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Nuclear Physics B269 (1986) 109–118
North-Holland, Amsterdam

LEPTONIC *CP*-VIOLATION AND MIRROR LEPTONS

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Received 18 October 1985

We study leptonic *CP*-violation within a single fermion generation in an extended electroweak model where new right-handed leptons, so-called mirror leptons, exist and mix with standard left-handed leptons. Essential differences are pointed out between the mixing of a left-handed neutrino with a mirror neutrino in such a model and the ordinary mixing of two left-handed neutrinos in the standard model. E.g. in the mirror case *CP*-violation may occur even if neutrinos are Dirac particles. The number of physical *CP*-violation phases is determined for different mixing schemes and parameterizations of the leptonic weak currents in terms of these phases are presented.

1. Introduction

As noticed by several authors [1, 2], the leptonic analogue of the Cabibbo matrix describing the mixing between two left-handed (LH) neutrino flavours ("flavour mixing") contains one *CP*-violating phase in a general Majorana neutrino case. For Dirac neutrinos such a phase can only appear at the three-generation level. The reason for this difference can be easily understood in the Majorana representation, where Majorana fields are real by definition and hence no complex phases can be absorbed into them from the rest of the theory. Two of the original three phases of the general Cabibbo matrix for two generations, on the other hand, can always be annihilated by the arbitrary phases of charged lepton fields.

In this paper we study how the situation is changed if, instead of two standard LH lepton generations, one considers the mixing between one LH lepton generation and one right-handed (RH) mirror lepton generation ("mirror mixing"). A particular motivation to investigate mirror fermions [3] arises from their natural and often unavoidable appearance in many theoretical ideas of present-day particle physics,

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0550-3213/86/$03.50

e.g. in the extended supersymmetries [4], some superstring models [5], Kaluza-Klein-type theories [6] and in models of family unification [3, 7]. For example, in the models unifying families within SO(18) gauge symmetry detailed work has been done recently [7] and it has been shown that in such models an equal number (three or four) of standard and mirror generations will exist at the mass scale below about 200–250 GeV. As mirror fermions feel the electroweak force with the usual strength, one expects to produce them copiously in the next-generation accelerators, so having a crucial test for the variety of theoretical models mentioned above. It is therefore important to study in detail properties and phenomenological consequences of mirror fermions so as to make it possible to distinguish them from the ordinary type of new fermions. In the present paper we concentrate on the question of *CP*-violation in the mirror mixing and find many striking differences as compared to the ordinary flavour mixing. Phenomenological implications, especially with regard to the experimental search for heavy leptons, will be discussed in a separate publication [8].

We divide our analysis into two parts according to whether neutrinos are Majorana (sect. 2) or Dirac particles (sect. 3). In both cases we determine the number of *CP*-violating phases and give a parameterization for the charged and neutral currents of leptons. The *CP*-conserving limit in the Majorana case, as well as the effect of a mixing between charged leptons, are discussed in detail. A summary of the results is presented in sect. 4.

2. Majorana neutrinos

Let us start with the simplest model incorporating mirror leptons, i.e. the standard $SU(2) \times U(1)$ theory where the ordinary LH lepton doublet and RH charged lepton singlet are accompanied by an RH mirror lepton doublet and an LH charged mirror lepton singlet. In other words, each enlarged lepton generation consists of the following set of $SU(2) \times U(1)$ multiplets:

$$\begin{pmatrix} \nu \\ \ell^- \end{pmatrix}_L \sim (2, -1), \qquad \ell^-_R \sim (1, -2);$$

$$\begin{pmatrix} N \\ L^- \end{pmatrix}_R \sim (2, -1), \qquad L^-_L \sim (1, -2), \tag{1}$$

where mirror leptons are denoted by capital letters. The singlet fields $\nu_R \sim (1, 0)$ and $N_L \sim (1, 0)$ need not exist if neutrinos are Majorana particles. (Or, if they exist, they may have decoupled from the rest of the theory.) In the Dirac case, which we will consider later on, they will naturally be included.

