

## New constraints on theories of Pc1 pearl formation

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**Abstract.** In this paper we study structured Pc1 pulsations (also called Pc1 pearls) observed on ground, concentrating on the relation between the pearl repetition period  $\tau$  and the wave frequency  $f$ . Earlier studies suggest that the product  $\tau f$  is roughly constant. We reexamine this relation and show that a simple inverse law is excluded. Instead, our observations suggest the relation  $\tau \propto f^{-p}$ , with  $p = 0.59 \pm 0.06$ , posing a new, strict constraint on theories of Pc1 pearl formation. We also study the  $L$  dependence of various combinations of  $\tau$  and  $f$  using a model  $L$  value and extract additional constraints from these combinations. We discuss these constraints in the bouncing wave packet model of pearl formation and determine the range of allowed parameter values in this model. We present two models of energetic ions with different  $L$  distributions and show that one of them can be excluded by the constraints derived. We also discuss how to further improve on these constraints to better test the bouncing wave packet model and other theories of Pc1 pearl formation.

### 1. Introduction

The first Pc1 waves were observed by *Harang* [1936] and *Sucksdorff* [1936]. *Sucksdorff* [1936] also introduced the “name pearl necklace” to denote a chain of separate, regularly repeated wave bursts or pearls. Pc1 pearls, also called structured Pc1, are frequently observed on ground, especially at mid- and low latitudes, where they are the dominant type of Pc1 activity [*Benioff*, 1960]. An example of a Pc1 pearl chain is seen in the sonogram of Figure 1.

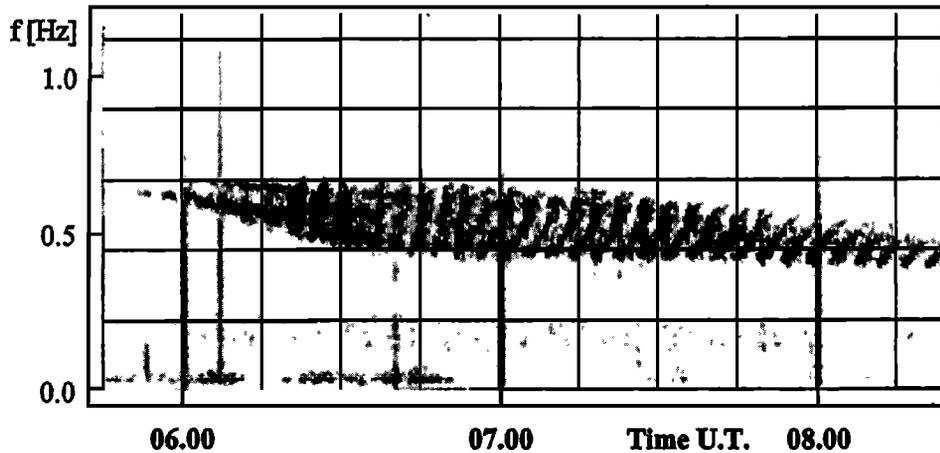
Later, during the second phase of Pc1 studies, it was reported that the individual wave packets constituting a pearl necklace are seen alternately in the two opposite hemispheres [*Yanagihara*, 1963; *Gendrin and Troitskaya*, 1965]. This soon led to the suggestion [*Obayashi*, 1965] that the pearl series represents a succession of echo signals of a wave that oscillates along geomagnetic field lines in a magnetospheric waveguide and is reflected from the ends of this waveguide in the opposite hemispheres. The losses of wave energy during reflec-

tion are expected to be compensated during subsequent crossings of the wave growth region around the equator. This hypothesis of a bouncing wave packet (BWP) has been the leading model of Pc1 pearls for more than 30 years.

However, several other theories of pearl formation have been proposed. For example, *Polyakov et al.* [1983] have proposed a feedback model where the ionosphere is modified by the Pc1 waves. This is basically a refinement of the BWP model including an active role by the ionospheres. Another model is based on the observation that a typical pearl repetition period is close to the bounce period of ions of appropriate energy. A group of phase-bunched ions could modulate the wave growth and raise Pc1 bursts at the ion repetition period. A third model suggests that long-period ULF waves could produce nonbouncing Pc1 bursts by affecting relevant plasma parameters and thus modifying the equatorial Pc1 growth rate. In fact, a few recent studies have presented evidence for Pc1 wave bursts that are phase locked with long-period ULF waves [*Fraser et al.*, 1992; *Rasinkangas et al.*, 1994; *Plyasova-Bakounina et al.*, 1996] or, at least, have a repetition period equal to the period of simultaneous ULF waves [*Mursula et al.*, 1997; *Rasinkangas and Mursula*, 1998].

