GEOPHYSICAL RESEARCH LETTERS, VOL. 25, NO. 11, PAGES 1837-1840, JUNE 1, 1998

A new method to determine the solar cycle length

K. Mursula

Department of Physical Sciences, University of Oulu, FIN-90570 Oulu, Finland

Th. Ulich

Sodankylä Geophysical Observatory, FIN-99600 Sodankylä, Finland

Abstract. The length of the solar cycle and its long-term variation have recently received additional significance due to their suggested connection to global climate. The cycle length is conventionally defined as the time difference between two successive sunspot minima. However, the sunspot minimum times sensitively depend on the way the sunspot numbers are averaged, i.e. whether one uses daily or monthly averaged values and whether and how the data are further smoothed. Using differently processed sunspot data, the sunspot minimum times vary typically by a few months, leading to a corresponding inaccuracy in solar cycle length. Here we propose a new method to define the solar cycle length as a difference between the median activity times of two successive sunspot cycles. The great advantage of this method is that the median times are almost independent of how the sunspot minima are determined. Therefore the method allows the solar cycle lengths to be calculated with a very small inaccuracy of a few days only. We show that the individual cycle lengths calculated from the conventional and the median method may differ by nearly a year. However, the long-term trend of cycle lengths remains roughly the same during modern times.

Introduction

Periodicities in solar activity at different time scales have been of great importance when trying to understand the long-term evolution of the Sun [see e.g. *Zirin,* 1988; *Kontor,* 1993; *Rozelot,* 1994]. It has been proposed that solar activity demonstrates a low-dimensional nonlinear (chaotic) behaviour [see e.g. *Kurths and Ruzmaikin,* 1990; *Ostryakov and Usoskin,* 1990; *Mundt et al.,* 1991; *Kremliovsky,* 1994; *Rozelot,* 1995]. However, this view was critized by *Price et al.* [1992] who emphasized that filtering of data affects such analyses, giving spurious evidence in favour of nonlinear behaviour. Thus, the question whether solar activity is a deterministic or stochastic process is still open, and simple proxies such as the length of the approximately 11-year solar cycle remain as important measures of long-term solar activity. The solar cycle length (SCL) has received additional significance from recent results relating the SCL variation to the long-term evolution of global climate [see e.g. *Friis-Christensen and Lassen*, 1991; *Lassen and Friis-Christensen*, 1995]. Therefore, it is important to define the varying cycle length as accurately as possible.

Sunspot numbers form the longest and most uniform index of solar activity, and are used to define the solar cycle and its minima and maxima. The conventional SCL definition is to calculate the time difference between two successive sunspot minima, or, occasionally, maxima. We will call these lengths the min-min and max-max lengths. These SCL's crucially rely on an accurate timing of these extremal values. However, since solar activity is strongly fluctuating, the times of minima and maxima depend sensitively on the way the sunspot numbers are processed, i.e. whether one uses daily or monthly averaged sunspot data and whether and how the data are further smoothed.

We will show here that conventional SCL estimates using differently processed sunspot data differ typically by 4-5 months. As an alternative to the conventional method, we will propose a new median-based SCL definition which is almost insensitive to an exact timing of extremes. We will calculate the median cycle lengths and show that they can be determined within an accuracy of about 3 days only. Although the individual cycle lengths calculated according to the conventional and the median method can differ by nearly a year, the long-term SCL variation remains roughly the same during modern times, sustaining the interesting similarity suggested between the SCL's and global temperature.

Difficulties in the conventional cycle length definition

The problem with the conventional SCL definition is illustrated in Figure 1 which depicts two sunspot minima: the minimum between solar cycles (SC) 12 and 13, and the latest minimum between SC 22 and 23. The strongly varying 27-day running mean of daily sunspot numbers and the 13-month running mean [*Gleissberg,* 1944; *Howard,* 1977] of monthly averaged sunspot numbers are plotted there for a couple of years around the respective minima. The 13-month curve has been used e.g. when the "official" times and values of sunspot minima and maxima have been determined [see e.g. *Brunner,* 1939; *NOAA*, 1996].