We will only consider the dominant lepton/mirror-lepton mixings within one enlarged generation of eq. (1) and neglect the possible intergenerational mixings for simplicity of our argument. The general Majorana mass lagrangian for the neutrinos ν_L and N_R is then given by

$$-\mathcal{L}_M = \hat{m}_\nu \nu_L^T C \nu_L + \hat{m}_N N_L^{cT} C N_L^c + 2\hat{m}_{\nu N} \bar{N}_R \nu_L + \text{h.c.}$$

$$= (\bar{\nu}_R^c, N_R) M \begin{pmatrix} \nu_L \\ N_L^c \end{pmatrix} + \text{h.c.}, \tag{2}$$

where, e.g. $\nu_R^c = C(\bar{\nu}_L)^T$, and the second equation defines the symmetric two-by-two mass matrix M. The diagonal terms in eq. (2) are Majorana mass terms and the off-diagonal ones are of Dirac-type. If either $\hat{m}_\nu$ or $\hat{m}_N$ is different from zero, the mass eigenstates will be Majorana neutrinos. In the opposite case ν_L and N_R will combine to form a pseudo-Dirac neutrino which couples through its LH part to ℓ^- and RH part to L^- with equal strengths.

Let us write the elements of the complex mass matrix M in a polar form as follows:

$$M = \begin{pmatrix} m_\nu e^{i\gamma_1} & m_{\nu N} e^{i\gamma_{12}} \\ m_{\nu N} e^{i\gamma_{12}} & m_N e^{i\gamma_2} \end{pmatrix}, \tag{3}$$

where the phases γ_1, γ_2 and γ_{12} are real numbers and the norms m_ν, m_N and $m_{\nu N}$ are real and non-negative. The symmetric matrix M can be diagonalized by a unitary matrix U through a congruent transformation

$$U^T M U = M_d = \text{diag}(m_1, m_2), \tag{4}$$

where the eigenvalues m_i $(i = 1, 2)$ can be taken real and non-negative, i.e. they are the physical masses of the corresponding neutrino eigenstates. The matrix U can be expressed in the following form [2]:

$$U = U'\Theta^+, \tag{5}$$

where U' is a unitary matrix given by

$$U' = \begin{pmatrix} \cos\theta\, e^{-i\gamma_1/2} & -\sin\theta\, e^{-i(\gamma_1 - 2\alpha)/2} \\ \sin\theta\, e^{-i(\gamma_2 + 2\alpha)/2} & \cos\theta\, e^{-i\gamma_2/2} \end{pmatrix} \tag{6}$$

and Θ is a diagonal matrix of phases

$$\Theta = \text{diag}(e^{i\theta_1/2}, e^{i\theta_2/2}). \tag{7}$$

The unitarity of U implies the relation

$$2m_{\nu N} e^{i\beta} \cos 2\theta = (m_\nu e^{i\alpha} - m_N e^{-i\alpha}) \sin 2\theta, \tag{8}$$

where

$$\beta = \gamma_{12} - \tfrac{1}{2}(\gamma_1 + \gamma_2). \tag{9}$$

The phases θ_i $(i = 1, 2)$ are determined by the requirement that m_i's in (4) are real and non-negative (see eq. (13) below).

The eigenstates of M are given by

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = U^+ \begin{pmatrix} \nu_L \\ N_L^c \end{pmatrix} + U^T \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} \tag{10}$$

according to eqs. (2) and (4). In our formalism the mass eigenstates χ_i hence obey the "clean" (phaseless) Majorana condition

$$\chi_i^c = \chi_i . \tag{11}$$

Consequently all CP-violation parameters will be visible in the lagrangian instead of being hidden in the field operators as "creation phase factors" [2, 9].

