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THE LORENTZ STRUCTURE OF LEPTONIC CHARGED WEAK INTERACTIONS

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We analyze all available data on leptonic, charge changing, weak interactions in terms of more general Lorentz structures than V − A. For this purpose we construct classes of models which follow the pattern of unified gauge theories but which are general enough so as to make use of the nearly complete experimental information. In a first class of models we assume the interactions to be mediated by the exchange of charged bosons whose couplings to leptons fulfill either strict universality or modified universality (for spin-zero exchanges). In a second class of models we investigate to which extent the data admit or require unconventional couplings in addition to V − A. A χ^2 analysis is presented for nine typical cases and limits on non-canonical couplings are given. As expected the data admit relatively large deviations from V − A. A few per mill measurement of the parameter ξ in μ decay as well as an absolute measurement of the muon neutrino helicity from pion decay would help to remedy this unsatisfactory situation.

1. Introduction

The Lorentz structure of purely leptonic charged weak interactions is not well known. As was pointed out repeatedly the data still admit relatively large deviations from the effective V − A coupling [1–3]. Neither can the data exclude sizeable contributions from effective V + A couplings nor even from couplings other than vector and axial vector. The precise form of the leptonic weak interactions plays a key role in testing gauge models which generalize the standard Glashow–Salam–Weinberg (GSW) model. More generally, the question about the nature of the leptonic weak couplings is important in the context of the possibility of non-vanishing neutrino masses and of the possible existence of processes that change lepton family numbers. Indeed, these problems which are intimately related, are the subject of renewed and intense theoretical and experimental efforts at present.

In a recent paper Maalampi et al. [3] gave a statistical analysis of muon decay, inverse muon decay, $\pi\ell 2$ and $K\ell 2$ decay, and polarization data in nuclear Gamow–

Teller transitions, within the framework of two specific unified gauge models: The left–right symmetric model based on $SU(2)_L \times SU(2)_R \times U(1)$, and the fermion–mirror fermion mixing model [3, 4]. Like most other models which have been proposed as extensions of the GSW theory, these models predict weak couplings of vector and axial vector nature and with real coupling constants. Models of this kind usually are very restrictive in having lepton universality, *CP* conservation, vector-axial vector couplings and (close to) maximal parity violation built in from the start. As a consequence, they do not make use of all of the available data on leptonic processes. For example, the parameter η and the transverse components of the electron polarization in μ decay which are known to be sensitive to small and specific deviations from $V-A$ [2], and for which precision data exist, are trivially zero in most of them.

In this work we analyze all available data on weak leptonic charge changing vertices in the spirit of unified gauge theories but without committing ourselves to any specific or too restrictive model. In a first step we assume that the charged weak interactions are mediated by the exchange of heavy charged bosons whose couplings to the external leptons need not be relatively real or universal. This leads, rather naturally, to a factorized, effective four-fermion interaction and allows us to compare the information from, say, $\pi \to e\nu_e/\pi \to \mu\nu_\mu$, μ decay and polarization data in nuclear Gamow–Teller transitions. In a second step, we relax the assumption of factorization and test for specific, partly complex, scalar, pseudoscalar, and/or tensor coupling terms which may be present in addition to the dominant, real $V-A$ coupling.

These kinds of analyses are general enough so as to make use of the nearly complete experimental information. At the same time they are guided by a well-defined physical picture of the weak leptonic processes. They are somewhat less general but more physical than the model independent parametrization in terms of the complete four-fermion interaction introduced by Kinoshita and Sirlin [5] and carried out in practice by Derenzo [1].

Our investigation of leptonic weak processes is motivated by the desire to test the basis on which current unified models of electroweak interactions are built. At the same time it is intended to provide a physical frame in which to analyze and understand the recent precision data on μ decay [6, 7] and the data that is expected from further experiments now in progress at the meson factories.

In this paper we consider exclusively interactions of the "charge changing" type i.e. interactions which contain the leptons of a given family f in the combination $(\bar{f}\nu_f + \text{h.c.})$. More complicated, mixed interactions containing both "charge changing" and "charge retention" vertices will be dealt with in a subsequent publication.

In sect. 2 we summarize the notation and formulae for the relevant leptonic reactions. In sect. 3 we discuss the models that we use in describing the leptonic vertices. Sect. 4 contains a description of the data, the fits and the results, whilst sect. 5 gives a summary and our conclusions.

2. Notation and reactions

The most precise body of data on purely leptonic charged weak interactions is provided by the decay

$$\mu^+ \to e^+ \nu_e \overline{\nu_\mu} . \tag{1}$$

Including the recent measurement of the complete polarization $\boldsymbol{P} = (P_L, P_{T1}, P_{T2})$ of the decay positron by the ETH-Mainz-SIN group [6, 7] we have at our disposal the total decay rate τ_μ plus nine real parameters (only eight are independent) which are obtained from the positron observables. These are: the spectrum parameters ρ, η, δ; the asymmetry ξ; the average longitudinal polarization $\langle P_L \rangle = \xi'$; the parameters α and β, as well as α' and β', which characterize the transverse components P_{T1} and P_{T2}, respectively. In ref. [2] these parameters are given in terms of the coupling constants C_i, C_i' of the general four-fermion interaction, written in charge retention form*. In this paper we discuss charge changing interactions which are all of the form

$$\mathcal{H} = \frac{G_0}{\sqrt{2}} \Sigma_i \{(\overline{e(x)} \Gamma_i \nu_e(x))[G_i(\overline{\nu_\mu(x)} \Gamma^i \mu(x)) + G_i'(\overline{\nu_\mu(x)} \Gamma^i \gamma_5 \mu(x))] + \text{h.c.}\} \tag{2}$$

where Γ_i denote the covariants S, P, V, A, and T and are defined as in eq. (3.9) of ref. [2]. The two sets of complex coupling constants $\{C_i, C_i'\}$ and $\{G_i, G_i'\}$ are equivalent and are related by Fierz transformations**.

