{total}}(P_{\mu 2})} = B_P \cdot r, \tag{9}$$

where B_P is the ratio

$$B_P = \frac{m_e^2 (m_P^2 - m_e^2)^2}{m_\mu^2 (m_P^2 - m_\mu^2)^2} (1 + \Delta_P), \tag{10}$$

and Δ_P is the $O(\alpha)$ radiative correction as calculated by Goldman and Wilson [12], $\Delta_\pi = -0.0347$ and $\Delta_K = -0.0372$. The factor r in eq. (9) is a ratio of terms depending on the parameters of the lepton currents. The expressions for R_P in the different models are:

$$\begin{aligned} \text{model a: } R_P &= \frac{\Gamma(P \rightarrow e \bar{\nu}_e) + \Gamma(P \rightarrow e \bar{N}_e)}{\Gamma(P \rightarrow \mu \bar{\nu}_\mu) + \Gamma(P \rightarrow \mu \bar{N}_\mu)} = B_P, \\ \text{model b: } R_P &= \frac{\Gamma(P \rightarrow e \bar{\nu}_e) + \Gamma(P \rightarrow e \bar{N}_e)}{\Gamma(P \rightarrow \mu \bar{\nu}_\mu)} = B_P \frac{2}{V_\mu^2 + A_\mu^2}, \\ \text{model c: } R_P &= \frac{\Gamma(P \rightarrow e \bar{\nu}_e)}{\Gamma(P \rightarrow \mu \bar{\nu}_\mu)} = B_P \frac{V_e^2 + A_e^2}{V_\mu^2 + A_\mu^2}, \\ \text{model d: } R_P &= \frac{\Gamma(P \rightarrow e \bar{\nu}_e)}{\Gamma(P \rightarrow \mu \bar{\nu}_\mu) + \Gamma(P \rightarrow \mu \bar{N}_\mu)} = B_P \frac{1}{2} (V_e^2 + A_e^2). \end{aligned} \tag{11}$$

The equalities $V_\ell^2 + \tilde{V}_\ell^2 = A_\ell^2 + \tilde{A}_\ell^2 = 1$ have been used in deriving these formulae.

The longitudinal polarizations of antineutrinos in π^- and K^- decay are given by

$$P_{\bar{\nu}_\ell} = \alpha_\ell, \quad P_{\bar{N}_\ell} = \tilde{\alpha}_\ell. \tag{12}$$

The longitudinal polarization P_{ℓ^-} of the negatively charged lepton in each subprocess is equal to the polarization of the antineutrino produced with it. In models b and

c only $\bar{\nu}_\mu$ is produced and one accordingly has

$$P_{\mu^-} = \alpha_\mu. \quad (13)$$

In models a and d the muon can be accompanied by both $\bar{\nu}_\mu$ and \bar{N}_μ . The longitudinal polarization of μ^- in these cases is

$$P_{\mu^-} = \beta_\mu. \quad (14)$$

It turns out that the ratio of the widths of electronic and muonic three-lepton decays of the tau lepton also measures the same quantity r , if the tau mirror neutrino is heavy. In the general case, however, two new parameters, V_τ and A_τ , will appear in the formulas. Since the data on the third family of leptons are rather inaccurate and scanty, we will restrict our analysis of the mixing model to the first two generations of fermions and mirror fermions. Therefore we also neglect the data on τ decay.

2.2. MUON DECAY

The effective lagrangian for the free muon decay in the charge changing form is given by

$$\mathcal{L}_{\text{eff}} = \sqrt{\frac{1}{2}} \tilde{G} [J_\alpha(\mu\nu_\mu) + J_\alpha(\mu N_\mu)]^\dagger [J^\alpha(e\nu_e) + J^\alpha(eN_e)]; \quad (15)$$

i.e. there are four possible channels in all:

$$(i) \quad \mu \rightarrow e\nu_\mu\bar{\nu}_e,$$

$$(ii) \quad \mu \rightarrow e\nu_\mu\bar{N}_e,$$

$$(iii) \quad \mu \rightarrow eN_\mu\bar{\nu}_e,$$

$$(iv) \quad \mu \rightarrow eN_\mu\bar{N}_e.$$

Kinematically allowed processes in model a are all the channels (i) to (iv); in model b, (i) and (ii); in model c, (i), and in model d, (i) and (iii). The energy spectrum of the decay electron can be parametrized in the Michel form with the usual parameters ρ , δ , ξ , η and G_F [13]. The different subprocesses contribute additively to these parameters. The decay rate is determined by the Fermi coupling constant G_F . The parameter η vanishes in our cases, where only V and A couplings exist in the charge changing form of the lagrangian. The angular asymmetry of the spectrum is given by ξ . It is important to note that the expression for ξ given in the standard references contains the implicit assumption that the decaying muon is totally polarized. The experiments, however, actually measure the quantity $P_\mu\xi$, where P_μ is the longitudinal polarization of the incident muon produced in pion or kaon

decay*. Separate measurements on the P_μ have been performed [14], but they are too inaccurate to be useful.