The neutrino mass lagrangian $\mathcal{L}_m$ reads in the mass eigenstate basis as follows:

$$-\mathcal{L}_m = m_1 \bar{\chi}_1 \chi_1 + m_2 \bar{\chi}_2 \chi_2 . \tag{12}$$

For $\cos 2\theta \neq 0$, the mass eigenvalues m_i (and an implicit definition of the phases θ_i) are given by

$$m_1 = \left(m_\nu \cos^2\theta - m_N \sin^2\theta \, e^{-i2\alpha}\right) e^{-i\theta_1} / \cos 2\theta ,$$

$$m_2 = \left(m_N \cos^2\theta - m_\nu \sin^2\theta \, e^{i2\alpha}\right) e^{-i\theta_2} / \cos 2\theta . \tag{13}$$

In the special case $\cos 2\theta = 0$, eq. (8) implies $m_\nu = m_N$ and $\sin\alpha = 0$ and the masses are then

$$m_{1,2} = \left(m_\nu \pm m_{\nu N} e^{i\beta}\right) e^{-i\theta_{1,2}} . \tag{14}$$

We now address the following question: how many of the original three phases (γ_1, γ_2 and γ_{12}) of the mass matrix M are physically significant? We will answer this question first in the simple case where the charged lepton ℓ and its mirror partner L do not mix with each other and consider the general case thereafter.

2.1. THE CASE OF NO ℓL MIXING

When the leptons ℓ and L do not mix the charged weak current is according to eq. (10) given by

$$J_{cc}^\mu = \bar{\ell}_L \gamma^\mu \nu_L + \bar{L}_R \gamma^\mu N_R = \bar{\ell}_L \gamma^\mu U_{1i} \chi_{iL} + \bar{L}_R \gamma^\mu U_{2i}^* \chi_{iL} . \tag{15}$$

Redefining the phases of the charged lepton fields as follows,

$$\ell \to e^{i(\gamma_1+\theta_1)/2}\ell ,$$
$$L \to e^{-i(\gamma_2+\theta_2)/2}L , \qquad (16)$$

we can make the dominant couplings in eq. (15) (i.e., those proportional to $\cos\theta$) real, but there remains then a complex phase factor in the subdominant terms:

$$J^{\mu}_{cc} = \bar{\ell}_L\gamma^{\mu}(\cos\theta\, \chi_{1L} - e^{i\delta}\sin\theta\, \chi_{2L})$$
$$+\bar{L}_R\gamma^{\mu}(e^{i\delta}\sin\theta\, \chi_{1R} + \cos\theta\, \chi_{2R}) . \qquad (17)$$

The phase δ is given by

$$\delta = \alpha + \tfrac{1}{2}(\theta_1 - \theta_2) . \qquad (18)$$

It is straightforward to see that this same phase appears automatically in the neutral current of neutrinos:

$$J^{\mu}_{nc} = \bar{\nu}_L\gamma^{\mu}\nu_L + \bar{N}_R\gamma^{\mu}N_R$$
$$= \bar{\chi}_{Li}U^*_{1i}U_{1j}\gamma^{\mu}\chi_{Lj} + \bar{\chi}_{Ri}U_{2i}U^*_{2j}\gamma^{\mu}\chi_{Rj}$$
$$= \tfrac{1}{2}\cos 2\theta[-\bar{\chi}_1\gamma^{\mu}\gamma_5\chi_1 + \bar{\chi}_2\gamma^{\mu}\gamma_5\chi_2]$$
$$+\sin 2\theta[-i\sin\delta\, \bar{\chi}_1\gamma^{\mu}\chi_2 + \cos\delta\, \bar{\chi}_1\gamma^{\mu}\gamma_5\chi_2] . \qquad (19)$$

In this view the redefinition (16) of the charged lepton fields proves to be a natural one. We notice that in (19) the phase δ again joins the θ-suppressed terms while the dominant diagonal couplings are CP-conserving.

Thus if the charged leptons ℓ and L do not mix, the lepton sector of the mirror model can contain at most one physical CP-phase, just as is the case in the standard flavour mixing. However, a remarkable difference between the two mixing schemes is that in the case of mirror mixing also neutral currents can be CP-violating. In the flavour mixing case non-diagonal couplings, and hence CP-violation, are absent in neutral currents due to the GIM mechanism.