In the case of the earlier minimum, the 27-day curve shows a minimum late in 1889. The 13-month running mean curve has two broad, almost equally low local minima, one at the turn of 1888/1889, the other, somewhat lower minimum in early 1890. The official minimum has been chosen to be in 1889.6, in the middle of these two minima of the 13-month curve, at the time of fairly strong sunspot activity. In the case of the latest cycle minimum, the 13-month curve had its minimum in May 1996. However, a long interval with no sunspots at all occurred later in the year, and the 27-day minimum took place in October 1996. These examples demonstrate that the question of the time of the "correct" minimum is very dependent on the way the data is treated,

Paper number 98GL51317.
0094-8534/98/98GL-51317$05.00

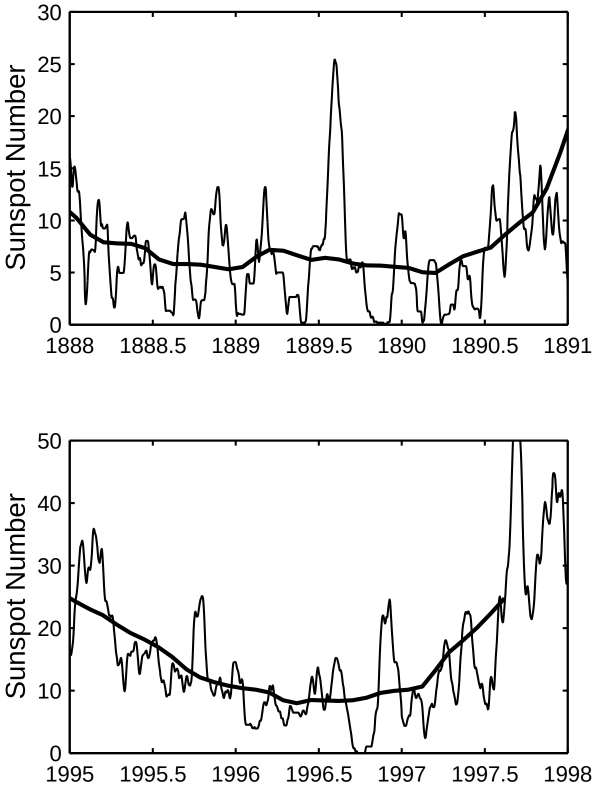

Figure 1. 27-day (thin line) and 13-month (thick line) running means of sunspot numbers around the sunspot minima of SC 12/13 (top) and SC 22/23 (bottom).

and that the choice of the official minimum within the conventional method remains fairly arbitrary. In the case of the two minima of Figure 1 the time difference between the minimum times determined from different data sets is five to six months.

The problem illustrated in Figure 1 applies to all solar minima and thus affects all min-min SCL estimates. We have calculated the SCL's using two alternative sets of minima and depicted the difference between these lengths and the official cycle lengths in Figure 2. We will restrict ourselves here to the recent 150 years where the sunspot numbers are reliable [*Eddy,* 1976] and where the solar cycles based on sunspot numbers and sunspot groups mainly agree [*Hoyt et al.,* 1994]. The official times of sunspot minima and maxima for these cycles are given in Table 1. One set of alternative cycle lengths was calculated using the minima formed by the lowest monthly sunspot numbers. (Very similar values were found when using 27-day running mean values). The other set corresponds to minima of the 13-month running mean curve. The numbers at the top of Figure 2 give the difference between the longest and the shortest of the three SCL estimates. The average value of these numbers is 143 days, implying that the typical size of the arbitrariness in min-min SCL determination is 4-5 months. The largest value of almost 11 months is found for the length of SC 12. We also note that the similar arbitrariness in max-max SCL's is even larger than in min-min lengths.