The longitudinal polarization of the final state electron is given by the parameter $\xi' = -P_{e^-}$. Transverse polarization, instead, is identically zero, because there are no S, P or T currents present in the charge changing form of the lagrangian and the neutrinos are assumed to be Dirac particles**.

We include the experimental constraints on ρ , δ , $P_\mu \xi$ and P_{e^-} in our fit. The expressions for these parameters in the different models in terms of the coupling constants [see eq. (7)] are given below:

$$\begin{aligned}
 \text{model a:} \quad \rho &= \frac{3}{8}(1 + \beta_e \beta_\mu), \\
 P_\mu \xi &= \beta_\mu (2\beta_e - \beta_\mu), \\
 \delta &= \frac{3}{8} \frac{\beta_e + \beta_\mu}{2\beta_e - \beta_\mu}, \\
 P_{e^-} &= -\beta_e;
 \end{aligned} \tag{16a}$$

$$\begin{aligned}
 \text{model b:} \quad \rho &= \frac{3}{8}(1 + \beta_e \alpha_\mu), \\
 P_\mu \xi &= \alpha_\mu (2\beta_e - \alpha_\mu), \\
 \delta &= \frac{3}{8} \frac{\beta_e + \alpha_\mu}{2\beta_e - \alpha_\mu}, \\
 P_{e^-} &= -\beta_e;
 \end{aligned} \tag{16b}$$

$$\begin{aligned}
 \text{model c:} \quad \rho &= \frac{3}{8}(1 + \alpha_e \alpha_\mu), \\
 P_\mu \xi &= \alpha_\mu (2\alpha_e - \alpha_\mu), \\
 \delta &= \frac{3}{8} \frac{\alpha_e + \alpha_\mu}{2\alpha_e - \alpha_\mu}, \\
 P_{e^-} &= -\alpha_e;
 \end{aligned} \tag{16c}$$

$$\begin{aligned}
 \text{model d:} \quad \rho &= \frac{3}{8}(1 + \alpha_e \beta_\mu), \\
 P_\mu \xi &= \beta_\mu (2\alpha_e - \beta_\mu), \\
 \delta &= \frac{3}{8} \frac{\alpha_e + \beta_\mu}{2\alpha_e - \beta_\mu}, \\
 P_{e^-} &= -\alpha_e.
 \end{aligned} \tag{16d}$$

* We realized this fact (also noted by Sakurai [24]) after completing our preceding paper, which is thus erroneous in this point. Using the correct formula, however, will not much change the numerical results given there.

** We also want to remark that the recent results [25] on the parameters α , α' , β and β' (see ref. [13]) that measure the transverse polarisation of the electron also have to be interpreted as the product of P_μ and the theoretical value of the parameter (e.g. $\alpha^{\text{exp}} = P_\mu \alpha$).

The Fermi coupling constant G_F has the expression $G_F^2 = \tilde{G}^2$, $G_F^2 = \frac{1}{2}\tilde{G}^2(V_\mu^2 + A_\mu^2)$, $G_F^2 = \frac{1}{3}\tilde{G}^2(V_e^2 + A_e^2)(V_\mu^2 + A_\mu^2)$ and $G_F^2 = \frac{1}{2}\tilde{G}^2(V_e^2 + A_e^2)$ in models a, b, c and d, respectively.

2.3. BETA DECAY

Measurement of the longitudinal polarization of the electron in nuclear β -decay provides a well-known test for the V – A hypothesis. Pure V – A predicts the value $P_{\beta^-} = -v/c$. The expression corresponding to the most general case is given e.g. by Marshak et al. [15]. In the models we are considering this reduces to the following forms:

$$\text{models a and b: } P_{\beta^-} = -\frac{v}{c}\beta_e, \quad (17)$$

$$\text{models c and d: } P_{\beta^-} = -\frac{v}{c}\alpha_e.$$

One thus measures here (aside from the factor v/c) the same quantity as in the electron longitudinal polarization in muon decay. The given formulae are valid for both pure Fermi transitions and pure Gamow–Teller transitions.