Of the nine experimental parameters only eight are independent because of the linear relation

$$\eta = \frac{1}{A}(\alpha - 2\beta) , \tag{3}$$

where A is a dimensionless combination of coupling constants which appears in the expression for the decay rate:

$$\frac{1}{\tau_\mu} = \frac{m_\mu^5 G_0^2}{192\pi^3} \left\{1 + \frac{e^2}{8\pi^2}\left(\tfrac{25}{4} - \pi^2\right)\right\} \left\{1 + 4\eta \frac{m_e}{m_\mu} - 8 \frac{m_e^2}{m_\mu^2}\right\} \frac{A}{16} . \tag{4}$$

The next datum is the reaction

$$\nu_\mu + e^- \to \mu^- + \nu_e , \tag{5}$$

for which an integrated cross section has been measured [8]. In appendix A, eq. (A.1), we give the full expression for the invariant cross section $d\sigma/dt$, as derived from the general interaction eq. (2)***. We transform eq. (A.1) to the variable

* Note that eqs. (3.15) and (3.17) of ref. [2] are misprinted. Eq. (3.15) defines a' not α', whilst eq. (3.17) should read $c' = -2\text{Re}\,(C_T C_T'^*)$.

** For convenience, we define here the effective couplings such that they are dimensionless numbers.

*** A special case of this expression was given in ref. [9].

$y := E_\mu/E_\nu$ (where E_μ and E_ν are the muon and neutrino energies in the laboratory system, respectively). Following ref. [8], we integrate over y from $y_{\min} = 10.93\ \mathrm{GeV}/E_\nu$ up to 1, as well as over the neutrino spectrum. Finally, dividing by the corresponding, integrated cross section for the pure V−A case, we obtain

$$\begin{aligned} S = \frac{1}{2A}\{&(1+c_1-2c_2)[|G_S|^2+|G'_S|^2+|G_P|^2+|G'_P|^2 \\ &-2h\ \mathrm{Re}\,(G_S G'^*_S + G_P G'^*_P)] + 2(1+c_1+2c_2) \\ &\qquad\qquad \times[|G_T|^2+|G'_T|^2-2h\ \mathrm{Re}\,(G_T G'^*_T)] \\ &-2(1-c_1)[\mathrm{Re}\,(G_S G^*_T + G'_S G'^*_T - G_P G^*_T - G'_P G'^*_T) \\ &-h\ \mathrm{Re}\,(G_S G'^*_T + G'_S G^*_T - G_P G'^*_T - G'_P G^*_T)] \\ &+2(1+c_1)[|G_V|^2+|G'_V|^2+|G_A|^2+|G'_A|^2+2h\ \mathrm{Re}\,(G_V G'^*_V + G_A G'^*_A)] \\ &+4(1-c_1)[\mathrm{Re}\,(G_V G^*_A + G'_V G'^*_A) + h\ \mathrm{Re}\,(G_V G'^*_A + G_A G'^*_V)]\}\,, \end{aligned} \tag{6}$$

where $c_1 = 0.375$ and $c_2 = 0.50$, and h denotes the helicity of the incoming ν_μ, from pion decay $\pi^+ \to \mu^+\nu_\mu$. We verify that for the V−A interaction

$$G_V = G_A = 1\,, \qquad G'_V = G'_A = -1\,, \qquad G_S = G'_S = G_P = G'_P = G_T = G'_T = 0\,, \tag{7}$$

we have

$$h = -1\,, \qquad A = 16\,, \qquad S = 1\,. \tag{8}$$

As it stands, eq. (2) is no more than an effective four-lepton contact interaction. It does not exhibit lepton universality nor is it obvious how to generalize it to the quark sector such as to enable us to include semileptonic processes in our analysis. If we assume that weak charged interactions are mediated by heavy, charged bosons with spin-zero, one, and possibly, two, the effective four-fermion interaction, valid at low energies, assumes the form

$$\mathcal{H} = \frac{G_0}{\sqrt{2}}\left\{\sum_k K^{(k)\dagger}K^{(k)} + \sum_k J^{(k)\dagger}_\alpha J^{(k)\alpha} + \sum_k T^{(k)\dagger}_{\alpha\beta} T^{(k)\alpha\beta}\right\} \tag{9}$$

where

$$K^{(k)} := \sum_f \{g^{(f)}_{Sk}(\bar f \mathbb{1} \nu_f) + g^{(f)}_{Pk}(\bar f \gamma_5 \nu_f)\}\,, \tag{10a}$$

$$J^{(k)}_\alpha := \sum_f \{g^{(f)}_{Vk}(\bar f \gamma_\alpha \nu_f) + g^{(f)}_{Ak}(\bar f \gamma_\alpha\gamma_5 \nu_f)\}\,, \tag{10b}$$

$$T^{(k)}_{\alpha\beta} := \sum_f \{g^{(f)}_{Tk}(\bar f \sigma_{\alpha\beta} \nu_f) + g^{(f)}_{T'k}(\bar f \sigma_{\alpha\beta}\gamma_5 \nu_f)\}\,. \tag{10c}$$

The sum over k runs over the number of charged bosons of a given spin, the sum over f runs over the various lepton and quark families. At any vertex we allow for an arbitrary amount of parity violation. The constants $g_{ik}^{(f)}$ are the coupling constants to the exchanged boson of mass m_k, except for the factor $2^{1/4}/m_k\sqrt{G_0}$. The effective constants G_i, and G_i' of eq. (2) are then expressed as follows:

$$G_S=\sum_k g_{Sk}^{(e)}g_{Sk}^{(\mu)*}, \qquad G_P=\sum_k g_{Pk}^{(e)}g_{Pk}^{(\mu)*},$$

$$G_S'=-\sum_k g_{Sk}^{(e)}g_{Pk}^{(\mu)*}, \qquad G_P'=-\sum_k g_{Pk}^{(e)}g_{Sk}^{(\mu)*},$$

$$G_V=\sum_k g_{Vk}^{(e)}g_{Vk}^{(\mu)*}, \qquad G_A=\sum_k g_{Ak}^{(e)}g_{Ak}^{(\mu)*},$$

$$G_V'=\sum_k g_{Vk}^{(e)}g_{Ak}^{(\mu)*}, \qquad G_A'=\sum_k g_{Ak}^{(e)}g_{Vk}^{(\mu)*},$$

$$G_T=\sum_k \{g_{Tk}^{(e)}g_{Tk}^{(\mu)*}-g_{T'k}^{(e)}g_{T'k}^{(\mu)*}\},$$

$$G_T'=-\sum_k \{g_{Tk}^{(e)}g_{T'k}^{(\mu)*}-g_{T'k}^{(e)}g_{Tk}^{(\mu)*}\}. \tag{11}$$