Let us make a comment concerning the Lorentz structure of the neutral currents in eq. (19). While the diagonal currents are purely axial ($\bar{\chi}\gamma_{\mu}\chi = 0$ for Majorana neutrinos), the non-diagonal ones are of more general vector-axial vector structure. Only in the CP-conserving case (see later) the non-diagonal currents have either pure vector ($\delta = \pm\frac{1}{2}\pi$) or axial vector ($\delta = 0$) couplings.

Before we move to discuss the consequences of having a non-vanishing ℓL mixing, let us consider the CP-conserving limit of the above case. We start by noticing that if β (eq. (9)) vanishes, CP is conserved. Indeed, for $\beta = 0$, eq. (8) implies $\sin\alpha = 0$ and, consequently, eq. (13) gives $\sin\theta_i = 0$. Hence the phase δ can only attain the values $\delta = 0$ and $\pm\frac{1}{2}\pi$ ($\delta = \pi$ corresponds to $\delta = 0$ with $\theta \to -\theta$). More explicitly, $\delta = 0$ when θ_1 and θ_2 are equal, which occurs (see eq. (13)) for

$$\tan^2(2\theta) < \frac{4m_\nu m_N}{(m_\nu - m_N)^2}\,. \tag{20}$$

By virtue of (8) one can rewrite this condition in the following more simple form (valid also for $\cos 2\theta = 0$)*:

$$m_{\nu N}^2 < m_\nu m_N\,. \tag{21}$$

When the inequalities (20) and (21) are not satisfied, the phase takes the values $\delta = \pm\frac{1}{2}\pi$. We should notice that in this latter case, although CP is conserved, the weak currents (17) and (19) of Majorana neutrinos contain purely imaginary couplings. Earlier confusion in the literature about a similar situation within the standard model was clarified by Wolfenstein [10].

2.2 THE CASE OF NON-VANISHING ℓL MIXING

We now turn to discuss the modifications that the inclusion of a non-vanishing mixing between charged leptons will imply for the results obtained above. There is again a fundamental difference between the flavour mixing and mirror mixing cases in this respect. As is well known, in the standard model one is always allowed to regard charged members of lepton doublets unmixed and one need only consider mixings among neutrinos. This freedom stems from the fact that the interaction lagrangian, e.g. the part involving the leptonic doublets L_i ($i = \mathrm{e}, \mu$), $\bar{L}_\mathrm{e}\gamma_\rho D^\rho L_\mathrm{e} + \bar{L}_\mu\gamma_\rho D^\rho L_\mu$, is as a norm invariant under arbitrary rotations in the (L_e, L_μ) space. In the mirror lepton case we do not have such freedom since the covariant derivatives D^ρ for leptons and mirror leptons are different. Accordingly, ℓL mixing is a genuine physical effect in the mirror model and will have consequences concerning our previous discussion.

Let us consider the general (Dirac) mass lagrangian for the charged leptons ℓ and L:

$$-\mathcal{L}_{\ell L} = \hat{m}_\ell \bar{\ell}_\mathrm{L}\ell_\mathrm{R} + \hat{m}_L \bar{L}_\mathrm{L} L_\mathrm{R} + \hat{m}_{\ell L}\bar{\ell}_\mathrm{L} L_\mathrm{R} + \hat{m}_{L\ell}\bar{L}_\mathrm{L}\ell_\mathrm{R} + \mathrm{h.c.}$$

$$= (\bar{\ell}_\mathrm{L}, \bar{L}_\mathrm{L}) M_{\ell L}\begin{pmatrix}\ell_\mathrm{R}\\ L_\mathrm{R}\end{pmatrix} + \mathrm{h.c.}. \tag{22}$$

* Notice that in case one diagonalizes the real form of the matrix M in eq. (3) via an orthogonal transformation as is done, e.g. in ref. [10], the eigenvalues of M will take the same sign if the condition (21) is satisfied and otherwise opposite signs. In the parlance of Wolfenstein [10], the two neutrino eigenstates will then have the same or the opposite CP quantum numbers, respectively.