2.4. INVERSE MUON DECAY

The last piece of data we use in our fits comes from the inverse muon decay

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e. \quad (18)$$

As was realized long ago [16], this process provides information about the V, A structure of the leptonic charged currents and the two-component neutrino theory, beyond what μ -decay without observation of the decay neutrinos gives.

Neglecting the electron mass compared to the incident neutrino energy we obtain for the differential cross section of reaction (18) in the laboratory system*

$$\begin{aligned} \frac{d\sigma}{dy} = \frac{\tilde{G}^2}{16\pi} s \left\{ (V_e^2 + A_e^2)(V_\mu^2 + A_\mu^2) \left(1 + y^2 - \frac{m_\mu^2}{s}(1+y) \right) \right. \\ + 4V_e A_e V_\mu A_\mu \left(1 - y^2 - \frac{m_\mu^2}{s}(1-y) \right) \\ - 2P_{\nu_\mu} \left[((V_e^2 + A_e^2)V_\mu A_\mu + (V_\mu^2 + A_\mu^2)V_e A_e) \left(1 - \frac{m_\mu^2}{s} \right) \right. \\ \left. \left. + ((V_e^2 + A_e^2)V_\mu A_\mu - (V_\mu^2 + A_\mu^2)V_e A_e) \left(y^2 - \frac{m_\mu^2}{s}y \right) \right] \right\}, \quad (19) \end{aligned}$$

where s is the c.m. energy squared, $y = E_\mu/E_{\nu_\mu}$ and P_{ν_μ} is the initial neutrino polarization.

* It is necessary to keep the muon mass terms since the average neutrino energy is $\bar{E}_\nu \approx 20\text{--}25$ GeV for the experiment considered [17] and so $m_\mu^2/\bar{s} = m_\mu^2/2m_e\bar{E}_\nu \sim 0.5$.

In the fermion–mirror fermion mixing model there are four processes that may contribute to the inverse muon decay:

$$\begin{aligned}
 \text{(i)} \quad & \nu_\mu + e^- \rightarrow \mu^- + \nu_e, \\
 \text{(ii)} \quad & \nu_\mu + e^- \rightarrow \mu^- + N_e, \\
 \text{(iii)} \quad & N_\mu + e^- \rightarrow \mu^- + \nu_e, \\
 \text{(iv)} \quad & N_\mu + e^- \rightarrow \mu^- + N_e.
 \end{aligned}
 \tag{20}$$

The differential cross sections for reactions (ii) to (iv) can be obtained from eq. (19) with obvious modifications. The allowed processes in the different models are the following: model a (i)–(iv), model b (i) and (ii), model c (i), and model d (i) and (iii).

In the two models a and d, where the muonic mirror neutrino N_μ is light, the neutrino beam consists of two coherently produced components, ν_μ and N_μ . The probability of finding either of these states interacting is an oscillatory function of their mass difference. If the mass difference is very small ($\Delta m_\nu^2 \equiv |m_N^2 - m_\nu^2| \ll O(1 \text{ eV}^2)$ for the CHARM experiment [26]) then the mass eigenstates add up completely coherently to form weak eigenstates. In this case there is, of course, no dependence on the mixing angle ϕ_μ . On the other hand, if the mass difference is large enough ($\Delta m_\nu^2 \geq 100 \text{ eV}^2$) one detects only the average of the oscillation and the beam is effectively incoherent. We have calculated the cross sections in these two models for both of these extreme cases which are independent of the oscillatory term and thus applicable for our analysis. We call them the coherent and incoherent case. As we will see, the results are almost the same for these two cases.

In the experimental studies the differential cross section is presented in the units of $G_{FS}^2/2\pi$, the value of the cross section in the V–A limit. The expressions for this quantity which we denote S , are in the different models given below.

$$\begin{aligned}
 \text{model a:} \quad S_{\text{coh}} = & \frac{1}{2}\{(\cos^4 \theta_\mu + \sin^4 \theta_\mu)(1+c) \\
 & + \beta_e(\cos^4 \theta_\mu - \sin^4 \theta_\mu)(1-c)\},
 \end{aligned}
 \tag{21a}$$

$$\begin{aligned}
 S_{\text{inc}} = & \frac{1}{2}(\cos^4 \phi_\mu + \sin^4 \phi_\mu)\{(\cos^4 \theta_\mu + \sin^4 \theta_\mu)(1+c) \\
 & + \beta_e(\cos^4 \theta_\mu - \sin^4 \theta_\mu)(1-c)\};
 \end{aligned}$$