So far we have not assumed lepton universality nor have we specified the number of bosons of a given type. For example, if there is only one charged boson with spin-zero and an arbitrary number of other bosons, the scalar and pseudoscalar constants of eq. (2) are not independent since in this case

$$G_S'G_P'=G_SG_P. \tag{12a}$$

Similarly, if there is only one charged vector boson, we have the relation

$$G_V'G_A'=G_VG_A. \tag{12b}$$

Clearly, the interaction (9), very much like the effective coupling (2), is too general for a meaningful analysis of the data. Furthermore, in trying to include the $\pi\ell 2$ decays into our study, we have to face the fact that each pseudoscalar and axial vector current introduce new and unknown hadronic form factors g_π and f_π, respectively:

$$\langle 0|K^{(k)}|\pi(q)\rangle=\frac{i}{(2\pi)^{3/2}}g_\pi^{(k)}, \tag{13a}$$

$$\langle 0|J_\alpha^{(k)}|\pi(q)\rangle=\frac{i}{(2\pi)^{3/2}}f_\pi^{(k)}q_\alpha. \tag{13b}$$

Therefore, we shall consider several classes of models which are comprehensive enough for an analysis of all available data and allow for a meaningful determination of the parameters, but which do not have the full complexity of eqs. (2) or (9). In models without tensor couplings we can also use the precise polarization data in

Gamow–Teller transitions because possible hadronic P terms vanish in the non-relativistic limit. The theoretical expression for this quantity is given in eq. (B.1) of appendix B. The data is quoted in table 1 below.

3. Models of leptonic interactions

3.1. FACTORIZED SINGLE BOSON MODELS WITH UNIVERSALITY (FSU)

These models are characterized by two assumptions.

(i) Only one term contributes to each product of covariants in eq. (9).

(ii) The coupling constants fulfill strict e-μ-universality: $g_i^{(e)} = g_i^{(\mu)}$, for all covariants S, P, V, A and T. In terms of the coupling constants in eq. (2) this implies that G_S, G_P, G_V, G_A are real and non negative, G_T is real and G_T' is imaginary

$$G_S' = G_P'^*, \qquad |G_S'| = \sqrt{G_S G_P},$$
$$G_V' = G_A'^*, \qquad |G_V'| = \sqrt{G_V G_A}. \tag{14a}$$

Thus, we may parametrize these latter constants by

$$G_S' = -\sqrt{G_S G_P} \cdot e^{i\alpha_S} = G_P'^*$$
$$G_V' = -\sqrt{G_V G_A} \cdot e^{i\alpha_V} = G_A'^*. \tag{14b}$$

TABLE 1

Experimental quantities used in our analysis

Quantity	Value	Ref.
ρ	0.7517 ± 0.0026	*
η	-0.12 ± 0.21	
$-\xi P_{\mu^+}$	0.975 ± 0.015	13
δ	0.7551 ± 0.0085	*
ξ'	1.008 ± 0.054	6
$\tau_\mu[\mu s]$	2.19714 ± 0.00007	*
S	0.98 ± 0.12	8
P_μ	0.99 ± 0.16	15
R_π	0.9953 ± 0.0097	14*
$-(c/v)P_\beta^{GT}$	1.001 ± 0.008	16
$R\bar{\nu}$	$0.000^{+0.035}_{-0}$	11

The first six refer to $\pi \to \mu \to e$ decay; S refers to inverse μ decay and is defined in eq. (6). P_μ and R_π denote the muon polarization and the reduced $\pi \to e\nu_e/\pi \to \mu\nu_\mu$ branching ratio (15), respectively. $-(c/v)P_\beta^{GT}$ is the electron polarization in nuclear Gamow–Teller decays, and $R_{\bar{\nu}}$ is the cross section ratio $\sigma_{SP}(\bar{\nu})/\sigma_{VA}(\bar{\nu})$ in reaction (22), as defined by eq. (25).

* Averaged values as given in the data tables [12]. References to the individual experiments are given there.

We now turn to the calculation of the branching ratio $B(\pi \to e\nu_e / \pi \to \mu\nu_\mu)$. Following ref. [3] we define a reduced ratio by dividing B by the theoretical value, valid for V and A couplings and strict $e-\mu$ universality, as well as by the radiative correction factor,

$$R_\pi := \frac{\Gamma(\pi \to e\nu_e)}{\Gamma(\pi \to \mu\nu_\mu)} \left\{ \frac{m_e^2}{m_\mu^2} \left(\frac{1-(m_e/m_\pi)^2}{1-(m_\mu/m_\pi)^2} \right)^2 (1+\Delta_{\mathrm{rad}}) \right\}^{-1} . \tag{15}$$

As to the ratio of the hadronic matrix elements, eqs. (13a) and (13b), we define the dimensionless quantity

$$x_\pi := \frac{g_\pi}{f_\pi m_\mu} , \tag{16}$$

g_π and f_π are assumed to be relatively real so that x_π is a real parameter. The reduced branching ratio (15) is then given by

$$R_\pi = \frac{F(x_\pi \cdot m_\mu / m_e)}{F(x_\pi)} \tag{17}$$

where the function F stands for

$$\begin{aligned} F(x) := {} & G_V + G_A + x^2(G_S + G_P) \\ & + 2x[\sqrt{G_V G_S} \cos \alpha_{VS} - \sqrt{G_A G_P} \cos(\alpha_V - \alpha_S - \alpha_{VS})] , \end{aligned} \tag{18}$$

and where α_{VS} is the relative phase between g_V and g_S. The helicity $\bar{h}$ of the antineutrino in the decay $\pi^- \to \mu^- + \bar{\nu}_\mu$. which equals the longitudinal polarization P_μ of the accompanying muon, is found to be

$$\begin{aligned} P_{\mu^-}(x_\pi) = \bar{h} = -h = 2\{ & \sqrt{G_V G_A} \cos \alpha_V - x_\pi^2 \sqrt{G_S G_P} \cos \alpha_S \\ & + x_\pi[\sqrt{G_S G_A} \cos(\alpha_V - \alpha_{VS}) - \sqrt{G_V G_P} \cos(\alpha_S + \alpha_{VS})]\} / F(x_\pi) . \end{aligned} \tag{19}$$

In the case of the pure V − A interaction (7) we have

$$\alpha_V = 0 \qquad R_\pi = 1 , \qquad P_{\mu^-} = \bar{h} = +1 .$$

Clearly, the same expressions (17), (18) and (19) hold also for $K\ell 2$ decays. However, unlike ref. [3], we do not use the measured value for R_K because in our models this quantity depends on still a new, unknown parameter x_K, the analogue of eq. (16).

