THE LOST SUNSPOT CYCLE: REANALYSIS OF SUNSPOT STATISTICS

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ABSTRACT

We have recently suggested (Usoskin et al., 2000) that one low sunspot cycle was possibly lost in 1790s, and argued (Usoskin et al., 2002) that the existence of such a cycle does not contradict with available solar proxies, like auroral observations and cosmogenic isotopes. However, some arguments based on a statistical analysis of sunspot activity have been presented against the lost cycle (Krivova et al., 2002). Since the consequences of a new cycle are significant for solar cycle studies, it is important to try to estimate the probability of such a cycle to exist. Here we present the results of a rigorous statistical analysis of all available sunspot observations around the suggested additional cycle minimum in 1792-1793. We show that the level of sunspot activity in 1792-1793 is statistically similar to that in the minimum phase, but significantly different from that in the mid-declining or maximum phases. Using the estimated uncertainties we also calculate new, weighted annual values of group sunspot numbers in 1790-1796 which show a clear minimum in 1792-1793 and a maximum in 1794-1795, supporting the idea of an additional weak cycle in 1790's.

Key words: Sunspot activity; Solar cycle; Dalton minimum.

1. INTRODUCTION

We have recently suggested (Usoskin et al. 2001) that one sunspot cycle was likely missed in 1790s. This suggestion was also shown to be supported by the observed auroral occurrence frequency (Usoskin et al. 2002) and by direct measurements of the magnetic declination range (Mursula et al. 2003) during that period. The concentration of cosmogenic ^{10}Be and ^{14}C isotopes in terrestrial archives does not exclude the existence of an additional cycle (Usoskin et al. 2002). Recently, a paper (Krivova et al. 2002) has been published where the authors criticize this idea claiming, e.g., that an additional sunspot minimum did not exist in 1792–1793. Since consequences of the new cycle would be significant for solar cycle studies, it is important to carefully estimate the probability of this cycle to exist. In this paper we reanalyze the sunspot statistics in 1790s using quantitative statistical tests, and show that the available record of sunspot observations in 1790s does not exclude but rather supports the possibility for an additional minimum in 1792–1793. The details of the analysis are given in (Usoskin et al. 2003a). We analyze here the group sunspot numbers (GSN, denoted as $R_{\rm g}$; see Fig. 1) provided by (Hoyt & Schatten 1998).

2. SUNSPOT OBSERVATIONS IN 1792-1793: STATISTICAL ANALYSIS

The observations made in 1792–1793 are mainly isolated daily observations by single observers (Hoyt & Schatten 1998). There are in total only 20 observations on 16 days during 1792–1793 (Usoskin et al. 2001) so that 12 of observations form a period of 8 consecutive days in Aug–Sep 1793, while the other 8 observations are quite randomly spread over the period so that they all fall on different months. Moreover, there are no two consecutive months with sunspot observations, except for Aug–Sep 1793. Using a simple analysis (Krivova et al. 2002) suggested that GSNs in 1792–1793 are typical for the mid-declining phase of a sunspot cycle and, therefore, exclude the possibility of an additional minimum at this time. They assumed implicitly that one isolated daily observation $R_{\rm d}$ adequately represents the corresponding monthly mean: $R_{\rm m} = R_{\rm d}$. However, such an assumption may lead to an error (Hoyt & Schatten 1998; Usoskin et al. 2002b). A new method has been recently developed for this analysis that more reliably allows to estimate the $R_{\rm m}$ value and its uncertainties $\sigma_{\rm m}$ from a single daily $R_{\rm d}$. While the details of the method are given elsewhere (Usoskin et al. 2002b) the basic idea of the method is as follows. All the daily GSN values for the period 1850–2000 of reliable sunspot measurements form a reference population