$$\text{model b:} \quad S = \frac{1}{4}\{(1 + \alpha_\mu^2)(1+c) + 2\beta_e\alpha_\mu(1-c)\};
 \tag{21b}$$

$$\text{model c:} \quad S = \frac{1}{4}\{(1 + \alpha_\mu^2)(1+c) + 2\alpha_e\alpha_\mu(1-c)\},
 \tag{21c}$$

$$\begin{aligned}
 \text{model d:} \quad S_{\text{coh}} = & \frac{1}{2}\{(\cos^4 \theta_\mu + \sin^4 \theta_\mu)(1+c) \\
 & + \alpha_e(\cos^4 \theta_\mu - \sin^4 \theta_\mu)(1-c)\},
 \end{aligned}
 \tag{21d}$$

$$\begin{aligned}
 S_{\text{inc}} = & \frac{1}{2}\{(\cos^4 \phi_\mu + \sin^4 \phi_\mu)\{(\cos^4 \theta_\mu + \sin^4 \theta_\mu)(1+c) \\
 & + \alpha_e(\cos^4 \theta_\mu - \sin^4 \theta_\mu)(1-c)\}.
 \end{aligned}$$

The constant c takes into account the muon mass terms and the integration over the energy distribution of the neutrino beam. We use the value $c = 0.375$ for the CERN SPS wide-band neutrino beam as given in ref. [17].

3. Left–right symmetric model

The charged current lagrangian of the (ordinary) fermions in the left–right symmetric electroweak model based on the flavor group $SU(2)_L \times SU(2)_R \times U(1)$ has the form

$$\mathcal{L}^{\text{cc}} = -\frac{g}{2\sqrt{2}}[W_L^\dagger (J_\ell^{\text{V}} - J_\ell^{\text{A}}) + W_R^\dagger (J_\ell^{\text{V}} + J_\ell^{\text{A}})] + \text{h.c.}, \quad (22)$$

where W_L and W_R are the charged gauge bosons corresponding to the subgroup $SU(2)_L$ and $SU(2)_R$, respectively, and the leptonic currents are given by

$$J_{\ell,\mu}^{\text{V}} = \bar{\ell} \gamma_\mu \nu_\ell, \quad J_{\ell,\mu}^{\text{A}} = \bar{\ell} \gamma_\mu \gamma_5 \nu_\ell. \quad (23)$$

Let the left–right symmetry be broken by an asymmetric vacuum, causing the gauge bosons W_L and W_R to mix and form two mass eigenstates W_1 and W_2 with unequal masses $m(W_1)$ and $m(W_2)$, respectively. We define the mixing angle ω by

$$\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} \cos \omega & -\sin \omega \\ \sin \omega & \cos \omega \end{pmatrix} \begin{pmatrix} W_L \\ W_R \end{pmatrix}. \quad (24)$$

In any specific left–right symmetric model, both the masses and the mixing angle are explicitly related to the vacuum expectation values of the Higgs fields responsible for the symmetry breaking. We will not commit ourselves to any specific model here, but consider the mass ratio $m(W_1)/m(W_2)$, and the mixing angle ω as free independent parameters.

The effective four-fermion lagrangian (in the charge changing form) in the low $|q^2|$ limit reads [9]

$$\mathcal{L}_{\text{eff}}^{\text{cc}} = -\sqrt{\frac{1}{2}} G_{\text{LR}} \{ J_\ell^{\text{V}\dagger} \cdot J_\ell^{\text{V}} + \eta_{\text{AA}} J_\ell^{\text{A}\dagger} \cdot J_\ell^{\text{A}} + \eta_{\text{AV}} [J_\ell^{\text{V}\dagger} \cdot J_\ell^{\text{A}} + J_\ell^{\text{A}\dagger} \cdot J_\ell^{\text{V}}] \}, \quad (25)$$

where

$$\begin{aligned} \sqrt{\frac{1}{2}} G_{\text{LR}} &= \frac{1}{8g^2} \left[\left(\frac{\sin \omega - \cos \omega}{m_{W_1}} \right)^2 + \left(\frac{\sin \omega + \cos \omega}{m_{W_2}} \right)^2 \right], \\ \eta_{\text{AA}} &= \frac{r + \varepsilon^2}{1 + r\varepsilon^2}, \quad \eta_{\text{AV}} = \frac{(r-1)\varepsilon}{1 + r\varepsilon^2}, \\ \varepsilon &= \frac{1 + \tan \omega}{1 - \tan \omega}, \quad r = \left(\frac{m_{W_1}}{m_{W_2}} \right)^2. \end{aligned} \quad (26)$$

The pure V–A results are recovered when $\varepsilon = 1$ and $r = 0$, or equivalently $\eta_{AA} = -\eta_{AV} = 1$.

