SIMULATING REGULARITY AND RANDOMNESS IN SUNSPOT ACTIVITY

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ABSTRACT

The time series of sunspot activity displays both regular features and randomness, and their interrelation has been studied during last decades. We present here a model of sunspot production which employs three components of solar magnetic field: the 22-year dynamo field, a weak constant relic field, and a randomly fluctuating field. Within this model, sunspots are produced when the total field exceeds the buoyancy threshold. This model can reproduce the main features of sunspot activity throughout the 400-year period of direct solar observations, including two different sunspot activity modes, the present, normal sunspot activity and the Maunder minimum. The two sunspot activity modes could be modeled by only changing the level of the dynamo field while keeping the other two components constant. We discuss the role of the three components and how their relative importance changes between normal activity and great minimum times. We found that the relic field must be about few per cent of the dynamo field in normal activity times. Also, we find that the dynamo field during the Maunder minimum was small but non-zero, being suppressed typically by an order of magnitude with respect to its value during normal activity times.

1. INTRODUCTION

The main feature of sunspot activity (SA) is the regular 11-year cycle due to the action of the dynamo mechanism modulated by long-term effects, such as the secular Gleissberg cycle. Sometimes sunspot activity is dramatically suppressed, leading to a so called great minimum, like the Maunder minimum (MM) in 1645-1715 when sunspot activity almost vanished. Earlier it was common to describe SA as a multiharmonic process with several fundamental harmonics superposed with each other (Sonett 1983; Vitinsky 1965). On the other hand, SA series also contains a significant random component, and it was common to study solar activity as an example of low-dimensional deterministic chaos described by a strange attractor (Ostryakov & Usoskin 1990; Mundt et al. 1991; Rozelot 1995). Most of earlier studies have concentrated on either the regular or the random (stochastic) component of SA, but some studies have included both components (Sonett 1982; Ruzmaikin 1997, 1998). However, only the normal sunspot activity level is studied in these papers. On the other hand, it has been suggested that the solar dynamo can be in a quite different mode during the great minima (Sokoloff & Nesme-Ribes 1994; Schmitt et al. 1996). Correspondingly, the relation between the regular and random components of SA can be very different during great minima and normal activity times. Here we present a unified model of sunspot production during the two different modes of sunspot activity level.

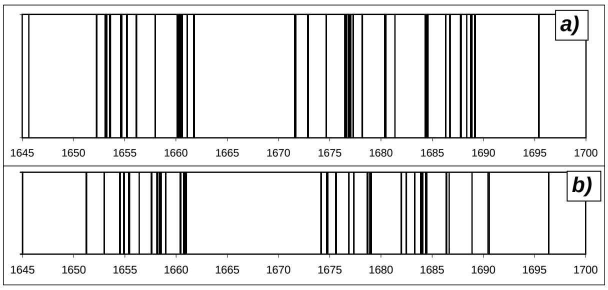

Figure 1. Days with sunspots during the deep Maunder minimum. a) actual observations according to the GSN series; b) a sample of simulation for $B_o = 0.05$, $\sigma_o = 3$, $A_{11} = 0.05$

2. PROPERTIES OF SUNSPOT ACTIVITY

As the SA index we use the group sunspot number (GSN) series (Hoyt & Schatten 1998) which covers the period since 1610 and, thus, includes the full MM period. During MM more than 95% of days were covered with sunspot observations (Hoyt & Schatten 1996). However, sunspots were registered in less than 2% of days during that period. Because of the sparse and seemingly sporadic occurrence of sunspots, only the information whether a sunspot was reported during a certain period or not is meaningful for MM (Usoskin et al. 2000). Days with observed sunspots during the deep MM in 1645-1700 are shown in Fig. 1a as vertical bars. Sunspot occurrence was grouped into two major intervals [1652-1662] and [1672-1689] with a high statistical significance (Usoskin et al. 2000). The "mass centers" of these intervals were in 1658 and 1679-1680, respectively, which forms together with SA maxima in 1639 and in 1705 a 22-year variation of SA during MM. A sub-dominant 11-year cycle in SA may have existed in the second half of MM (Ribes & Nesme-Ribes 1993). Thus, we can summarize the two main features of SA during the deep MM (1645-1699) as follows: (1) Sunspots occurred seldom, approximately on 2% of days; (2) Daily sunspot occurrence was grouped into two long intervals, in 1652-1662 and 1672-1689, with no activity outside these intervals.