During the last 30 years a number of papers have studied various properties of Pc1 pearls observed on the

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**Figure 1.** Sonogram showing a chain of Pc1 pearls registered at Oulu on September 1, 1993, from 0600-0800 UT. (The vertical bars are hour marks).

ground. One related empirical fact is that the relative variation of the product of the repetition period  $\tau$  and the wave frequency  $f$  is less than the relative variations of  $f$  and  $\tau$  separately, which has led to the long-held conclusion that the product  $\tau f$  is roughly constant at about 100 (where the times are given in seconds) (for early papers, see, e.g., *Troitskaya and Guglielmi [1967]*). However, we will show in this paper, e.g., that this strict inverse law between  $\tau$  and  $f$  is not valid, but, rather, a more complicated relation with negative correlation exists between the two variables. This demonstrates that many of the basic experimental features of Pc1 pearls are still on a rather loose basis and that new, detailed studies of Pc1 pearls are needed.

On the other hand, observations of repetitive Pc1 bursts by satellite instruments are few, mainly because of the limitations of the satellite orbit. *Fraser et al. [1989]* observed a structured Pc1 wave event with  $\tau = 135$ -160 s and  $f = 0.65$  Hz. *Erlandson et al. [1996]* studied an event with  $\tau = 154$  s and  $f = 0.6$  Hz. Somewhat shorter repetition periods of 55-60 s and higher frequencies ( $f = 1.5$  Hz) were observed by *Erlandson et al. [1992]* at lower latitudes. In all these cases, the  $\tau f$  product roughly agrees with each other and with the above mentioned ground-based value of about 100. This verifies the consistency between satellite and ground observations of Pc1 pearls. However, it does not give additional evidence for the bouncing wave packet model [*Mursula et al., 1997*]. Only *Perraut [1982]* has detected a chain of pearls close to the equator with a repetition period ( $\tau = 240$  s;  $f = 0.3$  Hz) half that observed on the ground, supporting the bouncing wave packet model.

The main objective of the present study is to reexamine the relation between the repetition period and the wave frequency of Pc1 pearls observed on the ground. In section 2 we analyze a number of Pc1 pearls observed in Finland. We find that  $\tau$  and  $f$  are not simply inversely related but, rather, have a more complicated

relation. We also study the  $L$  dependence of several combinations of  $\tau$  and  $f$ . In section 3 we derive the BWP model prediction for the  $\tau - f$  relation and for the  $L$  dependence of the various combinations, using a general power law assumption for the unknown parameters. Moreover, two simple models of energetic ions are presented, which lead to different  $L$  distributions of ion velocities. In section 4 we compare the observations with BWP model predictions, extract the range of parameter values allowed by the present study, and suggest improvements to be obtained from future studies. Section 5 concludes our main results.