We will determine the values for the parameters r and ω by making a fit to the data obtained from the muon decay, the nuclear beta decay, and the inverse muon decay. Since the form of the interaction lagrangian in this model is universal for all lepton families, the decay ratios of the electric and muonic channels in the pseudoscalar decays and in the tau decay will not yield any constraints on the parameters.

At this stage we want to point out that the two effective charged current lagrangians considered in this work and given in eqs. (15) and (25), are not equivalent, although both of them in the respective models correspond to a general form of V, A coupling. The basic difference is that in the mixing model there exists only one type of charged gauge boson, W_L whereas in the LR model both W_L and W_R are present. The effective lagrangian of the mixing model, given in eq. (15), can be rewritten in the form of the left–right symmetric effective lagrangian, eq. (25), only on the condition that $\eta_{AA} = \eta_{AV}^2$, which correspond in the LR model to the limit $m(W_R) \rightarrow \infty$, i.e. effectively a one gauge boson model. On the other hand, the mixing model is more general in allowing for breaking of generation universality.

Another important difference between the two models is that in the left–right symmetric model the structure of the effective couplings depends on the momentum transfer $-q^2$ of the process considered, whereas in the mixing model the relative strengths of the vector and axial vector couplings remain constant. When $-q^2$ increases the contribution of the V+A term in the left–right symmetric model grows and the weak interactions approach the parity conservation limit.

We now list the quantities to be used in the fit for the left–right symmetric model.

3.1. MUON DECAY

The decay parameters of the muon decay are given by

$$\begin{aligned} \rho &= \frac{3}{8} \frac{(1 + \eta_{AA})^2 + 4\eta_{AV}^2}{1 + \eta_{AA}^2 + 2\eta_{AV}^2}, \\ P_\mu \xi &= -\frac{2\eta_{AV}\eta_{AA}}{\eta_{AA}^2 + \eta_{AV}^2} \cdot \left[-\frac{2\eta_{AV}(1 + \eta_{AA})}{1 + \eta_{AA}^2 + 2\eta_{AV}^2} \right], \\ \delta &= \frac{3}{4}, \quad \eta = 0, \quad P_{e^-} = -\xi. \end{aligned} \tag{27}$$

The Fermi coupling constant is given by $G_F^2 = \frac{1}{4}G_{LR}^2(1 + \eta_{AA}^2 + 2\eta_{AV}^2)$.

3.2. BETA DECAY

The expression for the longitudinal polarization of the electron in the nuclear beta decay is now different for pure Fermi and pure Gamow–Teller transitions, in

contrast to the mixing model:

$$P_{\beta^-}^F = \frac{v}{c} \frac{2\eta_{AV}}{1 + \eta_{AV}^2},$$

$$P_{\beta^-}^{GT} = \frac{v}{c} \frac{2\eta_{AV}/\eta_{AA}}{1 + (\eta_{AV}/\eta_{AA})^2}. \quad (28)$$

3.3. INVERSE MUON DECAY

The differential cross section for the inverse muon decay process, eq. (18), now has the form

$$\frac{d\sigma}{dy} = \frac{G_{LR}^2}{16\pi} s \left\{ (1 + \eta_{AA}^2) \left(1 + y^2 - \frac{m_\mu^2}{s} (1 + y) \right) \right. \\ \left. + 2\eta_{AA} \left(1 - y^2 - \frac{m_\mu^2}{s} (1 - y) \right) + 4[\eta_{AV}^2 + P_{\nu_\mu} \eta_{AV} (1 + \eta_{AA})] \left(1 - \frac{m_\mu^2}{s} \right) \right\}, \quad (29)$$

where s is the invariant mass squared, $y = E_\mu/E_{\nu_\mu}$, and the initial neutrino polarization is given by

$$P_{\nu_\mu} = \frac{2\eta_{AV}\eta_{AA}}{\eta_{AA}^2 + \eta_{AV}^2}. \quad (30)$$

Proceeding as in the case of the mixing models, one finds for the normalized cross section S the expression

$$S = \frac{1}{4} \frac{(1 + \eta_{AA})^2 + 4\eta_{AV}^2 + 4P_{\nu_\mu} \eta_{AV} (1 + \eta_{AA}) + c(1 - \eta_{AA})^2}{1 + \eta_{AA}^2 + 2\eta_{AV}^2}, \quad (31)$$

where the constant $c = 0.375$ as before.