The main feature of SA during normal activity times is the 11-year Schwabe cycle. One important parameter of SA during these times is the ratio between SA maxima and minima attained during one cycle. This ratio varies from about 10 to 200 for the 12-month running averaged GSN series after 1800. Also, a persistent 22-year cyclicity was shown to exist (Mursula et al. 2001) in SA with a roughly constant amplitude of about 10% of the modern SA level. This 22-year cyclicity is the underlying feature behind the empirical Gnevyshev-Ohl (G-O) rule (Gnevyshev & Ohl 1948) according to which the sum of sunspot numbers over an odd cycle exceeds that of the preceding even cycle. The 22-year cyclicity in sunspot activity is naturally explained by the action of the 22-year solar dynamo cycle in the presence of a weak constant (relic) field (Mursula et al. 2001). In order to study the random component of SA during normal SA level, we have studied the normalized residual shown in Fig. 2a, $r_i = (R_i - \langle R \rangle_i)/\langle R \rangle_i$ where $\langle R \rangle_i$ is the 31-month running average of the raw monthly GSN series, R_i. Fig. 2b shows the histogram distribution of this normalized residual, which is nearly Gaussian centered around zero (mean -0.05, σ =0.3, $\chi^2_6 = 3.2$) corresponding to the correlated noise in SA (Oliver & Ballester 1996; Frick et al. 1997; Ruzmaikin 1998). Thus, we can summarize the three main features of SA during normal activity times as follows: (1) The dominant 11-year SA cycle with the ratio between the 12-month smoothed sunspot maxima and minima during one cycle being 10-200; (2) A persistent, roughly constant 22-year cycle in sunspot activity at about 10% level of present SA; (3) Random fluctuations of monthly GSN values around the running average forming a correlated noise.

3. THE SIMULATION MODEL

The magnetic field in the bottom of the convection zone is considered to be a superposition of a regular and a random component, and sunspots are produced if this total field exceeds a buoyancy threshold (Ruzmaikin 1997, 1998). In addition to the normal dynamo field, the regular component in our model also includes a constant magnetic field, corresponding to the relic solar magnetic field (Cowling 1945; Sonett 1982; Mursula et al. 2001). The relic field can, due to the amplification by the dynamo mechanism, play a significant role in sunspot occurrence (Boyer & Levy 1984; Boruta 1996). The total field is considered to consist of two parts:

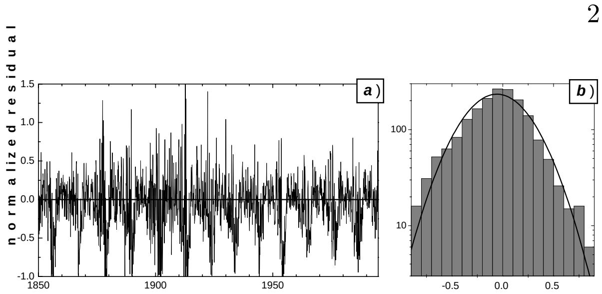

Figure 2. a) The normalized residual between the raw and smoothed monthly GSN series for the period 1849-1996. b) Distribution of the residual with the best fit Gaussian (mean=-0.05, standard deviation ≈ 0.3).

