



Stochastic simulation of cosmic ray modulation: Effect of a wavy HCS

ILYA G. USOSKIN¹, KATJA ALANKO-HUOTARI^{2,4}, KALEVI MURSULA², GENNADY A. KOVALTSOV³

¹*Sodankylä Geophysical Observatory (Oulu unit), POB 3000, University of Oulu, Finland*

²*Department of Physical Sciences, POB 3000, University of Oulu, Finland*

³*Ioffe Physical-Technical Institute, St.Petersburg, Russia*

⁴*Present affiliation: Specim, Spectral Imaging Ltd, POB 110, 90571 Oulu, Finland*

ilya.usoskin@oulu.fi

Abstract: We present a new method to include a 3D wavy heliospheric current sheet into a 2D numerical model of the heliospheric transport of galactic cosmic rays. Using an analytical solution for the flat sheet, we apply it to the wavy sheet assuming its local quasi-flatness. We study the effects of the current sheet in the cosmic ray spectrum and the dominant streaming patterns of cosmic rays in the heliosphere for different solar polarities and tilt angles of the current sheet.

Introduction

Although the heliospheric modulation of galactic cosmic rays (GCR) is well understood, some questions, e.g., on relative roles of different modulation mechanisms remain open. Here we concentrate on the effect of the heliospheric current sheet (HCS) drift, which is important for the modulation, but very difficult to model (e.g., [6, 2, 3]). We present here a quasi-steady 2D model of GCR transport which can be used to study the drift-dominated modulation during periods of low solar activity when the HCS is well organized and the heliospheric conditions are fairly quiet.

The model

The present model is based, as most modulation models, on a numerical solution of Parker's equation of GCR transport in the heliosphere [8], which cannot be (in a general case) solved analytically. This equation is usually solved numerically using the finite difference technique (e.g., [6, 3]), but an alternative stochastic simulation method has been developed recently (e.g., [5, 9, 10]). It is based on the equivalence between a Fokker-Planck differential equation and a set of ordinary differential equations [4], and reduces the 2D Parker's equa-

tion to the following set of stochastic differential equations (see [1] for details)

$$\begin{aligned}
 \Delta r &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \kappa_{rr}) \Delta t + (V_{sw} + \langle v_D \rangle_r) \Delta t \\
 &\quad + R_{n1} \sqrt{2\kappa_{\theta\theta}} \Delta t \\
 \Delta \mu &= \frac{1}{r^2} \frac{\partial}{\partial \mu} ((1 - \mu^2) \kappa_{\theta\theta}) \Delta t - \frac{\sqrt{1 - \mu^2}}{r} \cdot \\
 &\quad \cdot \langle v_D \rangle_\theta \Delta t + R_{n2} \sqrt{\frac{2}{r^2} (1 - \mu^2) \kappa_{\theta\theta}} \Delta t \\
 \Delta T &= -\frac{2 V_{sw} a T}{3 r} \Delta t
 \end{aligned} \quad , \quad (1)$$

where r and θ are the helio-distance and helio-latitude, T is the particle's kinetic energy, $\mu = \cos \theta$, κ the diffusion coefficient, $\langle v_D \rangle$ is the drift velocity, V_{sw} is the solar wind velocity, and R_{n1} and R_{n2} are normally distributed random numbers with unit deviation.

Two kinds of drifts are important: the drift caused by gradient and curvature of the HMF that has both radial and latitudinal components; and the HCS drift that drives GCR along the sheet and effectively operates in the radial direction. The two drifts complement each other, leading to a divergenceless situation. The drifts reverse their direction to the opposite with the changing sign of the HMF every 11 years.

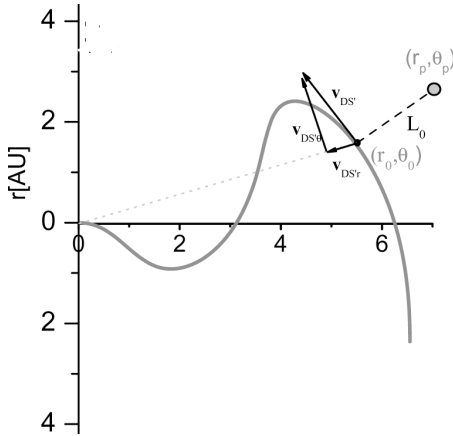


Figure 1: Wavy HCS (grey line) and the drift velocity vector for a particle at position (r_p, θ_p) .