The median method

The new SCL definition is based on the idea of determining the median time T_M, by which the integrated sunspot

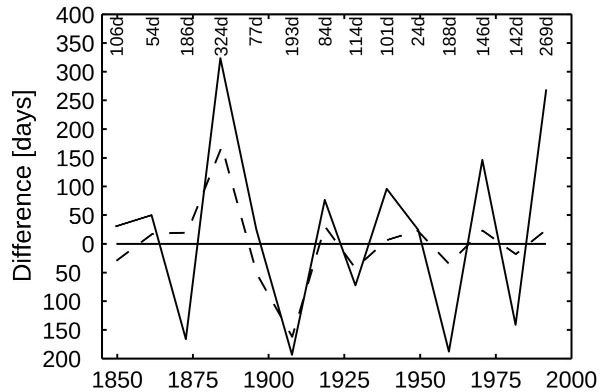

Figure 2. Difference between two alternative and the official cycle length estimates. All cycle lengths are determined in the conventional way by first locating the respective cycle minima. Solid line is determined from the minima of monthly averages, dashed line from the minima of the 13-month running mean curve.

number has, after a minimum, attained half of the value of the full cycle. Thus, the median time is defined by the equation

$$\sum_{t=m_i}^{T_M} R(t) = \sum_{t=T_M}^{m_{i+1}} R(t) \qquad (1)$$

where $R(t)$ is the daily sunspot number at time t, and m_i and m_{i+1} are times of two successive sunspot minima. Accordingly, the median time defines a kind of effective mass center of the cycle, dividing the cycle in two halves whose integrated sunspot numbers are equal. From the difference between two successive median times we can then obtain a new definition for the solar cycle length, the so called median cycle length, as the time difference between these cycle centers. Note that the median method does not allow the SCL to be defined for each cycle separately (if the latter are, as usual, defined from minimum to minimum) but

Table 1. Sunspot Cycle Extrema and Lengths

#	Min. [year]	L [a]	Max. [year]	L [a]	Med. [year]	L [a]
8	1833.9	9.6	1837.2	10.9	—	—
9	1843.5	12.5	1848.1	12.0	—	—
10	1856.0	11.2	1860.1	10.5	1861.019	10.393
11	1867.2	11.7	1870.6	13.3	1871.412	12.244
12	1878.9	10.7	1883.9	10.2	1883.656	10.713
13	1889.6	12.1	1894.1	12.9	1894.369	12.583
14	1901.7	11.9	1907.0	10.6	1906.952	11.084
15	1913.6	10.0	1917.6	10.8	1918.036	10.096
16	1923.6	10.2	1928.4	9.0	1928.132	10.413
17	1933.8	10.4	1937.4	10.1	1938.545	10.109
18	1944.2	10.1	1947.5	10.4	1948.653	9.918
19	1954.3	10.6	1957.9	11.0	1958.572	11.250
20	1964.9	11.6	1968.9	11.0	1969.822	11.054
21	1976.5	10.3	1979.9	9.7	1980.875	9.790
22	1986.8	9.6	1989.6	—	1990.665	—
23	1996.4	—	—	—	—	—

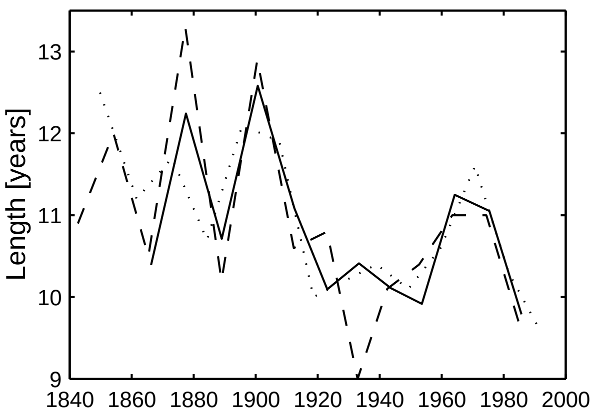

Figure 3. The solar cycle lengths of the median method (solid line), compared with those obtained from the official sunspot minima (dotted line; min-min curve) and maxima (dashed line; max-max curve).

rather gives a relative measure. In this respect the median cycle lengths resemble the max-max lengths of the conventional method. However, the median cycle lengths prove to be significantly more accurate than those obtained in the conventional method, justifying the new definition.