$$B_{tot} = B_{reg} + b, \tag{1}$$

where the regular B_{reg} corresponds to regular dynamo-related field, and b is the randomly fluctuating field generated by random motions (Ruzmaikin 1998). It is important that the regular field within the mean-field $\alpha - \Omega$ dynamo theories is below the threshold (Schüssler et al. 1994; Caligari et al. 1995), and therefore the random b-field is important in order to exceed the threshold. We take the 11-year oscillating field B_{11} field in the form of a 22-year sinusoid (Hale cycle) with amplitude A_{11} (Sonett 1982; Bracewell 1986), and accordingly the regular magnetic field in our model is

$$B_{reg}(t) = A_{11} \cdot sin(\pi \cdot t/T_{11}) + B_o, \tag{2}$$

where T_{11} = 11 years and B_o is the constant relic magnetic field. We adopted the probability distribution function of the solar random field in exponential form (Ruzmaikin 1998):

$$p(b) \propto exp(-|b|/\sigma) \tag{3}$$

We note that the normal (Gaussian) distribution of random values yields similar results (Usoskin et al. 2001). In accord with the correlated noise, the variance of the noise $\sigma(t)$ is assumed to be proportional to the regular component of the field at each time (Ostryakov & Usoskin 1990a):

$$\sigma(t) = \sigma_o \cdot |B_{reg}(t)| \tag{4}$$

where B_{reg} is given according to Eq. 2.

4. SIMULATION RESULTS

We numerically simulated SA separately for normal solar activity and for the great minimum. For each

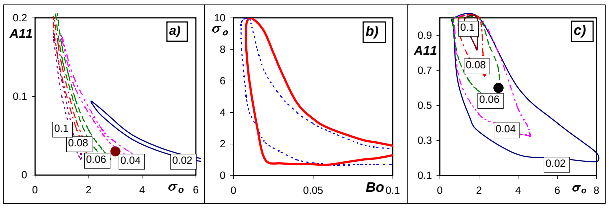

Figure 3. Area of possible values of model parameters. a) A_{11} vs. σ_o for the Maunder minimum. Value of B_o is fixed (as shown in boxes). b) σ_o vs. B_o. The allowed area of the parameter values is limited by the solid curve for the normal SA times, and by the dotted curve for the Maunder minimum. c) A_{11} vs. σ_o for the normal sunspot activity times. Value of B_o is fixed (as shown in boxes). The solid circle denotes values of parameters used for the sample simulation shown in Figs. 1 and 4.

day t, the values of B_{reg}, b, and B_{tot} were calculated using Eqs.(1-4) and a pseudo-random number generator. If the total field $|B_{tot}|$ exceeded the threshold B_{th}, sunspots occurred on that day, and the daily sunspot number was proportional to $(|B_{tot}| - B_{th})$. Field values are in arbitrary units with the value of the threshold, B_{th}, chosen to be unity. Accordingly, there are three independent parameters in the model: A_{11}, B_o and σ_o.

During the deep MM, the 11-year SA cycle is found to be very weak. Accordingly, we assume that A_{11} was small (but non-zero) during this time. A sample of simulated sunspot occurrence is shown in Fig. 1b. This sample shows a similar time behaviour to that of the actual sunspot occurrence. We have made 10^4 simulation sets of 20088 days (simulations) each, corresponding to the number days in the deep MM in 1645-1699. In the following we try to find the range of the model parameters which satisfies the two main features of SA during the deep MM, now given as the following two constraints. *Constraint I:* The number of simulated sunspot days was constrained to be 369 ± 57 out of 20088 days. *Constraint II:* The sunspot occurrence rate during the long spotless periods in 1645-1652, 1662-1672 and 1690-1699 (see Fig. 1a) is constrained to be significantly lower than during other periods of the deep MM. Not more than one sunspot day per year is allowed in these intervals.

Using these constraints we found areas of model parameter values which are shown in Fig. 3a and 3b. Fig. 3a presents the relationship between the amplitude A_{11} and the σ_o parameter for several values of the relic field B_o. The allowed area is prolonged and very narrow for a fixed B_o, reflecting the approximate inverse relation between the two parameters. This is mainly due to the effect of *constraint I.* The area of the possible values of σ_o and B_o (irrespective of the value of A_{11}) is limited by the two dotted lines in Fig. 3b.