Since the mass matrix $M_{\ell L}$ need not in this case be symmetric, we will usually have four phases to start with. Two unitary matrices, $\tilde{U}_{\rm L}$ and $\tilde{U}_{\rm R}$, are generally needed to diagonalize $M_{\ell L}$:

$$\tilde{U}_{\rm L}^{+} M_{\ell L} \tilde{U}_{\rm R} = M_{\ell L}^{\rm D} = \operatorname{diag}(m_\ell, m_L), \tag{23}$$

where m_ℓ and m_L are physical masses of the charged leptons. We will use for these matrices the following parametrization (K = L, R):

$$\tilde{U}_{\rm K} = \begin{pmatrix} \cos\tilde{\theta}_{\rm K}\, {\rm e}^{i\tilde{\phi}_{\rm K}} & \sin\tilde{\theta}_{\rm K}\, {\rm e}^{i(\tilde{\rho}_{\rm K}+\tilde{\xi}_{\rm K})/2} \\ -\sin\tilde{\theta}_{\rm K}\, {\rm e}^{i(\tilde{\rho}_{\rm K}-\tilde{\xi}_{\rm K})/2} & \cos\tilde{\theta}_{\rm K}\, {\rm e}^{i(\tilde{\rho}_{\rm K}-\tilde{\phi}_{\rm K})} \end{pmatrix}. \tag{24}$$

(We denote by tilde all parameters associated with the mixing of charged leptons.) The mass eigenstates ℓ' and L' are given by

$$\begin{pmatrix} \ell' \\ L' \end{pmatrix} = \tilde{U}_{\rm L}^{+} \begin{pmatrix} \ell \\ L \end{pmatrix}_{\rm L} + \tilde{U}_{\rm R}^{+} \begin{pmatrix} \ell \\ L \end{pmatrix}_{\rm R}, \tag{25}$$

where the primes will be dropped in what follows.

It is clear that the electromagnetic couplings of the charged leptons remain intact under the transformation (25). The weak isovector part J_3^μ of the neutral current, on the other hand, is different for the two fields and will therefore be affected by the mixing. The diagonal interactions in the eigenstate basis are real automatically; the non-diagonal ℓL couplings may in general contain complex phase factors. In order to find a minimal set of phases let us make a further redefinition of the charged fields, on top of the one made in (16):

$$\begin{aligned} \ell &\to {\rm e}^{i\tilde{a}}\ell, \\ L &\to {\rm e}^{i\tilde{b}}L. \end{aligned} \tag{26}$$

If we choose

$$\tilde{a} - \tilde{b} = \tilde{\phi}_{\rm L} - \tfrac{1}{2}(\tilde{\rho}_{\rm L} + \tilde{\xi}_{\rm L}), \tag{27}$$

the isovector current J_3^μ contains just one phase, $\tilde{\varepsilon}$, which appears in the RH part of the non-diagonal interaction:

$$\begin{aligned} J_3^\mu = {} & \tfrac{1}{2}\tilde{c}_{\rm L}^2 \bar{\ell}_{\rm L}\gamma^\mu \ell_{\rm L} + \tfrac{1}{2}\tilde{s}_{\rm R}^2 \bar{\ell}_{\rm R}\gamma^\mu \ell_{\rm R} \\ & + \tfrac{1}{2}\tilde{c}_{\rm R}^2 \bar{L}_{\rm R}\gamma^\mu L_{\rm R} + \tfrac{1}{2}\tilde{s}_{\rm L}^2 \bar{L}_{\rm L}\gamma^\mu L_{\rm L} \\ & + \left[\tilde{s}_{\rm L}\tilde{c}_{\rm L}\bar{\ell}_{\rm L}\gamma^\mu L_{\rm L} - \tilde{s}_{\rm R}\tilde{c}_{\rm R}{\rm e}^{i\tilde{\varepsilon}}\bar{\ell}_{\rm R}\gamma^\mu L_{\rm R} + {\rm h.c.}\right], \end{aligned} \tag{28}$$

where $\tilde{c}_K = \cos\tilde{\theta}_K$, $\tilde{s}_K = \sin\tilde{\theta}_K$ (K = L, R). The phase $\tilde{\varepsilon}$ is given by

$$\tilde{\varepsilon} = \tilde{\phi}_L - \tilde{\phi}_R + \tfrac{1}{2}(\tilde{\rho}_R - \tilde{\rho}_L + \tilde{\xi}_R - \tilde{\xi}_L). \quad (29)$$

