



Rolf Nevanlinna Institute

Multimapper/OUTBRED Reference Manual

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Bayesian QTL mapping software for outbred offspring data

Version 1.1 / for a backcross and F2 - full-sib family

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This software is a computer implementation of the Bayesian QTL mapping method that was presented in the paper “Bayesian mapping of multiple quantitative trait loci from incomplete outbred offspring data” by Sillanpää and Arjas (1999); see Sillanpää et al. (2004) for important comment. The method is an extension of the earlier method of Sillanpää and Arjas (1998) to more general experimental designs. The program implements the Metropolis-Hastings-Green (Metropolis et al. 1953, Hastings 1970, Green 1995) algorithm in estimation of the model parameters (see the above papers for further detail). The software is written in C-language and is designed for Unix or Linux environment but may work also in other environments.

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Improved Mixing Properties

Following updated sampling scheme (providing facilitated movement in the sample space) is now implemented in this new version 1.1. of the Multimapper/OUTBRED program. Only Step 2 (APPENDIX A; Sillanpää and Arjas 1999) is modified :

In each round, either (1) FAMILY BLOCK-UPDATE or (2) INDIVIDUAL UPDATE is conducted with equal probabilities.

(1) FAMILY BLOCK-UPDATE is similar as earlier version of Step 2 except for Step 2.5 : The grandparental origins are labeled for offspring alleles having a heterozygous parent but (instead of uniform distribution) are proposed from the prior (Equation 4) for alleles inherited from homozygotes. (The acceptance ratio is then modified accordingly.)

(2) INDIVIDUAL UPDATE : Offspring proposals covering all markers jointly are constructed as in Step 2 (Sillanpää and Arjas 1999) given the current configuration in parents, but the acceptance is tested separately for each haplotype in each individual. (The acceptance ratio is again modified accordingly.)

This updated version has recently been applied in some publications (Hurme et al. 2000; Maliepaard et al. 2001). The short description of this modification can be found in the Appendix of Hurme et al. (2000).

Technical details

You can "unpack" the files **MM_outbred.tar.Z** and **MMo_prepro.tar.Z** with the commands:

```
> uncompress MM_outbred.tar.Z
```

```
> tar -xvf MM_outbred.tar
```

and

```
> uncompress MMo_prepro.tar.Z
```

```
> tar -xvf MMo_prepro.tar
```

The files **o_data_structure.h** and **data_structure.h** include following lines for Sun UltraSparc workstations :

```
long lrand48(void);
```

```
#define rand() lrand48()
```

```
#undef RAND_MAX
```

```
#define RAND_MAX 2147483647
```

These lines should be commented out when the software is compiled in a Linux environment or in such Unix systems which have different **RAND_MAX** values. If these lines are omitted in Sun UltraSparc workstation or used in Linux, the software may not work properly and can give wrong results. Before compilation, please check what is a proper **RAND_MAX** value in your system. The Multimapper/OUTBRED software is compiled with a command **make**. One may need to edit the **Makefile** first. The preprocessor has its own **Makefile**. The utility programs **MMfreq.c** and **MMndist.c** need to be compiled separately. (Any **Makefile** does not control them).

You may want to redefine dimensions (maximum sizes) of some storages containing markers, phenotypes and so on. Dimensions are controlling how much memory is allocated for different parameters. These definitions can be found from the beginning of the file **o_data_structure.h**. One need to execute **make** after any such dimensional changes. (Note that preprocessor has its own dimensions defined in **data_structure.h**.)

Running Multimapper/Outbred Under Windows

As was pointed out to me by Dr. Peter Baker from Australia, one can use **cygwin** (available at <http://cygwin.com/>) to get gcc, make and other Unix utilities under Windows. By using cygwin gcc, one should be able to compile and run Multimapper/Outbred under Windows. To speed up the cygwin gcc downloading time, there are some mirror sites which are listed in (<http://cygwin.com/mirrors.html>) (hopefully they are also up to date).