A sample of simulated SA for normal activity times is shown in Fig. 4b. There is a good overall similarity with the actual GSN data (Fig. 4a) for the period of

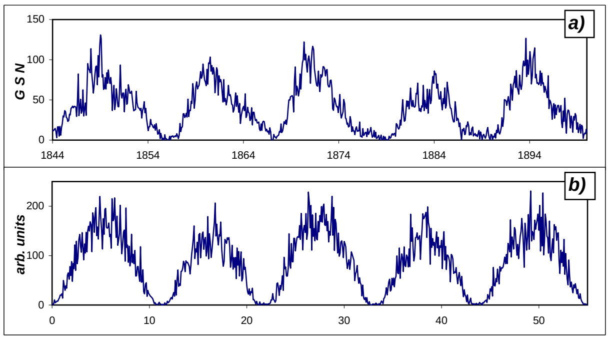

Figure 4. Sunspot activity: a) actual monthly group sunspot numbers for the period of roughly constant SA level; b) a sample of the monthly simulated SA for $A_{11} = 0.6$, $B_o = 0.05$, $\sigma_o = 3$.

fairly constant SA level (solar cycles 9-13). Contrary to real cycles, the simulated cycles are symmetric since we assumed a sinusoidal shape for the 11-year cycle. We simulated totally 1000 11-year solar cycles. The length of simulated cycles varied from 9.5 to 12.5 years, and the cycle amplitude changed by a factor of two, in good agreement with the real sunspot cycles. In accordance with the observed 22-year variation, the G-O rule was found to be valid throughout the entire simulated series.

In order to find the parameter range for SA during normal activity times we use the following two constraints. *Constraint I*: the ratio of the (12-month averaged) sunspot maxima and minima of a cycle is limited to be 10-200. *Constraint II:* the odd cycles are 10-30 % more intense than even cycles.

The relation between A_{11} and σ_o for fixed B_o is shown in Fig. 3c. The two parameters are in rough inverse relation, in analogy with the results for the deep MM period (see Fig. 3a). However, the area is now wider because no constraint was given to the SA cycle amplitude, contrary to MM. The area of the possible values of σ_o and B_o (irrespective of the value of A_{11}) is limited by the two solid lines in Fig. 3b. Finally, we smoothed the simulated sunspot series and studied the monthly residual. The normalized residuals for the sample shown in Fig. 4b has a Gaussian shape with the mean of -0.03 and σ of 0.26 similar to those obtained for the actual GSN series.

5. DISCUSSION

Our model can reproduce the time evolution of SA during both great minima and normal activity times (Figs. 1 and 4). The range of possible values of B_o and σ_o (Fig. 3) is essentially similar for these two very different modes of SA. It is important to note that the model can reproduce SA behaviour for the two modes with the same values of B_o and σ_o, only changing the amplitude of the 11-year cycle. This implies that the dynamo can be significantly suppressed during great minima while both the relic field and random component remain constant. Fig. 3 also shows that the value of σ_o must be larger than

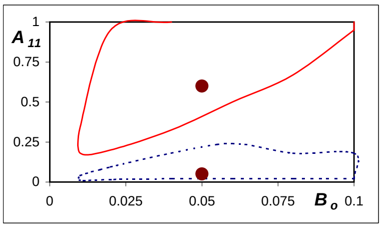

Figure 5. Area of possible values of model parameters A_{11} vs. B_o. The allowed area of the parameter values is limited by the solid curve for the normal SA times, and by the dotted curve for the Maunder minimum. Big solid circles denote parameters used for sample simulations in Figs. 1, 4.

about one, reflecting the fact that the fluctuating field is necessary in order to exceed the buoyancy threshold. The fact that measures of randomness of the simulated series are similar to those of the actual GSN data, suggests that we have correctly simulated the fluctuating field.