We have plotted three sets of SCL's in Figure 3. One curve (min-min) is obtained from the official sunspot minima in the conventional way, the other curve (max-max) from the official maxima. We have taken the latest, still unofficial minimum to be in 1996.4, corresponding to the minimum of the 13-month curve. Note that this is somewhat earlier than predicted by *Wilson et al.* [1996] based on the bimodal SCL distribution [*Wilson*, 1987], and the appearance of the first sunspots of a new cycle. The third curve shows the cycle lengths calculated according to the median method. The value of each SCL was positioned at the center of the respective time interval. The three SCL sets and the corresponding minimum, maximum and median times are given in Table 1.

It is interesting to note in Figure 3 that the median curve falls mostly, but not always, between the min-min and max-max curves. This demonstrates that the median method essentially verifies the long-term SCL variation attained by the conventional method. Overall, the median curve fits somewhat better with the min-min curve. However, despite this good agreement in the the long-term trend, the difference between the median length curve and the two conventional length curves can sometimes be quite sizable. There are two periods (cycles 11 and 16; see also Table 1) during the time depicted in Figure 3 where the difference between max-max curve and the median curve is more than one year, the largest difference during cycle 16 being more than one year and four months. Comparison with the min-min curve is more difficult because of the larger phase shift between the dates for which the cycle lengths were calculated. However, the difference between the curves (linearly interpolated points) attains a maximum of about one year in cycle 11. This is the cycle for which the conventional method gives the largest difference between the min-min and max-max values. We also note that the median curve has a considerably larger fluctuation than the min-min curve in late 19th century, but more recently the two curves follow fairly closely to each other.

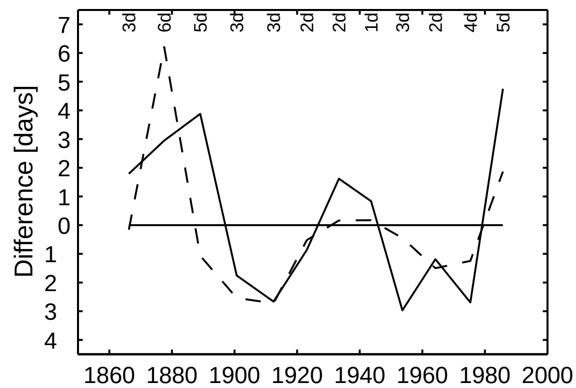

Figure 4. Variation of cycle length estimates of the median method using different minima as end points. Solid (dashed) line shows the difference of lengths obtained from monthly average (13-month running mean) minima and those from official minima.

Discussion

The essential benefit of the median method is that the median times are practically independent of the arbitrariness related to determining the times of the sunspot minima. This is due to the fact that the low sunspot numbers around minima contribute much less to the median sum (see Eq. 1) than a corresponding time interval around sunspot maxima. We have demonstrated this property in Figure 4 where we compare the SCL's obtained by the median method using different sunspot minima. We have calculated three sets of median cycle lengths, the first one using the official sunspot minima (shown in Table 1), the other using two alternative sets of minima. One alternative set corresponds to the minima of the monthly sunspot numbers, the other to the minima of the 13-month running mean curve. We have subtracted the cycle lengths of the first set (corresponding to official sunspot minima) from the latter two alternatives and plotted the differences in Figure 4.