4. Results of the fit

The experimental data used in the fit are presented in table 1. In the fermion-mirror fermion mixing model we combine the results from π and K decays [18]:

$$R_\pi / 1.239 \cdot 10^{-4} = 1.023 \pm 0.019$$

$$R_K / 2.474 \cdot 10^{-5} = 0.978 \pm 0.044$$

$$R = 1.016 \pm 0.017$$

Similarly, we combine the information on the longitudinal polarization of electron in muon decay [18, 19] and in pure Gamow-Teller nuclear beta decay [20]:

$$-P_{e^-} = 1.008 \pm 0.054$$

$$-c/vP_{\beta^-} = 1.001 \pm 0.008$$

$$P = 1.0012 \pm 0.0079$$

TABLE 1

V – A predictions and experimental values of the constraints used in the fit

Constraint	V – A value	Experimental value	Ref.
ρ	$\frac{3}{4}$	0.7517 ± 0.0026	[18]
δ	$\frac{3}{4}$	0.7551 ± 0.0085	[18]
$P_{\mu\xi}$	1	0.9722 ± 0.0140	[18]
$R_{\pi}/1.239 \cdot 10^{-4}$	1	1.023 ± 0.019	[18]
$R_{K}/2.474 \cdot 10^{-5}$	1	0.978 ± 0.044	[18]
$-P_e^-$	1	1.008 ± 0.054	[18, 19]
$-c/vP_{\beta}^{GT}$	1	1.001 ± 0.008	[20]
S	1	0.98 ± 0.18	[26]

We make a least squares fit to the data for the different models and determine the parameters at the one standard deviation (1σ) level. Generally the contours corresponding to a probability β (the β confidence level) in the space of n free parameters can be approximated in a least-squares fit by $\chi^2 = \chi_{\min}^2 + \Delta(\beta, n)$, where χ_{\min}^2 is the least square and $\Delta(\beta, n)$ is a constant [21]. The 1σ confidence level corresponds to $\beta = 0.683$, and the values of $\Delta(0.683, n)$ for one, two and three degrees of freedom are 1, 2.30 and 3.54, respectively. To find χ_{\min}^2 we have used the program MINUIT [22].

The results of the fit are presented in table 2. The table gives the best fit values of the mixing angles and the 1σ errors. For the mixing models the confidence region in the n -parameter space is n -fold symmetric around the V – A value. In model c the contour has a rather complicated shape (see below).

In the models a and b the theoretical formulae corresponding to the experimental constraints do not depend on the electron neutrino mixing angle ϕ_e , which consequently may obtain any value. There are no measurements of a purely charged current process which would limit this angle. Only elastic electron-antineutrino scattering $\bar{\nu}_e e \rightarrow \bar{\nu}_e e$, proceeding also via a neutral current diagram, would constrain ϕ_e , if incoherent scattering is justified. The neutral current analysis will be done in a separate work [27].

The quantity R is not fitted in models a, since R does not depend on the parameters of that model. The only data constraining ϕ_{μ} in models a is the incoherent inverse muon decay, S_{inc} . Model b fits R well, and compared with model a_{inc} R seems to force ϕ_{μ} down to lower values. The results in these two models are very similar. The main contribution to χ_{total}^2 in both models comes from the fit to $P_{\mu\xi}$, $\chi^2(P_{\mu\xi}) = 2.3$.

In models c and d_{inc} all four mixing angles θ_e , θ_{μ} , ϕ_e , and ϕ_{μ} enter. In model c the 1σ contour has a complicated structure. This is due to the fact that in the V – A limit one of the parameters is redundant. The V – A limit is obtained in the following