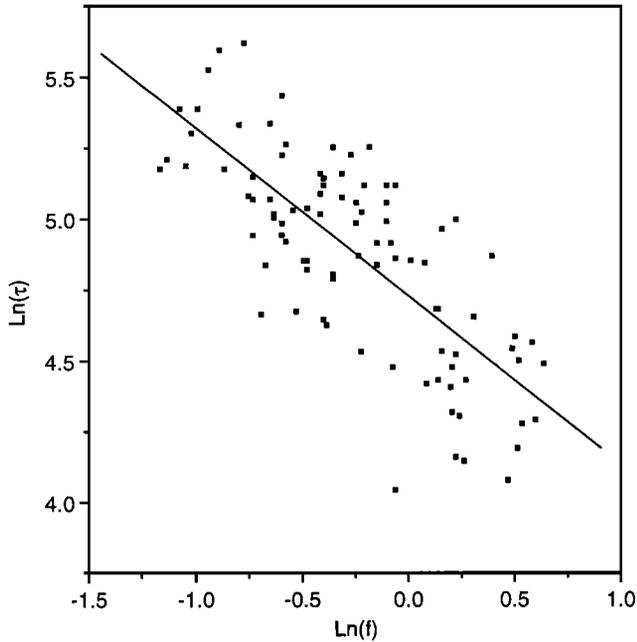
## 2. Data and Observations

In this study we have analyzed Pc1 pearls observed at Sodankylä ( $L = 5.1$ ) and Oulu ( $L = 4.3$ ) during the second half of 1993. Structured Pc1 pulsations were observed during 25 days, and the total amount of pearl events was 39. From these events, the midfrequency  $f$  and the repetition period  $\tau$  at that frequency were determined for a total of 94 samples. A sample is defined as follows: (1) it is a segment of a Pc1 pearl event, (2) the relative change of the midfrequency is no more than 10%, and (3) the number of wave packets (pearls) included is at least 20. The midfrequency ranged from 0.31 to 1.9 Hz, with a median at 0.73 Hz. The shortest (longest) repetition period was 57 s (276 s), and the median period was 142 s.

We have plotted the repetition period  $\tau$  as a function of  $f$  in a log-log plot in Figure 2. As expected from earlier results, there is a strong negative correlation between the two variables. The least squares fit to the data results in the following proportionality relation (line depicted in Figure 2):

$$\tau \sim f^{-0.593 \pm 0.054} \quad (1)$$

with a correlation coefficient  $R = -0.75$ . (The tilde will



**Figure 2.** Distribution of the repetition period  $\tau$  of the observed Pc1 pearls versus their frequency  $f$ . Note the double logarithmic scale. The line gives the best linear fit.

be used here to denote a proportionality relation). It is interesting to note that this result deviates by more than 7 standard deviations from the strict inverse relation between  $\tau$  and  $f$  reported earlier.

Let us now turn to study the  $L$  dependence of some combinations of  $\tau$  and  $f$ . Several ground-based studies [Heacock, 1971; Roth and Orr, 1975; Lewis et al., 1977; Baransky et al., 1981; Fraser et al., 1984; Webster and Fraser, 1985] have shown that the ionospheric foot point of Pc1 pearls is connected with the position of the plasmopause. Satellite observations of pearls are rather few, mainly because of the limitation mentioned above, but they are consistent with this result [Fraser et al., 1989; Erlandson et al., 1992; Mursula et al., 1994; Erlandson et al., 1996; Erlandson and Anderson, 1996]. Since plasma gradients are known to help waves remain guided in the field-aligned direction [Rauch and Roux, 1982; Mazur and Potapov, 1983; Thorne and Horne, 1992], the connection of Pc1 pearls with plasmopause is also theoretically well motivated.

Accordingly, with a lack of direct observations, we assume in the following that the  $L$  value of the Pc1 pearl source field line can be reasonably well approximated by the  $L$  value of the plasmopause, i.e.,  $L \simeq L_p$ . The location of the plasmopause was determined for each event from the model by Carpenter and Anderson [1992], according to

$$L_p = 5.6 - 0.46Kp_{\max}. \quad (2)$$

Here  $Kp_{\max}$  is the maximum  $Kp$  index during the last 24 hours (eight 3-hour values) before the event if it occurred in the magnetic local time (MLT) sector from

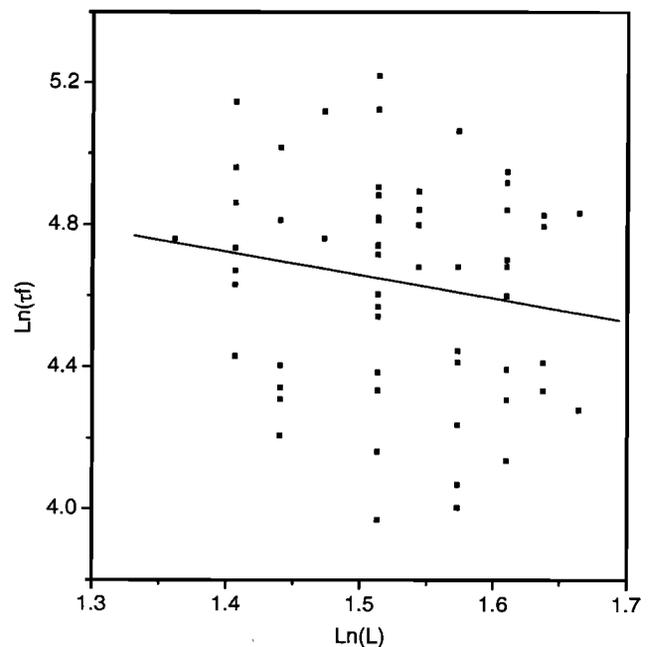
1500 to 0600 (nightside). In the 0600-0900, 0900-1200, and 1200-1500 MLT sectors the most recent, the two most recent, and the three most recent  $Kp$  values, respectively, are to be omitted when  $Kp_{\max}$  is determined [Carpenter and Anderson, 1992].