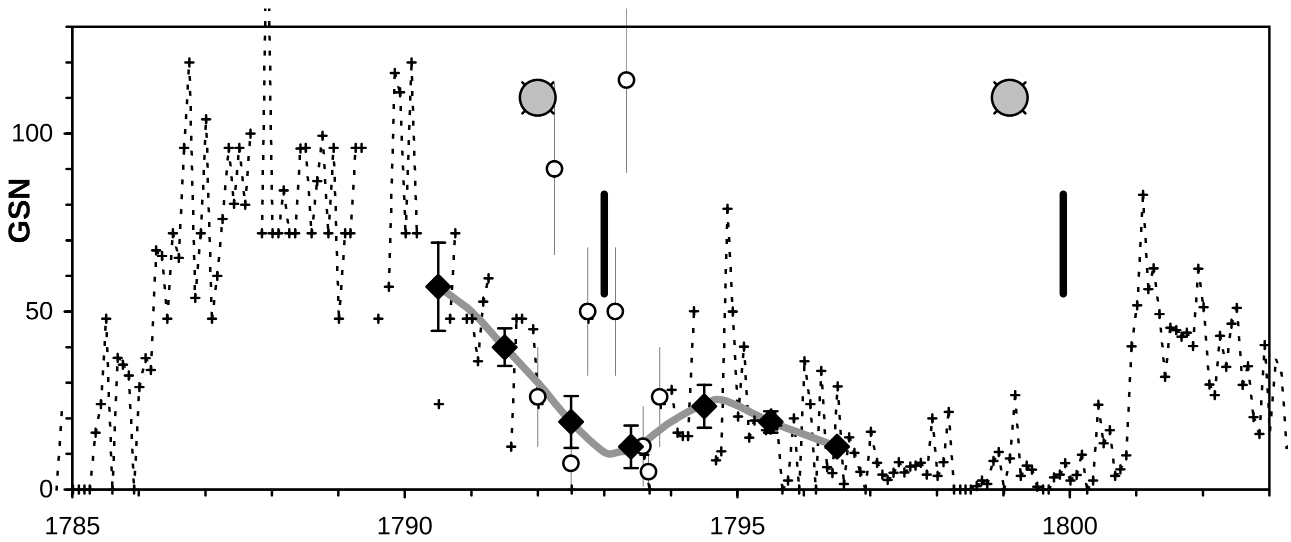

Figure 1. Sunspot activity in 1785-1803. The original and estimated (see Table 1) monthly GSN are shown by the dashed curve and open dots, respectively. The solid diamonds present the estimated weighted annual averages in 1790–1796 (Table 2) connected by the best-fit spline. Big grey dots correspond to the naked-eye sunspot observations (Yau & Stephenson 1988). Vertical solid bars indicate the suggested sunspot minimum times.

data set. For a given observed $R_{\rm d}$ in 1792–1793, all the days with the same daily GSN are collected from the reference population together with their corresponding actual monthly means $R_{\rm m}$. Then, from the distribution of collected monthly values, the mean $R_{\rm m}$ and its standard error $\sigma_{\rm m}$ can be estimated for the daily value in question. The monthly means and their errors estimated in this way for observations in 1792–1793 are given in Table 1 and shown in Fig. 1 as open dots with error bars.

Following the approach of (Krivova et al. 2002), we calculate the average GSN level in 1792–1793 (denoted as $\overline{R_{92-93}}$), i.e., around the suggested minimum, to compare it to the level of some later, better covered solar cycles. However, it is not correct to calculate $\overline{R_{92-93}}$ as a simple arithmetic average of monthly means $R_{\rm m}$ since they are of greatly unequal accuracy. Neither it is correct to calculate $\overline{R_{92-93}}$ from individual $R_{\rm d}$ since the latter are not independent in Aug-Sep 1793. In such a case $\overline{R_{92-93}}$ must be calculated as a weighted average of monthly means $R_{\rm m}$ with the weights $w_{\rm m} = 1/\sigma_{\rm m}^2$ (Agekyan 1972; Usoskin et al. 2003a). Our final estimate of $\overline{R_{92-93}}$ is 16.2 ± 7.6. Using the monthly means and their errors, we also calculated the weighted annual $R_{\rm g}$ values and their errors for the years 1790–1796 (see Table 2). The time profile of the annual $R_{\rm g}$ values depicted in Fig. 1 clearly suggests for an additional minimum at the turn of 1792–1793 and a maximum in 1794–1795.

We have plotted the obtained $\overline{R_{92-93}}$ together with the running 2-year mean of sunspots for the reference period 1850–1996 in Fig. 2. One can see that the value of $\overline{R_{92-93}}$=16.2±7.6 corresponds very well to the GSN values around solar minima. This can be tested using rigorous statistical tests (e.g., Agekyan 1972; Sachs 1972), the null hypothesis being that the small sample population (sunspot observations in 1792–1793) is statistically similar to a given reference population. Although the sample population's size is too small to analyze the shape of the distribution function, the hypothesis of the equality of means can be tested. We considered three reference populations from the reference period of 1850–1996: the min-, max- and mid- populations including all the daily GSN values in 2-year intervals around sunspot minima, maxima and in the mid-declining phase, respectively. We applied three different statistical tests for both the daily observations (16 and about 10000 points in the sample and each of the reference populations) and for the monthly averages (9 and about 320 points, respectively).