3.2. FACTORIZED SINGLE BOSON MODELS WITH WEAK UNIVERSALITY (FSWU)

In this class of models we assume, as before, that

(i) only one term contributes to each product of covariants;

(ii) the vector, axial vector and tensor couplings are universal, as in class (3.1); however, we assume "weak universality" in the following sense:

(iii) the scalar and pseudoscalar couplings are proportional to the mass of the charged lepton partner, viz.

$$g_{\mathrm{S}}^{(\mu)}/g_{\mathrm{S}}^{(\mathrm{e})}=g_{\mathrm{P}}^{(\mu)}/g_{\mathrm{P}}^{(\mathrm{e})}=m_{\mu}/m_{\mathrm{e}}\,. \tag{20}$$

Clearly, this latter assumption is inspired by the example of Higgs particle exchange with Yukawa couplings proportional to the fermion masses. This class of models has the remarkable property that R_π (as well as R_{K}) is trivially equal to one, independently of the magnitude of scalar and pseudoscalar couplings*. The neutrino helicity, however, is not trivial in these models. Defining, instead of x_π, eq. (16) the dimensionless quantity

$$z_\pi := \frac{g_\pi}{f_\pi\sqrt{m_{\mathrm{e}}m_\mu}} = x_\pi\sqrt{\frac{m_\mu}{m_{\mathrm{e}}}}\,, \tag{21}$$

we now find the longitudinal polarization of the muon to be given by $P_{\mu^-}(z_\pi)$, where P_{μ^-} is the expression (19).

3.3. V−A MODELS WITH ADMIXTURES OF OTHER COVARIANTS

In this class of models we assume that the interaction is dominated by simple $\mathrm{W_L}$ exchange with *CP*-conserving couplings, i.e. $G_{\mathrm{V}}=G_{\mathrm{A}}=-G'_{\mathrm{V}}=-G'_{\mathrm{A}}$ real. We then explore how large an amount of the other possible couplings, S, P and/or T, is compatible with the data, without constraining these couplings by factorization and universality conditions eqs. (14a).

Obviously, in this class the vector and axial-vector couplings factorize and are universal. The scalar and pseudoscalar couplings G_{S}, G'_{S}, G_{P}, G'_{P} may be assumed to be of the form given in eq. (11), with at least two terms in the sum over spin-zero exchanges. If the elementary couplings $g_{\mathrm{S,P}k}^{(\mathrm{f})}$ are neither universal nor proportional to m_{f} the four complex constants G_{S} to G'_{P} are independent. If strict universality, or weak universality in the sense of eq. (20), is assumed, then $G'_{\mathrm{P}}=G'^{*}_{\mathrm{S}}$. In either case it is possible, in principle, to compute R_π and $\bar{h}$ from the effective interaction (9). However, unlike the preceding cases, this class introduces too many parameters for a statistical analysis of the available data to be meaningful. Therefore, in this class of models we drop the information from the measured ratio R_π and helicity $\bar{h}$, by assuming the pseudoscalar hadronic form factor g_π, eq. (13a), to vanish in which case

$$R_\pi=1\,,\qquad \bar{h}=1\,,\qquad (g_\pi=0)\,.$$

As to the tensor couplings, finally, G_{T} and G'_{T}, eq. (11), are unrestricted, complex parameters if the elementary couplings are not universal, even for a *single* spin-2 exchange. If universality is assumed, however, G_{T} is real and G'_{T} is pure imaginary.

* This property was found independently by Shrock as well as by Williams and Li [10] (in a more restrictive form than here), in the context of models with Higgs exchanges.

3.4. FURTHER DISCUSSION OF THE MODELS

Before we turn to our fits we wish to add some further remarks to the models presented in subsect. 3.3.

(i) We do not consider models which assume only S, P, and T couplings and no V and A couplings, as these are definitely excluded by experiment [11]. This is the conclusion of a measurement of the decay asymmetry of positive muons in the inclusive reaction

$$\bar{\nu}_\mu + \mathrm{Fe} \to \mathrm{X} + \mu^+ . \tag{22}$$

This asymmetry, in fact, determines the quantity $\gamma = \bar{h} \cdot \langle P_{\mu^+} \rangle Q^2 \cdot \xi$ where $\langle P_{\mu^+} \rangle Q^2$ is the polarization of the muon from reaction (22) averaged over the momentum transfer squared. The published result $\gamma = 0.82 \pm 0.07(\mathrm{stat.}) \pm 0.12(\mathrm{syst.})$ at $\langle Q^2 \rangle = 4\ \mathrm{GeV}^2$, together with the information on the antineutrino helicity $\bar{h}$ from pion decay and on the asymmetry parameter ξ, implies that the leptonic vertex must have a dominant V and/or A component. In particular, the interaction cannot be S and P (or T) alone because in this case γ should have come out negative.

We quote, as an example, our expression for the muon polarization, for the case of S, P, V and A couplings, in FS(W)U models

$$\begin{aligned} \langle P_{\mu^+} \rangle = \{ & \bar{h}[(G_\mathrm{V} + G_\mathrm{A}) - y(G_\mathrm{S} + G_\mathrm{P})] + 2\sqrt{G_\mathrm{V} G_\mathrm{A}} \cos \alpha_\mathrm{V} - 2y\sqrt{G_\mathrm{S} G_\mathrm{P}} \cos \alpha_\mathrm{S} \} / \\ & \{ (G_\mathrm{V} + G_\mathrm{A}) + y(G_\mathrm{S} + G_\mathrm{P}) + 2\bar{h}[\sqrt{G_\mathrm{V} G_\mathrm{A}} \cos \alpha_\mathrm{V} + y\sqrt{G_\mathrm{S} G_\mathrm{P}} \cos \alpha_\mathrm{S}] \} , \end{aligned} \tag{23}$$

(in the limit of $\bar{h}$ close to +1). In this formula we have set

$$y := \langle \sigma_0 \rangle / \langle \sigma_1 \rangle , \tag{24}$$

where $\langle \sigma_1 \rangle$ denotes the hadronic V, A structure functions $W^{\alpha\beta}_\mathrm{VA}$ contracted with the leptonic tensor

$$p^{(\nu)}_\alpha p^{(\mu)}_\beta - g_{\alpha\beta}(p^{(\nu)} p^{(\mu)}) + p^{(\nu)}_\beta p^{(\mu)}_\alpha ,$$

appropriately averaged over the momentum transfer, and where $\langle \sigma_0 \rangle$ denotes the analogous S, P contribution $(p^{(\nu)} p^{(\mu)}) W_\mathrm{SP}$. (The tensor contributions are calculated in an analogous manner and have the same helicity pattern as the SP terms. They introduce still another unknown ratio $\langle \sigma_2 \rangle / \langle \sigma_1 \rangle$ of hadronic cross sections).