First, we studied the  $L$  dependence of the product  $\tau f$ . In Figure 3 this product is shown as a log-log scatterplot with respect to the model  $L$  value. The product varies from 53 to 185, with a median value of 111, in a good agreement with earlier observations. However, Figure 3 reveals a very large scatter of points, and only a weak decreasing dependence of  $\tau f$  on  $L$  can be observed. The best fit to the data gives the following relation:

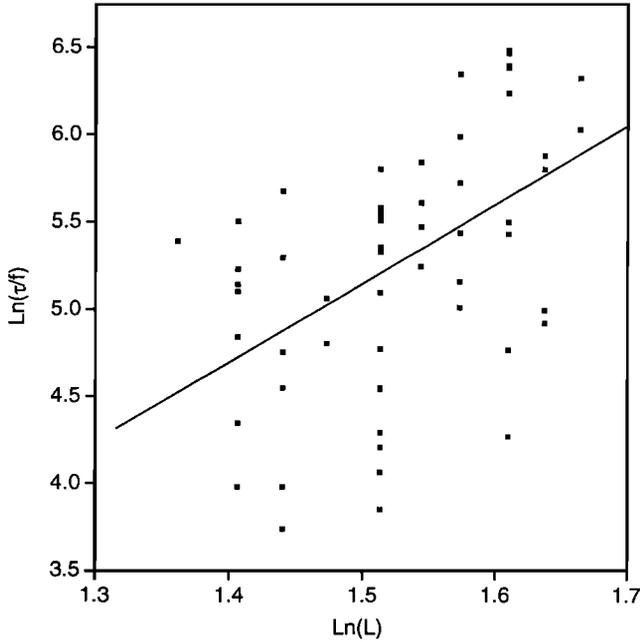
$$\tau f \sim L^{-0.648 \pm 0.509} \quad (3)$$

with  $R = -0.165$ . Note, however, that we have included in Figure 3 only those samples that started in the local time sector of 0500-1200 LT. These samples together form some 64 % of the whole set. Since the plasmopause is sharper and better defined in the morning sector, it is expected that the actual wave source location better corresponds there to the plasmopause location predicted by the model used. Without this local time limitation no correlation is observed and the power in (3) for the whole set is  $0.13 \pm 0.30$  ( $R = 0.045$ ), i.e., consistent with zero. In section 4 we discuss the experimental limitations for the observability of the  $L$  dependence of  $\tau f$  in more detail.

We have also studied the  $L$  dependence of some other combinations of  $\tau$  and  $f$ . An interesting combination is



**Figure 3.** Distribution of the  $\tau f$  product of observed Pc1 pearls versus the model  $L$  value. Only events that started in the 0500-1200 LT sector are included. Note the double logarithmic scale. The line gives the best linear fit.



**Figure 4.** Distribution of the  $\tau/f$  ratio of observed Pc1 pearls versus the model  $L$  value. Only events that started in the 0500-1200 LT sector are included. Note the double logarithmic scale. The line gives the best linear fit.

the  $\tau/f$  ratio, which is considerably more  $L$  dependent than the  $\tau f$  product. We have plotted the  $\tau/f$  ratio as a function of the model  $L$  value in a log-log scatterplot in Figure 4 and find the following best fit relation:

$$\tau/f \sim L^{4.52 \pm 1.06} \quad (4)$$

with  $R = 0.49$ . Accordingly, the observations indicate a strong  $L$  dependence on  $\tau/f$ . As in Figure 3, the result of Figure 4 is for the limited dependence sample set from the morning sector. This local time limitation considerably increased the  $L$  dependence and reduced the scatter in Figure 4. The power in (4) for the whole set is  $2.3 \pm 0.7$ , i.e., smaller but still significantly nonzero.