Modeling the HCS drift

Flat sheet

First we obtained an analytical HCS drift velocity, $\langle v_D \rangle_S$, in case of a flat sheet, in a way similar to [2]. It can be parameterized as a function of the ratio of the particle's Larmor radius L_0 to its distance to the HCS R_L , $y = L_0/R_L$:

$$\frac{\langle v_D \rangle_S}{v} = 0.45 - 0.40 \cdot y + 0.088 \cdot y^2, \quad (2)$$

whenever $y < 2$. Since this approximation is limited by an assumption on the homogeneity of HMF around HCS [3], we reduce the drift gradually with increasing rigidity, so that GCR particles with the rigidity P experience this drift only in an "effective" drift region of $r < D_D$, where

$$D_D/100\text{AU} = 1\text{GV}/P. \quad (3)$$

Drift in a wavy current sheet

The assumption of a flat sheet is realistic only during the solar minimum when the HCS tilt angle is small. A more realistic approach of a wavy HCS is needed at other times. However, as the tilt angle increases above about 40° , the HCS structure gets more complicated and HCS drift becomes ineffective. Here we present a new way to numerically calculate the HCS drift for a wavy sheet with the

tilt angle from 0 to 40° . HCS has a 3D-structure, but we reduce the problem to a 2D case by studying the drift at one longitude at a time and by averaging the final result over all longitudes.

Let us consider a GCR particle at the point with coordinates (r_p, θ_p, ϕ_p) in the vicinity of a HCS (see Fig. 1). First we find the minimum distance from the particle to the sheet (point (r_0, θ_0)). Then we calculate the HCS drift velocity of the particle assuming a locally quasi-flat sheet (guaranteed by Eq. 3) and applying Eq. 2. The drift velocity is tangential to the sheet and its direction depends on the polarity of HMF. Although the drift direction lies in the $r - \theta$ plane sufficiently far away from the Sun where the HMF lines are nearly azimuthal, the azimuthal component of the drift velocity is large in the inner heliosphere. However, the azimuthal component gets to zero when averaging over all longitudes. We take the changing spiral angle into account by multiplying the drift velocity by $\sin \psi$, where ψ is the spiral angle. Now we have calculated the particle's drift velocity at the position (r_p, θ_p, ϕ_p) . Next we slightly change the longitude ϕ_p and calculate the corresponding HCS drift velocity. We repeat this procedure for all longitudes, then sum up the radial and latitudinal components of the drift velocity vector and average the result. The average latitudinal component of the HCS drift appears negligible with respect to the radial component, and we study here only the effect of the radial component.

Results and Discussion

Using the full 2D model of GCR heliospheric transport and the newly developed model of HCS drift, we study streaming patterns of GCR in the heliosphere in the presence of drifts. We divided the 2D heliosphere into cells of equal size of $4 \text{ AU} \times 4 \text{ AU}$. We studied the cases of flat HCS (the tilt angle $\alpha = 2^\circ$) and wavy HCS (40°) for the two polarities of HMF. For illustration we studied GCR with the initial rigidity of $P = 2 \text{ GV}$, keeping the diffusion coefficient at $\kappa_0 = 3.0 \cdot 10^{-7} \text{ AU}^2 \text{ s}^{-1} \text{ GV}^{-1}$. Each time a particle left a cell, we recorded its "velocity" components $v_x = \Delta x / \Delta t$ and $v_y = \Delta y / \Delta t$. Then the streaming pattern was computed by averaging over the whole set of v_x