The same experiment also determined the maximal contribution σ_SP of scalar-pseudoscalar couplings to the total cross section $\sigma_\mathrm{SP} + \sigma_\mathrm{VA}$ with T couplings assumed to be absent. In our factorized models we find

$$R_{\bar{\nu}} := \frac{\sigma_\mathrm{SP}}{\sigma_\mathrm{VA}} = \frac{G_\mathrm{S} + G_\mathrm{P} + 2\bar{h}\sqrt{G_\mathrm{S} G_\mathrm{P}} \cos \alpha_\mathrm{S}}{G_\mathrm{V} + G_\mathrm{A} + 2\bar{h}\sqrt{G_\mathrm{V} G_\mathrm{A}} \cos \alpha_\mathrm{V}}\, y , \tag{25}$$

with y as defined above. In our models which have factorization but no tensor couplings we include the existing information on the ratio (25), see subsect. 4.1 below.

(ii) Models with V and A couplings only (such as considered in ref. [3]), on the other hand, are not suited for our purposes, even if we allow the four constants G_V, G_A, G'_V and G'_A to be arbitrary, complex and unrelated. This is so because in such models we have

$$\alpha = 0 = \beta = \alpha' = \beta', \qquad \forall G_V, G_A, G'_V, G'_A,$$

so that

$$\eta = 0, \qquad P_{T1} = 0, \qquad P_{T2} = 0.$$

Therefore, such models exclude an essential part of the experimental information from the start.

4. Data and fits

4.1. EXPERIMENTAL DATA

The full set of data which can be used to constrain the various parameters are given in tables 1 and 2. No model studied, however, uses the full set of data, because in each model some quantities are trivially independent of the parameters. This is discussed further in subsect. 4.2.

In the determination of some quantities related to reactions (1) and (5) one usually makes the implicit assumption that the beam lepton, μ^+ in reaction (1) and ν_μ in reaction (5), is fully polarized with polarization $P_{\mu^+} = P_{\nu_\mu} = -1$. In the more general models we study, this is no longer true, as was noted in ref. [3]. Thus we must take into account that the measurements [12, 13] of the positron asymmetry in reaction (1) really determine $P_\mu \xi$ rather than ξ. As to the transverse polarization

TABLE 2

Preliminary data obtained from the measurement of the transverse polarization of the positron in polarized μ^+ decay [7]

Quantities	Values (preliminary)	
$\dfrac{\alpha}{A}$	-0.053 ± 0.077	0 (input)
$\dfrac{\alpha'}{A}$	0.001 ± 0.077	0 (input)
$\dfrac{\beta}{A}$	0.021 ± 0.028	-0.014 ± 0.018
$\dfrac{\beta'}{A}$	-0.016 ± 0.028	0.002 ± 0.018

The second column shows unconstrained results for the four real parameters, as obtained from the measured P_{T1} and P_{T2}. Column 3 gives the values of β/A and β'/A which are obtained if $\alpha = \alpha' = 0$ is assumed.

parameters of the ETH-Mainz-SIN group, the muon beam polarization has been corrected for and we can use the data directly.

The results for α/A, β/A, α'/A, β'/A as shown in table 2 are still preliminary. The final results from the analysis of the full data set will be published elsewhere.

The values of these parameters as extracted from the measured transverse components P_{T1} and P_{T2}, respectively, depend on possible constraints within the models under consideration. Column 2 of table 2 shows the unconstrained values, whilst column 3 shows the values as obtained when $\alpha = \alpha' = 0$ is assumed. The experimental quantity used from the CDHS-CHARM experiment [11] on reaction (22) is

$$\frac{R_{\bar{\nu}}}{1+R_{\bar{\nu}}} = \sigma_{SP}(\bar{\nu})/\sigma_{tot}(\bar{\nu}) < 0.07\,, \qquad (CL = 95\%)\,. \tag{26}$$

In table 1 we convert this to a 68% CL upper limit for the ratio $R_{\bar{\nu}} = \sigma_{SP}(\bar{\nu})/\sigma_{VA}(\bar{\nu})$.

4.2. MODELS AND FITS

In the FSU models (subsect. 3.1) the 10 complex coupling constants G_i, G'_i, $i =$ S, P, V, A, T, are reduced to 8 real parameters by the conditions (14a). We take these to be

$$G_S,\, G_P,\, G_V,\, G_A,\, G_T,\, \alpha_S,\, \alpha_V,\, \mathrm{Im}\, G'_T\,. \tag{27}$$

Since G_V and G_A are found to be very strongly, negatively correlated in all fits, we use the subsidiary empirical constraint

$$G_V + G_A = 2\,, \tag{28}$$

to reduce out one parameter. In addition the parameters x_π and α_{VS} appear in eqs. (16)–(19).

We shall now discuss the fits of various models in the order of their appearance in table 3. The parameter errors in this work correspond to an increase of χ^2 of 1.0.

Model FSU-1 contains only V, A, and T couplings. Thus it depends only on the parameters G_A, α_V, G_T, Im G'_T. The parameters x_π and α_{VS} do not enter, as can be seen from eqs. (18)–(19). The following four quantities have trivial values:

$$\beta = \beta' = 0\,, \qquad \delta = \tfrac{3}{4}\,, \qquad R_\pi = 1\,.$$

The last two quantities in table 1 cannot be used because of the presence of tensor interactions. Thus there are $15 - 6 = 9$ experimental quantities constraining the 4 parameters, or 5 degrees of freedom. We note from the values of the fitted parameters in table 3 that this solution does not differ significantly from a pure V−A solution, but that substantial deviations are allowed within one standard deviation.