### 3. Model Estimates

We now study the bouncing wave packet model and its predictions for the above relations between  $\tau$ ,  $f$ , and  $L$ . According to this model, the repetition period of bouncing ion cyclotron waves can be calculated from the integral [see, e.g., Nishida, 1978; Guglielmi and Pokhotelov, 1996]:

$$\tau = \int_0^l \frac{dl}{v_g} \sim l/c_A, \quad (5)$$

where  $l$  is the bounce length along the field line between conjugate ionospheres. The group velocity of waves propagating along the field line, in the case of a single-component plasma, is

$$v_g = c_A(1 - \omega/\Omega)^{3/2}(1 - \omega/2\Omega)^{-1} \quad (6)$$

Here  $c_A$  is the Alfvén velocity,  $\omega$  is the wave (angular) frequency, and  $\Omega$  is the ion gyrofrequency. The last (proportionality) part of (5) is valid in the low-frequency (Alfvén) limit used here, neglecting the frequency dependence of the wave velocity and approximating the wave group velocity  $v_g$  by the Alfvén velocity  $c_A$ .

On the other hand, the resonance condition of Pc1 excitation [Cornwall, 1965] in the same limit gives the following estimate for the wave frequency:

$$f \sim f_0(c_A/v) \quad (7)$$

where  $f_0 = \Omega_0/2\pi$  ( $\Omega_0 = eB/m_i$  is the equatorial gyrofrequency of an ion with charge  $e$  and mass  $m_i$ ) and  $v$  is the parallel velocity of resonant protons. Now, using (5) and (7), we find the following proportionality equation of the product:

$$\tau f \sim f_0(l/v) \sim \frac{1}{vL^2} \sim L^{\nu-2} \quad (8)$$

where the approximation  $l \sim R_E L$  ( $R_E$  is the Earth's radius) was used. In the last form of (8) we have assumed a general power law form for the  $L$  dependence of the parallel velocity  $v \sim L^{-\nu}$ . The theoretical value of the coefficient  $\nu$  is discussed in sections 3.1. and 3.2. Note also that the Alfvén velocity and thus the dependence on equatorial density cancel in the  $\tau f$  product.

Similarly, the  $\tau/f$  ratio attains the following approximate form:

$$\tau/f \sim \frac{lv}{f_0 c_A^2} \sim v N_0 L^{10} \sim L^{10-\nu-d} \quad (9)$$

where we have taken a general power law form for the  $L$  dependence of the nearequatorial proton density  $N_0 \sim L^{-d}$ . (The power law dependence with  $d \simeq 4$  is favored by observations [Farrugia et al., 1989].) Next we discuss two simple models that yield a different  $L$  dependence for the parallel velocity  $v$ .

#### 3.1. Adiabatic Heating Model

This model makes the assumption that the first adiabatic invariant is approximately conserved during the inward drift of particles from the outer magnetosphere (magnetopause). Using this assumption, one obtains

$$v \sim V_{sw} \sqrt{\Omega/\Omega_m} \sim L^{-3/2} \quad (10)$$

where  $\Omega_m$  is the ion (proton) gyrofrequency at the magnetopause and  $V_{sw}$  is the solar wind velocity. Accordingly, in this model  $\nu = 3/2$  and (8) and (9) attain the form:

$$\tau f \sim L^{-0.5} \quad (11)$$

$$\tau/f \sim L^{8.5-d}. \quad (12)$$

(Note also that these proportionality relations are valid, whatever the source or initial energy of particles may be, as long as the adiabatic evolution is followed, since the  $L$  dependence does not depend on the initial velocity of particles before adiabatic drift.)