CP is now conserved only for the trivial values $\tilde{\varepsilon} = 0$ or π. It is evident from eq. (29) that $\tilde{\varepsilon}$ vanishes, for example, if the mass matrix $M_{\ell L}$ is hermitian, because then one can choose $\tilde{U}_R = \tilde{U}_L$.

Let us now consider the charged currents of leptons in the presence of an ℓL mixing. It is straightforward but somewhat tedious to show that if one fixes the phases of the charged leptons by keeping the choice of eq. (27) and defining $\tilde{a}$ to be

$$\tilde{a} = \tilde{\phi}_L + \tfrac{1}{2}(\gamma_1 + \theta_2), \quad (30)$$

so as to annihilate the phase of the $\ell_L \chi_{1L}$ coupling, the charged current attains the following form (c = cos θ, s = sin θ):

$$\begin{aligned} J^{\mu}_{cc} = {} & c\tilde{c}_L \bar{\ell}_L \gamma^{\mu} \chi_{1L} - s\tilde{s}_L e^{i\delta} \bar{L}_L \gamma^{\mu} \chi_{2L} \\ & - s\tilde{c}_L e^{i\delta} \bar{\ell}_L \gamma^{\mu} \chi_{2L} + c\tilde{s}_L \bar{L}_L \gamma^{\mu} \chi_{1L} \\ & - s\tilde{s}_R e^{i\chi} \bar{\ell}_R \gamma^{\mu} \chi_{1R} + c\tilde{c}_R e^{i(\chi - \delta - \tilde{\varepsilon})} \bar{L}_R \gamma^{\mu} \chi_{2R} \\ & - c\tilde{s}_R e^{i(\chi - \delta)} \bar{\ell}_R \gamma^{\mu} \chi_{2R} + s\tilde{c}_R e^{i(\chi - \tilde{\varepsilon})} \bar{L}_R \gamma^{\mu} \chi_{1R}, \end{aligned} \quad (31)$$

where the phases δ and $\tilde{\varepsilon}$ are defined in eqs. (18) and (29), respectively and the new phase χ is given by

$$\chi = \alpha + \theta_1 + \tfrac{1}{2}(\gamma_1 + \gamma_2) + \tilde{\phi}_L - \tfrac{1}{2}(\tilde{\rho}_R - \tilde{\xi}_R). \quad (32)$$

We have hence found that out of the altogether seven original phases of the neutrino and charged lepton mass matrices M and $M_{\ell L}$ only three genuine *CP*-violating phases can ensue. *CP* is conserved in the full theory for the values $\beta = 0$, $\tilde{\varepsilon} = 0, \pi$, $\chi = 0, \pi$. Let us notice that our previous form for J^{μ}_{cc}, given in eq. (17), can be obtained from eq. (31) by choosing $\tilde{\theta}_L = \tilde{\theta}_R = 0$ and redefining $L \to \exp i(\chi - \delta - \tilde{\varepsilon})L$. On the other hand, if the ℓL mixing is nonvanishing (i.e., $\tilde{\theta}_L \neq 0$ or $\tilde{\theta}_R \neq 0$) but does not involve any complex parameters (i.e., all phases with tilde vanish), we are left with two *CP*-phases, δ and χ.