TABLE 3

Fits using our single boson models as defined in subsects. 3.1 (models FSU-1, -2, -3) and 3.2 (models FSWU-1, -2)

Model	$\chi^2/\langle\chi^2\rangle$	G_S	G_P	α_V	G_A	G_T	Im G'_T	x_π or z_π	y	Constraints not used
FSU-1	2.63/5	—	—	$0.073^{+0.06}_{-0.18}$	$0.974^{+0.16}_{-0.10}$	$-0.03^{+0.17}_{-0.11}$	$-0.09^{+0.22}_{-0.05}$	—	—	$\delta\beta\beta' R_\pi P_\beta^{GT} R_{\bar\nu}$
FSU-2	3.13/4	$0.10^{+0.19}_{-0.10}$	$0.04^{+0.25}_{-0.04}$	$0^{+0.15}_{-0.15}$	$0.912^{+0.24}_{-0.06}$	$-0.08^{+0.22}_{-0.06}$	$0^{+0.13}_{-0.13}$	0	—	$\delta\beta' R_\pi P_\beta^{GT} R_{\bar\nu}$
FSU-3	3.10/8	$0.10^{+0.06}_{-0.10}$	$=G_S$	0.04 ± 0.08	$1.06^{+0.07}_{-0.19}$	—	—	0	—	$\delta\beta' R_\pi R_{\bar\nu}$
FSWU-1	2.17/4	$0.09^{+0.20}_{-0.09}$	$=G_S$	$0.015^{+0.01}_{-0.01}$	$1.01^{+0.13}_{-0.14}$	0.03 ± 0.11	0.04 ± 0.10	$0.26^{+\infty}_{-0.10}$	—	$\delta\beta' R_\pi P_\beta^{GT} R_{\bar\nu}$
FSWU-2	1.50/6	$0.10^{+0.06}_{-0.10}$	$=G_S$	$0^{+0.10}_{-0.06}$	1.00 ± 0.10	—	—	$0.27^{+\infty}_{-0.25}$	$0.15^{+\infty}_{-0.15}$	$\delta\beta' R_\pi$

In addition to the restrictions (14a) which follow from our assumptions, we use the empirical constraint $G_V+G_A=2$. The parameter α_V is defined by the second of eqs. (14b); the parameters x_π/z_π and y are defined in eqs. (16)–(21) and (24), respectively. Constraints not used means that these constraints have their canonical values, independently of all free parameters.

Model FSU-2 contains V, A, S, P, T interactions, however with the subsidiary conditions $\alpha_S = 0$, $x_\pi = 0$. It then follows that α_{VS} does not enter. It also follows that β is no longer trivially zero. Thus the free parameters are G_S, G_P, G_A, G_T, α_V, Im G'_T and the useful data sets are the ones shown in tables 1 and 2, with β', δ, and the last three of table 1 excluded. This gives 4 degrees of freedom. We note from table 3 that G_S and G_P obtain values 1σ away from zero, their V−A value. The same model with α_S free shows that the data set contains very little information on this parameter. In what follows we therefore keep it fixed at $\alpha_S = 0$.

Model FSU-3 is a special case of the previous model: V, A, S, and P interactions are assumed, but no T interactions. In this case the data set can be enlarged by the electron polarization in Gamow–Teller transitions, P_β^{GT}. We also make the specific assumption that quarks do not have S and P couplings. Then $x_\pi = 0$, $R_\pi = 1$ and $\sigma_{SP}(\bar{\nu})/\sigma_{VA}(\bar{\nu}) = 0$. As in the previous model, also β' and δ have trivial values. This leaves us with 11 constraints.

The free parameters are now G_S, G_P, G_A, α_V. However, the fit wants $G_S \simeq G_P$; by making this an identity we can gain one degree of freedom.

The results of models FSU-2 and FSU-3 are rather similar: G_S deviates by 1σ from zero, and so does G_T in FSU-2. The errors are of course larger in FSU-2 since it has more parameters. The sign of the deviation $(G_A - 1)$ is of no significance: there are always symmetric solutions around 1 for G_A and G_V. Thus it is a mere accident that FSU-2 finds $G_A < 1$ whilst FSU-3 finds the solution $G_A > 1$.

Let us now turn to the case of a non-vanishing x_π. As can be seen from eqs. (17)–(18), this parameter enters into the product

$$x_\pi^2 \left(\frac{m_\mu}{m_e}\right)^2 (G_S + G_P)\,.$$

Since the mass ratio is a very large number, $(m_\mu/m_e)^2 \simeq 4.3 \cdot 10^4$, it follows from the experimental value of R_π alone, that $x_\pi^2 G_S$ and $x_\pi^2 G_P$ must be very small numbers indeed, about 10^{-7}. Thus we may conclude, alternatively:

(i) $x_\pi \simeq 0$, thus no pseudoscalar boson couples to pions; or

(ii) $G_S \simeq G_P \simeq 0$, no scalar or pseudoscalar boson effectively contributes; or

(iii) the scalar and pseudoscalar couplings are proportional to the mass of the charged lepton partner as in eq. (20). Then $R_\pi = 1$ trivially. This alternative we have called FSWU in subsect. 3.2.

Model FSWU-1 then depends on the same parameter as FSU-2, and in addition on z_π and α_{VS}. The general analysis shows that α_{VS} is very small and that G_S and G_P are strongly correlated. We therefore make the subsidiary assumptions:

$$G_S = G_P\,, \qquad \alpha_{VS} = 0\,, \tag{29}$$

thereby achieving a fit with 4 degrees of freedom. The most interesting result of this fit is that

$$z_\pi = 0.26^{+\infty}_{-0.10}$$

is significantly different from zero.

Note that z_π only appears in the expression for P_μ, eq. (19), R_π being trivially 1. The value of z_π is determined exclusively by the experimental values of $-\xi P_{\mu^+}$ and P_μ. Since P_μ is poorly determined and close to 1, z_π is determined mainly by ξ. For example, if ξ had the experimental value 1, P_μ still being 0.99, one would find $z_\pi = 0.05$. The upper limit of z_π is unbounded because z_π multiplies G_S, and $G_S = 0$ is included within errors. The other parameters are qualitatively similar to the FSU-2 fit, except α_V which is now significantly better constrained.