### 3.2. Resonant Particle Model

Here we present a model of resonant particles, which leads to a different  $L$  dependence of the parallel velocity  $v$  and thereby to different relations between  $\tau$  and  $f$ . It is known that the size of plasmasphere depends on the large-scale convection electric field within the magnetosphere. In particular, the plasmapause position at 1800 LT is determined by the following relation [Nishida, 1978]:

$$L_p = 1/\sqrt{E_c}. \quad (13)$$

Here the convection electric field  $E_c$  is measured in units of  $\Omega_{\oplus} M_{\oplus}/cR_E^2 = 14.4$  mV/m, where  $\Omega_{\oplus}$  is the rotational angular velocity of the Earth and  $M_{\oplus}$  is its magnetic moment. The resonant particle model relates to the fact that ions of appropriate energy can penetrate from the plasma sheet into the plasmasphere, i.e., down to  $L < L_p$  in the evening sector. Therefore a “wedge”-shaped (quasi-discrete) energy spectrum of protons can be formed in the evening sector, as observed, e.g., by the Explorer 45 satellite [Smith and Hoffman, 1974]. The edge of the wedge was found to penetrate into the plasmasphere to  $L = L' < L_p$ , where the ion energy distribution function has roughly the form of a delta function  $\delta(\varepsilon - \varepsilon')$ . Guglielmi and Pokhotelov [1996] made a theoretical model to relate these parameters to the value of the convection electric field as follows:

$$\varepsilon' = a\sqrt{E_c}, \quad L' = b/\sqrt{E_c} \quad (14)$$

where the theoretical estimates of the parameters are  $a \approx 1.2$  and  $b \approx 0.4$  and the energy  $\varepsilon'$  is measured in units of  $e\Omega_{\oplus} M_{\oplus}/cR_E = 92$  keV.

One obtains from (14) the velocity distribution  $v \propto L^{-1/2}$ ; that is,  $\nu = 1/2$ , when the resonant particle model yields

$$\tau f \sim L^{-1.5} \quad (15)$$

$$\tau/f \sim L^{9.5-d} \quad (16)$$

## 4. Discussion

### 4.1. $L$ Dependence

Note that the observed  $L$  dependence of the  $\tau f$  product (equation (3)) has a negative power, as predicted by both models. (Of course, owing to the poor correlation of  $R = -0.165$  only, this result is on a very weak basis.) Actually, the value of the slope in (3) is rather close to the value ( $\nu = 0.5$ ) given by the resonant particle model. However, the experimental error is so large that the adiabatic heating model almost fits within the  $1\sigma$  (one standard deviation) error. Therefore, while being in overall agreement with the two presented models,

the  $L$  dependence of the  $\tau f$  product alone does not yield a strict constraint on these models.

There are a number of factors that contribute to the large scatter of points in Figure 3. As seen above, the dependence of  $\tau f$  on  $L$  is different in the two models, but it is not very strong in either. A very large fraction (about 95%) of Pc1 samples have a very small range of about  $L = 4 - 5$  only. Supposing this is the actual distribution of Pc1 source location, the variation in  $\tau f$  expected in the two models would be rather small, only about 13% and 38%, respectively. This considerably limits the experimental observability of the  $L$  dependence. The limited  $L$  range has several sources. First, the observed  $L$  range corresponds to the  $L$  values of the two stations used. This suggests that the observability of Pc1s having their foot point close to the stations is considerably favored compared with those farther away. Second, some 90% of samples occurred during moderate magnetic activity with  $Kp_{\max} \leq 3+$  (median value of  $Kp_{\max}$  was  $2+$ ). During such moderate geomagnetic conditions the plasmapause position remains at a rather limited  $L$  range. Third, the plasmapause model [Carpenter and Anderson, 1992] may not be sufficiently exact to be used in the present study since it gives only the average plasmapause position.

As seen above, the  $\tau/f$  ratio, contrary to the  $\tau f$  product, also depends on the equatorial density, i.e., on the slope parameter  $d$  (see (9)). On the other hand, the  $\tau/f$  ratio is considerably more  $L$  dependent than the  $\tau f$  product. Therefore, the problem of the small dynamic range (variation) of the  $L$  value discussed above is greatly alleviated, and the  $L$  dependence of the  $\tau/f$  ratio can be reliably determined. Accordingly, the observed value of the  $L$  dependence of the  $\tau/f$  ratio (equation (4)) can constrain the value of  $d$  in the two models. In the adiabatic heating model the power  $d$  can roughly vary between 3 and 5, in the resonant particle model it varies between 4 and 6. (The accurate constraints are depicted also in Figure 5.) These values are fairly reasonable when compared with the observed equatorial plasma distributions in the plasmasphere, where a typical value is  $d \approx 4$  [Farrugia et al., 1989].