3. Dirac neutrinos

We will now give a brief account of the case where neutrinos ν and N are Dirac particles. The general Dirac mass matrix for neutrinos assumes a form completely analogous to the one given in eq. (22) for the charged leptons. The diagonalization procedure (eqs. (23)–(25)) and the phase redefinitions (eqs. (26)–(27)) are directly applicable for the νN mixing, with obvious changes. (We use for the νN mixing the same parameters as for the ℓL mixing except for the tilde). The general structure of the neutral current of Dirac neutrinos is then given by an expression similar to eq. (28), thus containing one *CP*-violating phase, ε (corresponding to the phase $\tilde{\varepsilon}$ of the charged lepton sector). It has hence turned out that in both the Majorana and Dirac case the neutral currents of neutrinos contain one *CP*-violating phase. As far as the Lorentz structure of the currents is concerned, however, these two cases are strikingly different, as can be seen by comparing eqs. (19) and (28).

Also the charged current part can be elaborated following the same procedure as used in the Majorana neutrino case. Fixing the phase of the $\bar{\ell}\nu_L$ term to be zero, together with eq. (27) and its analogue for neutrinos, completely determines relative phases of the charged leptonic currents. It turns out that, contrary to the Majorana case (cf. eq. (31)), all LH currents have then real couplings and *CP*-violation only takes place in the RH sector:

$$\begin{aligned} J^{\mu}_{cc} &= c_L\tilde{c}_L\bar{\ell}_L\gamma^{\mu}\nu_L + s_L\tilde{s}_L\bar{L}_L\gamma^{\mu}N_L \\ &+ s_L\tilde{c}_L\bar{\ell}_L\gamma^{\mu}N_L + c_L\tilde{s}_L\bar{L}_L\gamma^{\mu}\nu_L \\ &+ s_R\tilde{s}_R e^{i\phi}\bar{\ell}_R\gamma^{\mu}\nu_R + c_R\tilde{c}_R e^{i(\phi+\varepsilon-\tilde{\varepsilon})}\bar{L}_R\gamma^{\mu}N_R \\ &- c_R\tilde{s}_R e^{i(\phi+\varepsilon)}\bar{\ell}_R\gamma^{\mu}N_R - s_R\tilde{c}_R e^{i(\phi-\tilde{\varepsilon})}\bar{L}_R\gamma^{\mu}\nu_R\,. \end{aligned} \tag{33}$$

In addition to those two angles, ε and $\tilde{\varepsilon}$, which also appear in the neutral currents, the charged current thus contains only one new phase, ϕ, given by

$$\phi = \tilde{\phi}_L - \phi_L + \tfrac{1}{2}(\rho_R - \tilde{\rho}_R - \xi_R + \tilde{\xi}_R)\,. \tag{34}$$

Accordingly, the maximal number of *CP*-violating phases is the same, three, independent of whether neutrinos are Majorana or Dirac particles. Let us remind ourselves for comparison that in the standard flavour mixing only one *CP*-phase can be effective if neutrinos are Majorana particles, and for Dirac neutrinos the theory is completely *CP*-conserving.

4. Summary

Our analysis shows that the leptonic *CP*-violation can have remarkably different forms depending on whether one considers mixing between standard left-handed lepton flavours or between standard leptons and right-handed mirror leptons. One can trace the dissimilarities back to the fact that in the mirror mixing case the leptonic GIM mechanism is not in force due to the different transformation properties of leptons and mirror leptons under the $SU(2) \times U(1)$ gauge group. Summarizing, when one standard and one mirror lepton generation mix, the number of physical *CP*-violating phases in the charged current is in general three (cf. eqs. (31) and (33)) and in the neutral current one (eqs. (19) and (28)), irrespective of whether the two neutrinos are Majorana or Dirac particles. This is to be compared with one phase for Majorana neutrinos (present only in the charged current) and no phase for Dirac neutrinos in the standard flavour mixing. On the other hand, the place of appearance of the *CP*-phases as well as the Lorentz structure of the weak currents in the mirror mixing case are different for Majorana versus Dirac neutrinos. Although our analysis is restricted to the case of one mirror and one standard lepton generation, analogous results would follow if further generations are included.

The main part of this work was done while one of us (K.M.) stayed at SIN (Villigen); he wishes to thank the SIN Theory Group for a pleasant working atmosphere. One of us (J.M.) has benefitted from a previous collaboration with A. Barroso (Lisboa).

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