FSWU-2 is similar to model FSU-3, depending only on V, A, S, and P. Since T is absent we can again use the P_β^{GT} data. In contrast to FSU-3 we no longer assume that the quarks have no S and P interactions. These couplings are now described by the parameters z_π, α_{VS}, and y, defined in eqs. (21) and (24). Using the subsidiary assumptions (29), we are left with the parameters $G_S, G_A, \alpha_S, \alpha_V, z_\pi, y$. To determine these we have the maximal data set, all constraints except for δ, β' and R_π. The fit determines α_S, z_π, and y poorly. The value of α_S is 0 ± 2.8 radians. The upper limit of z_π is again unbounded. Similarly the upper limit of y is unbounded because it is multiplied by an expression which can vanish in the limit $G_S = G_P = 0$. The other parameters are close to the V−A situation.

Note that the improvement in χ^2 for the FSWU models in comparison with the FSU models is only due to the fact that the parameter z_π is able to fit any value of $-\xi P_\mu$.

We now turn to the V−A models with admixtures of other covariants. As discussed in subsect. 3.3 we then have

$$G_V = G_A = -G'_V = -G'_A \qquad \text{real},$$

all other couplings possibly complex, and we cannot use the constraints R_π, P_μ, P_β^{GT}, or $R_{\bar\nu}$.

Model V−A(I) includes real scalar interactions. We find, however, that Re $G_S \simeq$ Re G'_S; by making this an identity we can gain one degree of freedom. The remaining parameters are then G_V and Re G_S only. The number of available experimental constraints are reduced by the trivial relations

$$\rho = \delta = \tfrac{3}{4}, \qquad \alpha' = \beta' = 0 .$$

This fit yields a G_S value significantly different from zero, see table 4. Replacing G_S by G_P shows that the model is perfectly S–P symmetric.

Model V−A(II) includes real tensor couplings. We find again an approximate equality: Re $G'_T \simeq -$Re G_T. Choosing this to be an identity we can eliminate Re G'_T. The model then contains only the parameters G_V and Re G_T. The number of constraints are reduced by the trivial relations

$$\eta = \alpha = \alpha' = \beta = \beta' = 0 .$$

We find that the fit wants Re G_T to differ from zero by about 1σ. Allowing G_T to be complex under the condition $G'_T = -G_T$ does not change the fit, and Im G_T comes out much smaller than Re G_T.

Model V – A (III) includes real scalar and tensor couplings. Using the subsidiary conditions

$$G_S = G'_S , \qquad G_T = -G'_T$$

we have all data except

$$\alpha' = \beta' = 0 .$$

As can be seen from table 4, we find that G_S is significantly different from zero whereas G_T is consistent with zero. The amount of G_T allowed by its errors is not more than what was allowed in V – A II, having tensor couplings alone.

Model V – A (IV) finally has complex scalar interactions and $G'_S = G_S$. The data set excludes the trivially true constraints

$$\rho = \delta = \tfrac{3}{4} , \qquad \alpha = \alpha' = 0 .$$

The fit shows that the allowed ranges of Re G_S and Im G_S are wide. The errors are, however, comparable.

5. Summary and conclusions

In the present work we analyze all available data on weak leptonic charge changing vertices in order to study more general Lorentz structures than V – A. Specifically we study structures which include S or P or T couplings in addition to the dominant V and A currents. We do not study general V, A structures without S, P, T, as this has been done elsewhere [3].

In a first set of models we assume that the charged weak interactions are mediated by the exchange of at most one charged boson of each kind whose couplings to the external leptons need not be relatively real. This leads to a factorized effective four-fermion interaction. Assuming e–μ universality, the number of free parameters becomes manageable. Such factorized, single boson, universal (FSU) models lead to the following conclusions.

In the absence of S and P couplings, the T couplings can be as large as 0.2 (68.3% confidence level), cf. model FSU-1.

The experimental ratio of the rates $\pi \to e\nu_e$ and $\pi \to \mu\nu_\mu$ requires the S and P couplings to pions to be extremely small, or else these couplings break universality. We find three ways to interpret this situation.

TABLE 4

Fits using the class of V − A models of subsect. 3.3 which are characterized by the assumption $G_V = G_A = -G'_V = -G'_A$ real

Model	$\chi^2/\langle\chi^2\rangle$	G_V	Re G_S	Im G_S	G_T	Constraints not used
V − A (I)	0.97/5	0.996 ± 0.004	$-0.28^{+0.13}_{-0.10}$	0	0	$\rho\delta\alpha'\beta'$
V − A (II)	1.18/4	0.998 ± 0.003	0	0	$0.09^{+0.04}_{-0.08}$	$\eta\alpha\beta\alpha'\beta'$
V − A (III)	1.04/6	0.997 ± 0.004	$-0.27^{+0.20}_{-0.21}$	0	-0.04 ± 0.11	$\alpha'\beta'$
V − A (IV)	1.68/4	0.999 ± 0.003	$0.19^{+0.11}_{-0.17}$	$0.03^{+0.18}_{-0.20}$	0	$\rho\delta\alpha\alpha'$

The parameters not shown in the table have been fixed or do not enter the fits (see text). Constraints not used means that these constraints have their canonical values. As explained in the text the constraints R_π, P_μ, P_β^{GT}, and $R_{\bar{\nu}}$ cannot be used in these models.

(i) There are substantial S and P couplings to leptons but none to quarks; cf. model FSU-2.

(ii) There are no S and P couplings.

(iii) The S and P couplings are proportional to the mass of the charged lepton partner. We call this case "weak universality". In this case the pion pseudoscalar coupling constant g_π is

$$\begin{aligned} g_\pi &= f_\pi \sqrt{m_e m_\mu} \cdot (0.26 \pm 0.10) \\ &= f_\pi m_\pi (0.014 \pm 0.005) \end{aligned}$$

cf. model FSWU-1.

In cases (i) and (iii) above, the S or P couplings are of the order of $0.1^{+0.2}_{-0.1}$.

The very precise determination of $R_{\bar\nu} = \sigma_{SP}(\bar\nu)/\sigma_{VA}(\bar\nu)$ in reaction (22) turns out not to be very useful because a vanishing $R_{\bar\nu}$ value can arise in three ways:

(i) the S and P couplings of leptons vanish; then reaction (22) gives no information on the S and P couplings of quarks;

(ii) exchange the words "leptons" and "quarks" above;

(iii) in the limit $G_S = G_P$, $\alpha_S = \pi$ and for a $\bar\nu$ helicity near +1, then one concludes as under (i).