Finally, we would still like to emphasize that all the above  $L$ -dependent results are based on using the model  $L_p$  value as a Pc1 source field line. This assumes that the pearl source follows the plasmapause and that the plasmapause location is sufficiently accurately given by the Carpenter and Anderson [1992] model. Considering the simplicity of these assumptions, it is quite remarkable that the observations yield results that are in fairly good agreement with theoretical estimates. In a recent study, Erlandson and Anderson [1996] analyzed the location of Pc1s observed in the ionosphere with respect to the plasmapause, using the same model as a proxy for the plasmapause position. They found that 70% of the events were within  $2 R_E$  from the plasmapause. This implies either that the Pc1s are not always exactly at the plasmapause but can often be found  $1-2 R_E$

away from it or that the plasmopause position given by the model deviates from the correct value by the same amount. It is clear that the  $L$  dependence can be tested more reliably only when a better, model-independent estimate of the pearl source field line is available. This would require a very dense ground-based magnetometer network or an extensive satellite-ground conjugate study.

#### 4.2. The $\tau - f$ Relation

Let us now study the direct relation between  $\tau$  and  $f$ . As seen in Figure 2, these two variables are strongly (negatively) correlated, but the observed relation in (1) deviates from the conventionally assumed, strictly inverse relation by more than 7 standard deviations. Having found this new, nontrivial relation, it is interesting to study whether the BWP model can accommodate this result. Note that using a direct,  $L$ -independent relation, we can avoid the above discussed problem of not knowing the correct  $L$  value. Let us now derive the relation between  $\tau$  and  $f$  within the BWP model. Using the above definitions, the  $L$  dependence of the wave frequency  $f$  (equation (7)) can be found:

$$f \sim f_0(c_A/v) \sim L^{\nu+d/2-6}. \quad (17)$$

Unless the power  $\nu + d/2 - 6$  is zero, we can invert this equation as follows:

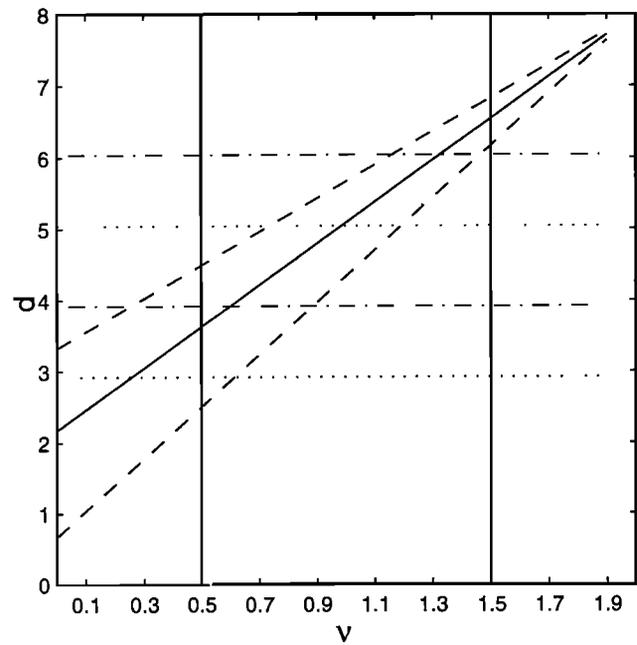
$$L \sim f^{\frac{2}{2\nu+d-12}}. \quad (18)$$

Using this relation, we finally obtain from (5):

$$\tau \sim l/c_A \sim L^4 \sqrt{N_0} \sim L^{4-d/2} \sim f^{-\frac{d-8}{2\nu+d-12}} \quad (19)$$

Accordingly, the BWP model predicts a highly nontrivial relation between  $\tau$  and  $f$ , which clearly deviates from strictly inverse proportionality. As seen in (19), the  $\tau - f$  relation depends on the two parameters  $\nu$  and  $d$ . Therefore the experimental result for the slope in (1) will give a constraint relating these two variables. This constraint is depicted in Figure 5. (Only values for  $\nu < 2$  are shown in Figure 5. For  $\nu \approx 2$  the power in (17) becomes zero, and no inversion is possible. There is another allowed region at  $\nu > 2$ , but this region is not physically reasonable because of  $d$  values that are too high. The allowed parameter values are located in the rather narrow region between the two dashed ( $1\sigma$  limit) lines.