First, we applied the Single-Sample Sign test to the null hypothesis. To each point of the sample population, a sign "-" or "+" is assigned depending on whether it is smaller or greater than the mean value of the reference population, respectively. Then the number of "+" elements N^+ and "-" elements N^- is counted and the value of a is calculated:

$$a = \frac{\min(N^-, N^+) - (n-1)/2}{\sqrt{n/2}}, \qquad (1)$$

where n is the size of the sample population. If the sample population has the same mean as the reference population, the mathematical expectation of a is zero. From the value of a the probability S_s of a false rejection of the null hypothesis is calculated. If a is significantly different (at the level of $\beta = 1 - S_s$) from zero then the null hypothesis of the equality of the two means should be rejected at the significance level β. Note that the value of $S_s < 0.05$ indicates that the two populations have significantly different means (at the significance level of 0.95). This test

Table 1. Estimated monthly means R_m and their errors σ_m corresponding to daily sunspot observations R_d in 1792–1793.

month	R_d	R_m	σ_m
Jan-92	24	26	14
Apr-92	96	90	24
Jul-92	0	7.3	7.7
Oct-92	48	50	18
Mar-93	48	50	18
May-93	123	115	26
Aug-93	24, 15, 0, 0	21.5	16
Sep-93	5*0	5	5
Nov-93	24	26	14

Table 2. The formal (Hoyt & Schatten 1998) and weighted annual averages of R_g and their errors in 1790's.

year	formal	weighted	σ
1790	61.5	57	12.5
1791	43.2	39.5	5.3
1792	42	19.2	7.3
1793	41	12.4	5.3
1794	30.2	23.4	5.2
1795	15.7	18.8	3
1796	13.7	12.9	3.8

gives a reliable estimate only if the sample size is significantly larger than 10 elements. Only the daily data set fulfills this requirement, and the calculated values of S_s are given in Table 3.

Next we applied the t-test which computes the t value of Student's statistics:

$$t = \frac{\overline{x} - \overline{y}}{\sqrt{\sigma_x^2/n_x + \sigma_y^2/n_y}}, \quad (2)$$

where the subscript indices x and y denote the sample and reference populations, respectively. The significance level S_t to accept the null hypothesis is calculated from the $t-$statistics, and given in Table 3.

As a third test we applied the non-parametric Wilcoxon Rank Sum test which tests the null hypothesis of the relative unbiasedness of the two populations. The $z-$statistics is computed

$$z = \frac{m_u - U}{\sigma_u}\ , \quad (3)$$

where U is the rank sum of the sample population x, and $m_u = \frac{n_x(n_x+n_y+1)}{2}$ and $\sigma_u = \sqrt{\frac{n_x n_y(n_x+n_y+1)}{12}}$ are the mathematical expectations of the mean and standard deviation of U. The probability S_W to accept the null hypothesis is calculated from z statistics, and shown in Table 3.

The results of all the above tests are consistent with each other and suggest that only the min-population may have the same mean as the 1792–1793 sample, while the hypothesis of the equality of the means should be rejected for both the max and mid reference populations at a high significance level. This result is robust and reliable, as confirmed by three different and independent statistical tests. The first and third tests do not require any statistical estimates of the sample population, and are therefore independent of our analysis of this population presented above. Moreover, the third test is even independent of the statistical estimates of the reference populations. This implies that the sample population of 1792–1793 is statistically similar to the minimum-like reference population and significantly different from both the maximum and mid-declining phase populations.

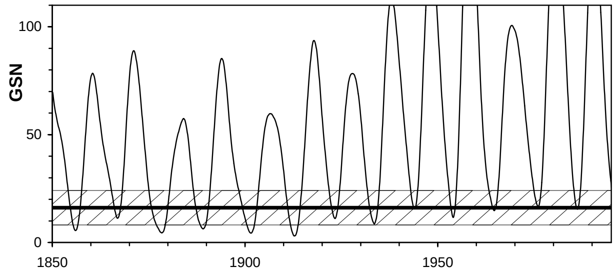

Figure 2. The 2-year smoothed group sunspot numbers (thin curve). The solid horizontal line with hatched area corresponds to $\overline{R_{92-93}} = 16.2 \pm 7.6$.