Thus a vanishing $R_{\bar\nu}$ cannot be used to set limits on any S or P coupling constants.

In a second step, we relax the assumption of factorization and test for specific real or complex S, P, T coupling terms due to at least two charged spin-zero bosons and/or one spin-two boson (without universal coupling) in addition to the dominant W_L exchange.

The large number of parameters force us to study only some simple extensions of the V − A model. We present the following results.

Purely real S or P interactions may be present at the level −0.30, significant to $1-2\sigma$, cf. models V − A (I), V − A (III).

T interactions may be present at the level 0.1, significant at 1σ or less, cf. models V − A (II), V − A (III).

Complex S or P interactions may have large real and imaginary parts, cf. model V − A (IV).

We note that the only experimental data in significant disagreement with a pure V − A interaction is the positron asymmetry parameter ξ in muon decay, see table 1. Our conclusions that significant S, P, and T couplings may be present, are strongly influenced by this datum alone.

In particular, the pion pseudoscalar coupling constant g_π is able to adjust itself to any value of ξP_μ as long as P_μ is ill-determined and only the product ξP_μ is measured. It would be of great importance to have ξP_μ, if not ξ, remeasured at a few per mill precision level, in order to test current ideas of gauge models which generalize the standard electroweak model. Finally, we emphasize that a more precise, absolute measurement of the neutrino helicity in pion decay (i.e. of its possible *deviation* from −1) is of greatest importance for these analyses.

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Appendix A

CROSS SECTION FOR $\nu_\mu e^- \to \mu^- \nu_e$

We give here the general expression for the invariant differential cross section for the reaction $\nu_\mu e^- \to \mu^- \nu_e$, eq. (5), for the case of the most general effective interaction eq. (2). Let $s = (p^{(\nu_\mu)} + p^{(e)})^2$ and $t = (p^{(\nu_\mu)} - p^{(\mu)})^2$ be the standard Mandelstam variables, and let h denote the helicity of the incoming muon neutrino. We write the differential cross section as a sum of contributions from different Lorentz covariants and interference terms, viz

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_{VA} + \left(\frac{d\sigma}{dt}\right)_{SP} + \left(\frac{d\sigma}{dt}\right)_{T} + \left(\frac{d\sigma}{dt}\right)_{SP-T} + \left(\frac{d\sigma}{dt}\right)_{SP-VA} + \left(\frac{d\sigma}{dt}\right)_{VA-T} . \quad (A.1)$$

The individual terms are given by the following formulae.

$$\left(\frac{d\sigma}{dt}\right)_{VA} = \frac{\frac{1}{2}G_0^2}{8\pi s^2}\{(|G_V + G_A|^2 + |G'_V + G'_A|^2)g_1(s) + (|G_V - G_A|^2 + |G'_V - G'_A|^2)g_2(s,t) + 2h[\mathrm{Re}\,((G_V + G_A)(G'^*_V + G'^*_A))g_1(s) + \mathrm{Re}\,((G_V - G_A)(G'^*_V - G'^*_A))g_2(s,t)]\} , \quad (A.2)$$

$$\left(\frac{d\sigma}{dt}\right)_{SP} = \frac{\frac{1}{2}G_0^2}{16\pi s^2} g_1(t)\{|G_S|^2 + |G'_S|^2 + |G_P|^2 + |G'_P|^2 - 2h\,\mathrm{Re}\,(G_S G'^*_S + G_P G'^*_P)\} , \quad (A.3)$$

$$\left(\frac{d\sigma}{dt}\right)_{T} = \frac{\frac{1}{2}G_0^2}{8\pi s^2}[2g_1(s) + 2g_2(s,t) - g_1(t)]\{|G_T|^2 + |G'_T|^2 - 2h\,\mathrm{Re}\,(G_T G'^*_T)\} , \quad (A.4)$$

$$\left(\frac{d\sigma}{dt}\right)_{SP-T} = -\frac{\frac{1}{2}G_0^2}{8\pi s^2}[g_1(s) - g_2(s,t)]\{\mathrm{Re}\,((G_S - G_P)G^*_T + (G'_S - G'_P)G'^*_T) - h\,\mathrm{Re}\,((G_S - G_P)G'^*_T + (G'_S - G'_P)G^*_T)\} , \quad (A.5)$$

$$\left(\frac{d\sigma}{dt}\right)_{SP-VA} = -\frac{\frac{1}{2}G_0^2}{4\pi s^2} u m_e m_\mu \{\mathrm{Re}\,(G_V G^*_S + G_A G^*_P - G'_V G'^*_S - G'_A G'^*_P) - h\,\mathrm{Re}\,(G_V G'^*_S + G_A G'^*_P - G'_V G^*_S - G'_A G^*_P)\} , \quad (A.6)$$

$$\left(\frac{d\sigma}{dt}\right)_{VA-T} = -\frac{\frac{1}{2}G_0^2}{8\pi s^2} 3um_e m_\mu \{\mathrm{Re}\,((G_V - G_A)G_T^* - (G_V' - G_A')G_T'^*) - h\,\mathrm{Re}\,((G_V - G_A)G_T'^* - (G_V' - G_A'^*)G_T^*)\}, \tag{A.7}$$

where the functions g_1 and g_2 stand for the expressions

$$g_1(x) = (x - m_\mu^2)(x - m_e^2), \qquad (x = s \text{ or } t) \tag{A.8}$$

$$g_2(s,t) = (s+t)(s+t-m_\mu^2 - m_e^2), \qquad u = m_e^2 + m_\mu^2 - s - t. \tag{A.9}$$

Appendix B

ELECTRON POLARIZATION IN β DECAY

In the absence of T couplings the electron polarization in Gamow–Teller transitions is given by

$$\frac{P^{GT}}{v/c} = -\frac{2\sqrt{G_V G_A}}{G_V + G_A}\cos\alpha_V. \tag{B.1}$$

This expression holds in the factorized models with universality or weak universality. In principle, the nuclear matrix element contains both P and A form factors. However, in the non-relativistic limit and for low momentum transfer the pseudoscalar form factor vanishes. Therefore, the expression (B.1) contains no nuclear form factor, even if S and P couplings are present. Actually, the same formula (B.1) also holds for the electron polarization in Fermi transitions, in factorized universal models with P, V, A and T couplings, but no S-couplings. However, the existing data is of much lesser precision than for Gamow–Teller transitions and is not useful for our fits.

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