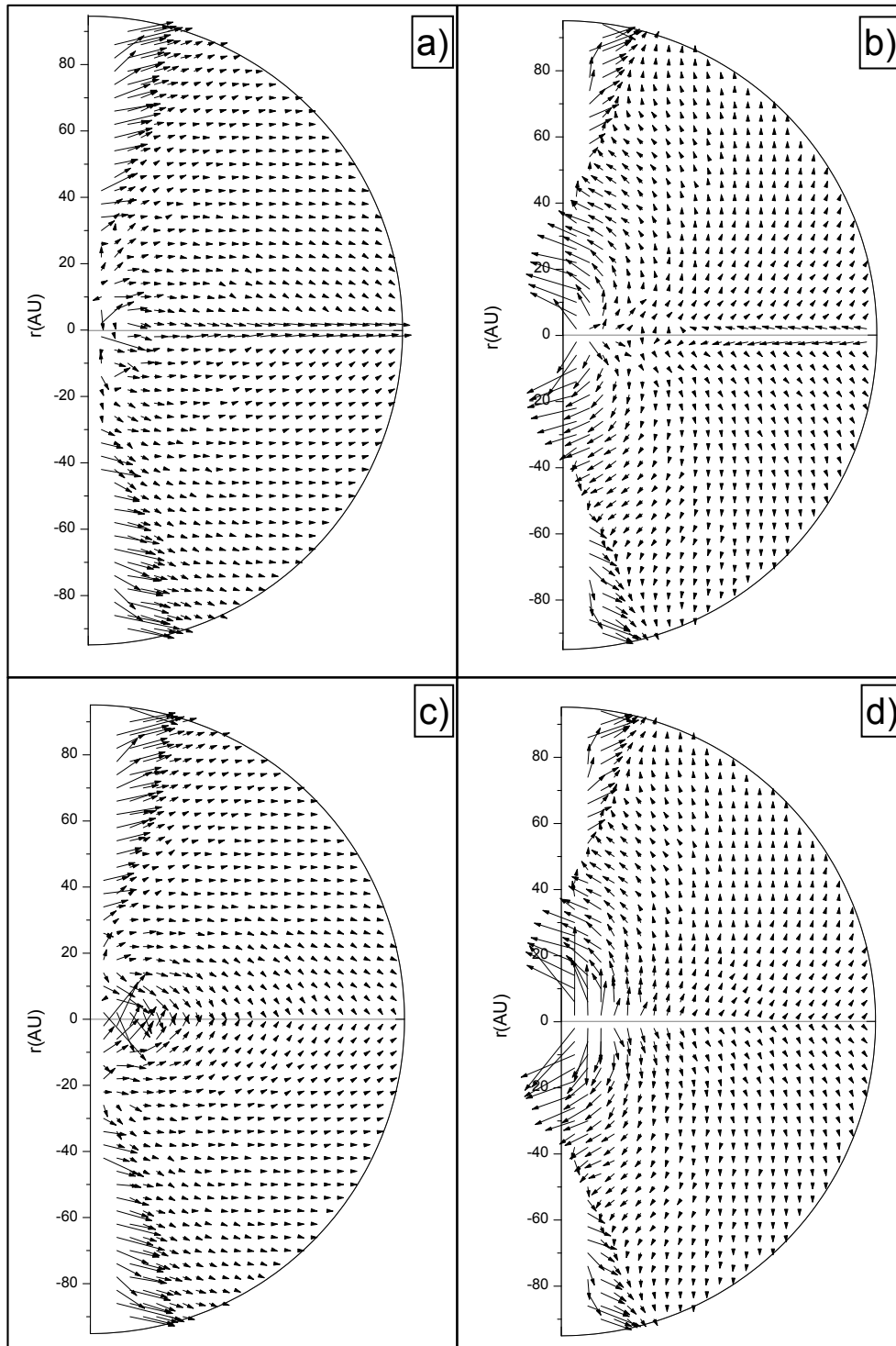


Figure 2: Streaming patterns of 2 GV particles for relatively low modulation ($\kappa_0 = 3.0 \cdot 10^{-7} \text{ AU}^2 \text{ s}^{-1} \text{ GV}^{-1}$) in a) $qA > 0$ conditions with $\alpha = 2^\circ$, b) $qA < 0$ conditions with $\alpha = 2^\circ$, c) $qA > 0$ conditions with $\alpha = 40^\circ$, d) $qA < 0$ conditions with $\alpha = 40^\circ$.