Figure 4 shows that the median cycle lengths determined using the three sets of sunspot minima differ from each other only by a few days. The numbers at the top of Figure 4 give the difference between the longest and shortest of the three cycle length estimates, in analogy with the numbers given in

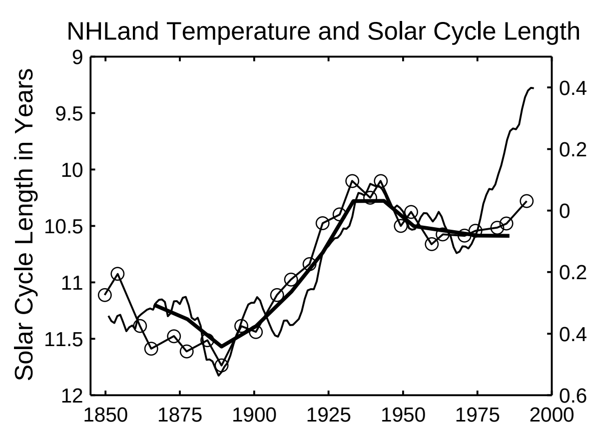

Figure 5. The median (thick line) and official (thin line with circles) cycle lengths (left vertical axis) together with the northern hemisphere land surface temperature (thin line with no marks; right vertical axis).

Figure 2. The average value of this difference is about three days, and even the largest difference is less than a week. This shows that the SCL estimates of the median method are almost insensitive as to how the sunspot minima are determined. This result expresses the great advantage of the median method as a SCL definition when compared to the conventional method where a typical inaccuracy in SCL is a few months rather than a few days (see Figure 2).

Some years ago the long-term evolution of SCL's (calculated in the conventional method) was shown to be in a good agreement with the northern hemisphere land (NHL) temperature [*Friis-Christensen and Lassen,* 1991], suggesting that climate change is strongly influenced by long-term solar variations. *Friis-Christensen and Lassen* [1991] also noted that the SCL gives a better agreement with temperature than e.g. the cycle amplitude since the amplitude curve (but not the SCL curve) is lagging with respect to the NHL temperature curve, leading to a causality problem. Taking into account the above demonstrated arbitrariness in conventional SCL estimates (see Figure 2), it is interesting to study the connection proposed by *Friis-Christensen and Lassen* [1991] using the new median SCL's.

We have plotted in Figure 5 the NHL temperature together with the conventional SCL's and the median SCL's. Both SCL curves were obtained by averaging the original series with a 5-point (1-2-2-2-1) filter as explained in *Lassen and Friis-Christensen* [1995]. (The end points were calculated here using a partial filter). As already shown in Figure 3, the long-term trends of the conventional and median SCL's are quite similar. Accordingly, we find that the results based on the median method support the correlation proposed by Friis-Christensen and Lassen, leaving the long-term evolution of the SCL curve and, in particular, its phase almost unchanged when compared to official cycle lengths. A similar agreement was also found by *Ochadlick et al.* [1993], using wavelet analysis of solar activity. However, wavelet analysis can not give the cycle lengths within an accuracy comparable to the median method.

Conclusions

We have demonstrated that the arbitrariness in sunspot minimum and maximum times is a few months, leading to a corresponding inaccuracy in the conventional estimate of the solar cycle length. As an alternative, we propose here a new, median based definition for the solar cycle length whereby the inaccuracy in cycle length can be reduced to a few days only. This is by a factor of 30-50 smaller than in the case of the conventional method.

We find that while the long-term trend of cycle lengths of the new median method mainly follows the official lengths, the difference in certain cycles can be as large as one year. The median cycle lengths fluctuate more than the official (min-min) cycle lengths in the 19th century, but during the recent cycles the two estimates agree well. The median cycle lengths verify the correlation suggested by *Friis-Christensen and Lassen* [1991] between the solar cycle length and the northern hemisphere land surface temperature.

A similar median based cycle length can be calculated for other solar parameters such as e.g. the 10.7 cm radio flux or solar irradiance. For these parameters, the median method offers a more physical interpretation than the conventional definition, since it takes into account the total effect of the various parts of the cycle with their corresponding weight.