The overall relation between A_{11} and B_o is shown in Fig. 5. One can see that there is a lower limit for B_o of about 0.01. We note that a lower limit of the same order of magnitude is found from constraints of both modes of sunspot activity. Accordingly, the existence of the relic field is necessary in both modes of SA in order to satisfy the model constraints. Since the constraints are based on rigorous observational facts, this result gives further evidence for the existence of the relic field. Moreover, we obtain a new estimate for the magnitude of the relic field of about 1-10% of the threshold field. Fig. 5 also shows that the amplitude of the dynamo field, A_{11}, can not be less than about 20% of the threshold during normal SA times, while the value of A_{11} during MM is limited to within 0.02-0.2. The lower limit on A_{11} during MM implies that the dynamo has to operate at some level even during the lowest sunspot activity times. However, it was suppressed during the deep MM by a factor of 10-30 with respect to the normal SA mode. Also, the upper bound on the relic field amplitude B_o is roughly linearly dependent on the value of A_{11} for normal activity times being roughly 10% of the dynamo field. The fact that the areas of possible parameters in Fig. 5 for the two modes of SA do not overlap implies that the dynamo was really in different modes during MM and in normal SA times.

Concluding, we have shown that the main features of sunspot activity throughout the entire period of direct solar observations, including two different sunspot activity modes ("normal" sunspot activity and great minimum times), can be reproduced by a simple model consisting of the 22-year dynamo field, a weak constant relic field and a random field. The two SA modes could be modeled by only changing the level of the dynamo field while keeping the other two parameters (relic field amplitude and variance of random field) constant. We have studied the role of the three components in sunspot production and discussed how their relative importance changes between normal activity times and great minima. We found that, in order to explain the observed level of 22-year cyclicity in sunspot activity (Mursula et al. 2001), the relic field must be about 3-10% of the dynamo field in normal SA times. Also, we find that the dynamo field during the Maunder minimum was small but non-zero, being suppressed typically by an order of magnitude with respect to its value during normal activity times.

REFERENCES

Bracewell, R. N.: 1986, Nature, 323, 516.

Boruta, N.: 1996, Astrophys.J., 458, 832.

Boyer, D.W., & E.H. Levy: 1984, Astrophys.J., 277, 848.

Caligari, P., F. Moreno-Insertis, & M. Schüssler: 1995, Astrohys. J., 265, 1056.

Cowling, T.G.: 1945, Mon.Not.Roy.Astron.Soc., 105, 167.

Frick, P., et al.: 1997, Astron.Astrophys., 328, 670.

Gnevyshev, M.N., & A.I. Ohl: 1948, Astron.Zh., 25(1), 18.

Hoyt, D.V., & K.H. Schatten: 1996, Solar Phys., 165, 181.

Hoyt, D.V., & K.H. Schatten: 1998, Solar Phys., 181, 491.

Mundt, M.D., H.W.B. Maguire II, R.P. Chase: 1991, J.Geophys.Res., 96(2), 1705.

Mursula, K., I.G. Usoskin, G.A. Kovaltsov, : 2001, Solar Phys., 198, 51.

Oliver, R., & J.L. Ballester: 1996, Solar Phys., 169, 215.

Ostryakov, V.M. & I.G. Usoskin: 1990, Solar Phys., 127, 405.

Ostryakov, V.M. & I.G. Usoskin: 1990a, Sov. Tech. Phys. Lett., 16(9), 658.

Ribes, J.C., & E. Nesme-Ribes: 1993, Astron.Astrophys., 276, 549.

Rozelot, J. P.: 1995, Astron.Astrophys., 297, L45, 1995.

Ruzmaikin, A.: 1997, Astron.Astrophys., 319, L13.

Ruzmaikin, A.: 1998, Solar Phys., 181, 1.

Schmitt, D., M. Schüssler, & A. Ferriz-Mas: 1996, Astron. Astrophys., 311, L1.

Schüssler, M.P., et al.: 1994, Astron.Astrophys., 281, L69.

Sokoloff, D., & E. Nesme-Ribes: 1994, Astron.Astrophys., 288, 293.

Sonett, C.P.: 1982, Geophys.Res.Lett., 9, 1313.

Sonett, C.P.: 1983, Nature, 306, 670.

Usoskin, I.G., K. Mursula, & G.A. Kovaltsov: 2000, Astron.Astrophys., 354, L33.

Usoskin, I.G., K. Mursula, & G.A. Kovaltsov: 2001, Solar Phys., 199, 187.

Vitinsky, Yu. I.: 1965, Solar Activity Forecasting, Israel Program for Scientific Translations, Jerusalem.