and v_y in each cell. This averaged streaming component is shown in Fig. 2. In $qA > 0$ conditions (panels a and c) the particle streaming around the HCS is preferably equatorward due to the gradient-curvature drift. In the flat sheet case the drift along the HCS is quite strong, and the particles preferably escape from the heliosphere along the sheet. In the case of wavy HCS the effect of HCS drift is quite small and the gradient-curvature drift seems to dominate even at low latitudes. In $qA < 0$ conditions (panels b and d) the gradient-curvature drift causes poleward streaming of particles, especially at high latitudes. At equatorial regions the HCS drift dominates in the flat sheet case and the particles are effectively driven towards the Sun. However, for $\alpha = 40^\circ$ only a small oppositely oriented HCS effect can be seen due to the low HCS drift velocity.

Conclusions

We have presented a 2D model of GCR transport in the heliosphere, based on the stochastic simulation technique. A special emphasis is paid to the effect of drifts on the GCR transport. We have applied both the flat and wavy current sheet in the full 2D-model. We have simulated the effect of HCS drift in the case of a flat and wavy HCS for the two polarities. We have studied the streaming patterns of GCR particles of 2 GV rigidity for relatively low modulation conditions. In $qA > 0$ case, the main pattern is that for a flat HCS the particles are driven by the gradient-curvature drift equatorwards, and away from the Sun by the HCS drift. For $qA < 0$, the particles are driven along the HCS toward the Sun, and polewards by the gradient-curvature drift. With a larger tilt angle, the HCS effect is minor compared to the gradient-curvature drift which dominates the whole heliosphere. This is in qualitative agreement with earlier studies (e.g., [7]) and allows for a qualitative estimates of the effect.

An important application of the model would be to compare the model results to the actual cosmic ray measurements, i.e., to the neutron monitor count rates. This will require, however, lengthy calculations where the diffusion coefficient and the HCS tilt angle are changed as a function of time.

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References

- [1] Alanko-Huotari, K., I.G. Usoskin, K. Mursula, G.A. Kovaltsov, Stochastic simulation of cosmic ray modulation including a wavy HCS, *J. Geophys. Res.* 2007 (in press).
- [2] Burger, R. A., H. Moraal, G. M. Webb, Drift theory of charged particles in electric and magnetic fields, *Astrophys. Space Sci.* 116: 107-129, 1985.
- [3] Burger, R. A., M. S.Potgieter, The calculation of neutral sheet drift in two-dimensional cosmic-ray modulation models, *Astrophys. J.* 339: 501-511, 1989.
- [4] Gardiner, C.W., Handbook of Stochastic Methods, Springer Verlag, Berlin, Germany, 1989.
- [5] Gervasi, M., P.G. Rancoita, I.G. Usoskin, G.A. Kovaltsov, Monte-Carlo approach to galactic cosmic ray propagation in the heliosphere, *Nucl. Phys. B (Proc. Suppl.)*, 78: 26-31, 1999.
- [6] Jokipii, J. R., E. H. Levy, Effects of particle drifts on the solar modulation of galactic cosmic rays, *Astrophys. J.* 213: L85-L88, 1977.
- [7] Jokipii, J. R., B. Thomas, Effects of drift on the transport of cosmic rays. IV - Modulation by a wavy interplanetary current sheet, *Astrophys. J.* 243: 1115-1122, 1981.
- [8] Parker, E. N., The passage of energetic charged particles through interplanetary space, *Planetary and Space Sci.*, 13: 9-49, 1965.
- [9] Usoskin, I.G., K.Alanko, K. Mursula, G.A. Kovaltsov, Heliospheric modulation strength during the neutron monitor era, *Solar Phys.*, 207: 389-399, 2002.
- [10] Zhang, M., A Markov stochastic process theory of cosmic-ray modulation, *Astrophys. J.* 513: 409-420, 1999A.