Although this study was restricted to cycles for which a complete series of daily sunspot numbers exists, the median method can be applied to the more sporadic data of earlier solar cycles, even more reliably than the conventional method. A related study is under way.

References

Brunner, W., Tables on sunspot-frequency for 1749-1938, J. Geophys. Res., 44, 247-256, 1939.

Eddy, J. A., The Maunder minimum, Science, 192, 1189-1202, 1976.

Friis-Christensen, E. and K. Lassen, Length of the solar cycle: an indicator of solar activity closely associated with climate, Science, 254, 698-700, 1991.

Gleissberg, W., A Table of Secular Variations of the Solar Cycle, J. Geophys. Res., 49, 243-244, 1944.

Howard, R., Solar cycle, solar rotation and large-sale circulation, in Illustrated Glossary for Solar-Terrestrial Physics, ed. A. Bruzek and C. J. Durrant, D. Reidel Publ. Co. Boston, Mass., p. 7-12, 1977.

Hoyt, D. V., K. H. Schatten, and E. Nesme-Ribes, The one hundredth year of Rudolf Wolf's death: Do we have the correct reconstruction of solar activity?, Geophys. Res. Lett., 21, 2067-2070, 1994.

Kontor, N. N., Long term variability of solar activity, Adv. Space Res., 13, (9)417-(9)427, 1993.

Kremliovsky, M. N., Can we understand time scales of solar activity?, Solar Phys., 151, 351-370, 1994.

Kurths, J., and A. A. Ruzmaikin, On forecasting the sunspot numbers, Solar Phys., 126, 407-410, 1990.

Lassen, K. and E. Friis-Christensen, Variability of the solar cycle length during the past five centuries and the apparent association with terrestrial climate, J. Atmos. Terr. Phys., 57, 835-845, 1995.

Mundt, M. D., W. B. Maguire II, and R. R. P. Chase, Chaos in the sunspot cycle: Analysis and prediction, J. Geophys. Res., 96, 1705-1716, 1991.

NOAA, Smoothed monthly mean sunspot numbers (1749–1996), available on magnetic tape, CD-ROM, via anonymous FTP (ftp.ngdc.noaa.gov) or WWW (www.ngdc.noaa.gov) from the National Oceanic and Atmospheric Administration (NOAA), Boulder, Colorado, USA, 1996.

Ochadlick, A. R. Jr., H. N. Kritikos, and R. Giegengack, Variations in the period of the sunspot cycle, Geophys. Res. Lett., 20, 1471-1474, 1993.

Ostryakov, V. M., and I. G. Usoskin, On the dimension of solar attractor, Solar Phys., 127, 405-412, 1990.

Price, C. P., D. Prichard, and E. A. Hogenson, Do the sunspot numbers form a chaotic set?, J. Geophys. Res., 97, 19113-19120, 1992.

Rozelot, J. P., On the chaotic behaviour of the solar activity, Astron. Astrophys., 297, L45-L48, 1995.

Rozelot, J.P., On the stability of the 11-year solar cycle period (and a few others), Solar Phys., 149, 149-154, 1994.

Wilson R. M., On the distribution of sunspot cycle periods, J. Geophys. Res, 10101-10104, 1987.

Wilson R. M., D. H. Hathaway, and E. J. Reichmann, On the behaviour of the sunspot cycle near minimum, J. Geophys. Res., 101, 19,967-19,972, 1996.

Zirin, H., Astrophysics of the Sun, Cambridge University Press, Cambridge, U.K., 1988.

Kalevi Mursula, Department of Physical Sciences, University of Oulu, FIN-90571 Oulu, Finland; e-mail: kalevi.mursula@oulu.fi

Thomas Ulich, Sodankylä Geophysical Observatory, FIN-99600 Sodankylä, Finland; email: thomas.ulich@sgo.fi

(Received December 21, 1997; revised March 4, 1998; accepted March 18, 1998.)