3. DISCUSSION AND CONCLUSIONS

The above analysis confirms the existence of a sunspot minimum in 1792-1793. This additional minimum is rather close to the "official" minimum in 1798.3 and would lead to a very short new cycle 4' of only about 5.4 years (Krivova et al. 2002). However, the official minimum was calculated from the Wolf sunspot series, which is different from the GSN series analyzed here. Applying the standard 13-month running mean (e.g., Gleissberg 1944; Harvey & White 1999) to the GSN series we found the minimum starting cycle 5 to be in 1799.9, implying that the lost cycle 4' was longer, about 7 years. As shown in (Usoskin et al. 2002) the new cycle of this length does not distort the cycle length distribution or the Waldmeier length-vs-amplitude statistics (Waldmeier 1961).

A naked-eye sunspot observation was reported in 1792 (Yau & Stephenson 1988, see also Fig. 1) which was used by (Krivova et al. 2002) as an argument against the new cycle. However, another naked-eye observation was made in Feb 1799 ((Yau & Stephenson 1988)) which falls between the official and the suggested minimum of cycle 5, in a period which was well covered by sunspot observations and when sunspot activity was lower than the average level in 1792 (see Fig. 1). Also, many naked-eye sunspot observations are listed during the Maunder minimum when the sparseness of sunspots is well documented (Hoyt & Schatten 1998). It was

Table 3. The probability of a false rejection of the null hypothesis that the sample and the reference populations have equal means, according to the Single-Sample Sign test S_s, the $t-$test S_t and the Wilcoxon Rank Sum test S_W.

	minimum	mid-decl	maximum
daily R_g	10 ± 12.3	55 ± 34	115 ± 48
S_s	0.85	0.05	0.03
S_t	0.16	0.0043	10^{-6}
S_W	0.28	$3 \cdot 10^{-5}$	0
monthly R_g	10 ± 8.3	55 ± 27	115 ± 39
S_t	0.03	10^{-10}	0
S_W	0.18	10^{-5}	0

concluded that naked-eye observations alone are not a reliable indicator of sunspot activity (Eddy 1976, 1983). Accordingly, the naked-eye sunspot observation in 1792 does not exclude the possibility of low GSN values in 1792–1793.

A visual analysis of the cosmogenic ^{10}Be and ^{14}C time series shows no evidence for an additional cycle in 1790s. However, as demonstrated in (Usoskin et al. 2002) employing numerical modeling of ^{10}Be production by cosmic rays, the expected difference in ^{10}Be concentration between the two cases (with or without the new cycle) is significantly below observational errors. Therefore the ^{10}Be data are not able to distinguish between them. Moreover, the radiocarbon ^{14}C isotope is even less sensitive than ^{10}Be to the fast and rather small changes of solar activity in 1790s implied by the new cycle. Therefore the cosmogenic radionuclide data can neither prove nor disprove the existence of the suggested new cycle in 1790s.

An analysis of auroral observations in 1790's reveals a small but distinct peak of auroral activity in 1796–1797 (Usoskin et al. 2002; Krivova et al. 2002). Although the existence of this peak which appears in three independent auroral data series is beyond doubt, its origin can be questioned. In (Usoskin et al. 2002) we interpreted this peak as the main peak of auroral activity in cycle 4'. This would be in accordance with the common situation where auroral maxima often occur a couple of years after the sunspot cycle maximum. On the other hand, (Krivova et al. 2002) regarded it to be due to the recurrent activity caused by high speed streams occurring very late in the cycle 3'. We note that the recurrent streams usually occur earlier in the cycle and lead to a much higher peak in auroral activity, often higher or of the same order of magnitude as the main peak. Rather, the peak in 1796–1797 was only about 10% of the main auroral activity peak of cycle 3' and occurred just prior to the official minimum. Therefore, the existence of the new cycle is not contradicted but rather favored by the auroral data.

Concluding, we have performed a careful statistical analysis of the sunspot observations in 1790s in order to further study the possibility of a lost cycle at this time (Usoskin et al. 2001). Using three independent statistical tests, we have shown that, contrary to recent claims (Krivova et al. 2002), the average level of sunspot activity in 1792–1793 is similar to that around sunspot cycle minima during the more recent, well observed years (1850–1996), but is significantly different from the activity either in the mid-declining phase or around sunspot maxima. Our results show that the existence of a new cycle in 1790s does not contradict with any available sunspot observations or indirect solar proxies (see also Usoskin et al. 2001, 2002). Even more importantly, our refined analysis of sunspot activity in 1790s gives additional evidence for a sunspot minimum in 1792-93, supporting the existence of a new cycle in 1790's.

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