TABLE 2
Results of the fit for the fermion-mirror fermion mixing models

Model	Best fit values for the parameters (with 68.3% confidence limit)	Best fit values (with χ^2 contribution)					$\chi^2_{\text{total}}/\text{d.f.}$	
		ρ	δ	$P_{\mu\xi}$	R	S		P
a_{coh}	$ \theta_e = 2.3^\circ (<3.8^\circ)$ ϕ_e free $ \theta_\mu = 0.0^\circ (<2.7^\circ)$ ϕ_μ free	0.749 (1.29)	0.754 (0.02)	0.993 (2.28)		0.999 (0.01)	0.997 (0.33)	3.9/3
	$ \theta_e = 2.3^\circ (<4.1^\circ)$ ϕ_e free $ \theta_\mu = 0.0^\circ (<3.2^\circ)$ $ \phi_\mu = 5.6 (<29^\circ)$	0.749 (1.29)	0.754 (0.02)	0.993 (2.28)		0.980 (0.00)	0.997 (0.33)	3.9/2
b	$ \theta_e = 2.3^\circ (<4.1^\circ)$ ϕ_e free $ \theta_\mu = 7.2^\circ (<12^\circ)$ $ \phi_\mu = 0.0^\circ (<12^\circ)$	0.749 (1.29)	0.754 (0.02)	0.993 (2.28)	1.016 (0.00)	0.999 (0.01)	0.997 (0.33)	3.9/3
	$ \theta_e = 30^\circ$ $ \phi_e = 4.1^\circ$ $ \theta_\mu = 31^\circ$ $ \phi_\mu = 0$ fixed	0.749 (1.29)	0.754 (0.02)	0.993 (2.28)	1.016 (0.00)	0.999 (0.01)	0.997 (0.33)	3.9/3
c	$ \lambda_e - 1 = 0.09 (<0.15)$ $ \lambda_\mu - 1 = 0.00 (<0.11)$ $ \kappa - 1 = 0.04 (<0.08)$	0.749 (1.29)	0.754 (0.02)	0.993 (2.28)	1.016 (0.00)	0.999 (0.01)	0.997 (0.33)	3.9/3
	$ \theta_e = 0.0^\circ (<8.0^\circ)$ $ \phi_e = 0.0^\circ (<8.1^\circ)$ $ \theta_\mu = 0.0^\circ (<3.2^\circ)$ ϕ_μ free	0.750 (0.43)	0.750 (0.36)	1.000 (3.94)	1.000 (0.89)	1.000 (0.001)	1.000 (0.02)	5.7/3
d_{inc}	$ \theta_e = 0.0^\circ (<9.0^\circ)$ $ \phi_e = 0.0^\circ (<8.1^\circ)$ $ \theta_\mu = 0.0^\circ (<3.1^\circ)$ $ \phi_\mu = 0$ fixed ($<21^\circ$)	0.750 (0.43)	0.750 (0.36)	1.000 (3.94)	1.000 (0.89)	1.000 (0.001)	1.000 (0.02)	5.7/3(2)

¹ Complicated 68.3% contour (see the text).

four special cases [8]:

- (i) $\theta_e = \theta_\mu = 0$, $|\phi_e| = |\phi_\mu| = \alpha$,
- (ii) $\phi_e = \theta_\mu = 0$, $|\theta_e| = |\phi_\mu| = \alpha$,
- (iii) $\theta_e = \phi_\mu = 0$, $|\phi_e| = |\theta_\mu| = \alpha$,
- (iv) $\phi_e = \phi_\mu = 0$, $|\theta_e| = |\theta_\mu| = \alpha$,

where α is an arbitrary positive angle. The contour consists of a large region around the point $\theta_e = \phi_e = \theta_\mu = \phi_\mu = 0$ and narrow strips along the V-A asymptotes. The

points corresponding to the minimum of χ^2 form a continuous curve in the parameter space. The best fit values given in table 2 correspond to the point $\theta_e = 30^\circ$, $\phi_e = 4.1^\circ$, $\theta_\mu = 31^\circ$ and $\phi_\mu = 0^\circ$ lying on this curve.

To get a better idea of the size of the deviation from the V–A limit in model c, we have refitted the data using the parameter $\lambda_i = A_i/V_i$ ($i = e, \mu$) and $\kappa = V_\mu/V_e$ instead of the mixing angles. The results of this fit are also presented in table 2. The fit in model c is equally good as in model b, i.e. $\chi^2_{\text{total}}/\text{d.f.} = 3.9/3$. As in all the models, the main contribution to χ^2_{total} comes from the fit to $P_\mu\xi$.

In model d_{inc} the angle ϕ_μ is again constrained by S_{inc} only. The fit is poor in both cases, $\chi^2_{\text{total}}/\text{d.f.} = 5.7/3$ and gives exactly the V–A values. Models d are far less probable than others.

In the left–right symmetric model the fit is made to the experimental quantities ρ , $P_\mu\xi$, P_{e^-} , S , and P_{β^-} given in table 1. There are two theoretical parameters in this model, the mass ratio m_{W_1}/m_{W_2} of the charged gauge bosons and the W_L – W_R mixing angle ω . The results of the fit are presented in table 3. The value of $\chi^2_{\text{total}}/\text{d.f.}$ for the best fit is 3.1/3. Thus the left–right symmetric model gives a somewhat better fit than the mixing models. This is mainly caused by the improvement in the fit to the parameter $P_\mu\xi$.