The two parameters  $\nu$  and  $d$  are strongly correlated in the allowed region. Accordingly, in the resonant particle model ( $\nu = 0.5$ ) a fairly small value of about  $d = 2.5 - 4.5$  is required in order to agree with observations. On the other hand, a much higher value of about  $d = 6 - 7$  is needed in the adiabatic heating model ( $\nu = 1.5$ ). Note also that the range of allowed values of  $d$  (for fixed  $\nu$ ) is decreasing with  $\nu$ , making the constraint more tight for the adiabatic heating model. The experimental information on equatorial plasma density



**Figure 5.** Constraint on the parameters of the bouncing wave packet model ensuing from the observed  $\tau - f$  relation (solid line) (dashed lines present the  $1\sigma$  limits). Vertical lines denote the two ion distribution models ( $\nu = 0.5$ , resonant particle model;  $\nu = 1.5$ , adiabatic heating model). Horizontal lines denote the constraint ( $1\sigma$  limits) ensuing from the  $\tau/f$  ratio on the two models (resonant particle model, dash-dotted line; adiabatic heating model, dotted line).

in the plasmasphere suggests a value of about  $d \approx 4$  [Farrugia *et al.*, 1989] and therefore favors the resonant particle model. On the other hand, the adiabatic model disagrees with observations significantly (by nearly 4 standard deviations) in the case of such a low value of  $d$ .

Let us now compare the constraint obtained from the direct  $\tau - f$  relation with the constraint following from the  $L$  dependence of the  $\tau/f$  ratio. As shown above, the theoretical value of the  $\tau/f$  ratio depends, in both models discussed above, on the density parameter  $d$  (see (12) and (16)). The limits on  $d$  in the two models following from the observed value of this ratio (equation (4)) are depicted in Figure 5 as two horizontal bands at about  $d = 3 - 5$  and  $d = 4 - 6$  for the adiabatic heating model and the resonant particle model, respectively. We can see that, in the case of the adiabatic heating model, the two constraints are in disagreement (at the  $1\sigma$  confidence level). On the other hand, in the resonant particle model, the two constraints are consistent and favor the upper range ( $d \approx 4 - 4.5$ ) of the region allowed by the  $\tau - f$  relation.

Overall, we can conclude that the observed nontrivial  $\tau - f$  relation can be accommodated within the bouncing wave packet model for reasonable parameter values. We have used here two different models of ion velocity distributions as a demonstration of two different phys-

ical mechanisms, drift and acceleration. In fact, both mechanisms are effective in the real magnetosphere, and therefore the actual value of  $\nu$  may be intermediate to the two cases presented here, i.e.,  $0.5 < \nu < 1.5$ . As seen in Figure 5, the constraints from the  $\tau - f$  relation and the  $\tau/f$  ratio are in good agreement if  $\nu \approx 1$ . Therefore one would need a third constraint in order to test the bouncing wave packet idea in this parameter range. As seen in (8), the  $\tau f$  product is a direct measure of the parameter  $\nu$ . Therefore an improvement of this constraint in future studies of Pc1 pearls by including a model-independent identification of Pc1 source location is well motivated.

## 5. Conclusions

In conclusion, we have reanalyzed the relation between the repetition period  $\tau$  and the wave frequency  $f$  of Pc1 pearls. Contrary to earlier observations, we find that a simple inverse relation between  $\tau$  and  $f$  is excluded. Instead, the observations suggest the relation  $\tau \propto f^{-p}$ , where  $p = 0.59 \pm 0.06$ , posing a new, nontrivial constraint on theories explaining the formation of Pc1 pearls. We discussed this constraint in the bouncing wave packet model of Pc1 pearls, using general power law forms for the  $L$ -dependent ion velocity and plasma density distributions. Other constraints were extracted from the  $L$  dependence of the  $\tau f$  product and the  $\tau/f$  ratio. We presented two different models of ion distribution, the adiabatic heating model and the resonant particle model, and showed that the latter is in better agreement with observations. We derived the range of allowed parameter values in the bouncing wave packet model and discussed how to improve these limits with future studies. The obtained experimental results can be used as constraints for all theories of Pc1 pearl formation.

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