5. Conclusions and discussion

In this paper we have investigated the charged leptonic weak currents in connection with two models, the fermion–mirror fermion mixing model and the left–right symmetric $SU(2)_L \times SU(2)_R \times U(1)$ flavor model. Both of these models allow deviations from the standard V–A theory and lead to more general V, A structures for the leptonic charged currents.

In the mixing model the V+A component of the weak coupling arises from the assumption that physical fermions are mixtures of the weak interaction eigenstates of V–A fermions and V+A mirror fermions. In the left–right symmetric model the right-handed vector boson corresponding to the subgroup $SU(2)_R$ mediates the V+A interaction of fermions. According to the different origins of the V+A component, these models do not lead to an equivalent parametrization of the

TABLE 3
Results of the fit for the left–right symmetric model

Best fit values (with 68.3% confidence limit)	Best fit values (with χ^2 contribution)					$\chi^2_{\text{total}}/\text{d.f.}$
	ρ	$P_\mu\xi$	P_{e^-}	P_β	S	
$m_{W_1}/m_{W_2} = 0.22$ (<0.29) $ \omega = 2.8^\circ$ ($<4.2^\circ$)	0.749 (0.97)	0.988 (1.30)	0.983 (0.21)	0.995 (0.58)	0.991 (0.00)	3.1/3

charged weak currents, but provide two independent modifications of the $V-A$ theory.

Mirror fermions are predicted by many family unification models (e.g. $SO(2n)$, $SO(2n+1)$, $SU(n)$ ($n > 5$) and E_8 models). To be compatible with the experimental observations, the charged mirror leptons should be heavier than about $20 \text{ GeV}/c^2$. The mirror neutrinos, on the contrary, may be light. We have considered four separate cases with different mass configurations of the electron mirror neutrino N_e and the muon mirror neutrino N_μ . In models a and d we used two different formulas for the inverse muon decay scattering, corresponding to the two different mass scales for the light muonic mirror neutrino. The fits are almost as good in all the models, except for models d, which seem to be slightly disfavored.

Charged current data from the following processes were included in the fit: pseudoscalar meson decay, muon decay, nuclear beta decay, and inverse muon decay. In π and K decays only the ratio of widths to the electronic and muonic two lepton channels was considered. This ratio is independent of the hadronic part of the process, e.g. the Cabibbo angle. Similarly, in nuclear β -decay we used data on longitudinal polarization of the electron in pure Gamow-Teller transitions which do not depend on the nucleon couplings.

The values of the best fit parameters were observed generally to lie within one standard deviation from their $V-A$ values, but substantial departures from the standard $V-A$ results are allowed. The mixing angles in the fermion-mirror fermion mixing models are not always all constrained by the data used; the mixing angle of the ν_e and the corresponding mirror neutrino N_e may in some cases obtain an arbitrarily large value. In the value of the mixing angle of the muonic neutrino and its mirror particle there is also arbitrariness, since in some cases this angle is constrained only by the inaccurate data of the inverse muon decay.

The left-right symmetric model turned out to give a slightly better fit than the mixing models. The decisive experimental parameter seems to be ξ or the product $P_\mu \xi$ of the muon decay, where P_μ is the longitudinal polarization of the muon in pseudoscalar decay and ξ is the angular asymmetry factor of the final state electron spectrum. This quantity deviates noticeably from the $V-A$ prediction whereas the Michel parameter ρ is in good agreement with the $V-A$ prediction. More accurate measurements of ξ might allow one to discriminate between the $SU(2)_L \times U(1)$ flavor model and the left-right symmetric $SU(2)_L \times SU(2)_R \times U(1)$ flavor model.

We obtained a lower limit for the mass of the dominantly right-handed gauge boson W_2 . At the 68.3% confidence level $m(W_2) > 3.4 m(W_1)$. For the $W_L - W_R$ mixing angle the fit gives $|\omega| < 4.2^\circ$ (68.3% c.l.). Curiously enough, a limit of the same order has been obtained for the mass of the second neutral vector boson from neutral current processes [23]. We can see that the accuracy of neutral currents is already as good as that of charged currents and that a unified treatment of the data of both reaction types may be justified.

Finally, for a further discrimination between the mixing models and the left-handed flavor model accurate measurements of the electron longitudinal polarization in nuclear beta decays associated with pure Fermi transitions are required. In the left-right symmetric model different expressions are obtained in the case of pure Fermi and pure Gamow-Teller transitions, whereas in the mixing models these processes measure the same theoretical quantity.

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