Registration & Mailing list

No official registration for the execution of the program is required, however, one may inform me that one is willing to join for the mailing list. It would be appreciate if one would sent me an e-mail message subjected as 'participation to MM/OUTBRED mailing list' where she/he would indicate his/her name, institution and e-mail address. I will update given names to the mailing list. This way I could notify the people in the list about a newer version of the program and possible errors in the program. (One may find the same information from my web-page as well.) This information would also give me some feedback how many persons are using the program or are interested in it.

Genetic map and marker data

The input files for the genetic linkage map *<filename>.map*, and offspring data *<filename>.cro* are the same as those for QTL Cartografer (Basten et al. 1996). The data, which could come from controlled experiment or a simulation, will consist of markers, traits and other explanatory variables. The details of the file formats are well documented in the QTL Cartografer manual and thus not presented here. Users are acquaint themselves with these formats by reading the appropriate sections of the QTL Cartografer manual, which is available at no charge from North Carolina State University via anonymous ftp at [statgen.ncsu.edu](ftp://statgen.ncsu.edu). Any questions concerning QTL Cartografer should be directed to Dr. Christopher J. Basten (basten@statgen.ncsu.edu).

Similar file format in Multimapper/OUTBRED and in QTL Cartografer has several advantages:

- (1) Genetic linkage maps constructed under MAPMAKER/EXP program are applicable in Multimapper/OUTBRED after file conversion with QTL Cartografer package.
- (2) Many QTL Cartografer support programs (e.g., Qstats) can be utilized.

Preprocessing of the data

You need to preprocess data with the program named `prep_main` before doing any statistical QTL analysis. This program deduces what can be known for unknown genotypes and linkage phases based only on observed data in family structure. The program needs three input files (1) `<filename>.map` , (2) `<filename>.cro` , and (3) `codesystem.file` (fileformat of this is given later in the text). Additionally, two voluntary input files are possible : (1) parental genotype file (`parents.cro`) and (2) parental linkage phase file (`parents.hap`). Preprocession creates two output files : (1) `geno_dedu.out` , and (2) `geno_pair.file` , which are used later in the statistical QTL analysis.

Deduction rules (listed below) are similar to the genotyping rules of Wijsman (1987) and they are applied to data sequentially until no new assignments occur. The rules are following :

- (1) If an offspring is homozygote, say AA, then
 - (1.1) the corresponding allele origins can be assigned randomly, and
 - (1.2) if there is any uncertainty in the parental genotypes, allele A is assigned to both parents with certainty.
- (2) If the parent is homozygote, say AA, then
 - (2.1) that parent will be the origin of allele A in all offspring genotypes, and
 - (2.2) if there is any uncertainty in the offspring genotypes, then allele A is assigned to all the offspring with certainty.
- (3) If an offspring allele A is not present in the known genotype of parent 1, then the (certain) origin of that allele (A) is parent 2 (error check is here possible).

(4) If an offspring has allele A with a known origin, then the corresponding parent will have that allele in its genotype with certainty.

(5) If an offspring has allele A, then both parents might have that allele.

(6) If the parent is heterozygote, say AB, then all offspring might have (i.e., will have at least one of the) alleles A and B.

(7) If a parent might have allele A, then all the offspring might have that allele too. ("Might" is included for future use with more complex pedigree structures).

Rules (6) and (7) are needed only for the deductions between grandparents and parents.

File `parents.cro` includes genotype information at marker loci of each chromosome. First that information is given for a father, and then for a mother. Information for each chromosome is given its own line. Genotype codes for markers are given in three-character-fields separated by space-bars. These codes have been introduced by an analyst in the `codesystem.file`. One line, that is containing only one number and is appearing before parental genotype lines, has to be there but it is not used for any purpose.

File `parents.hap` includes linkage phases, i.e, allelic origin information corresponding alleles in each marker genotype. This information is given in similar order as than in file `parents.cro`. Only the first allele origin of each marker genotype is stored into the file. The second allele origin can be determined from that as a complement.

Following codes are used : 0 = the first allele is paternal origin

1 = the first allele is maternal origin

-2 = origin of the first allele is unknown

These codes can be used systematically to describe haplotypic arrangements in linkage phases, even if their actual allelic origins in reality would be unknown. If all marker linkage phases are unknown in parents, this file is not needed.

Example files of 2 chromosomes having 11 markers in each are shown below. Genotype codes, in the `parents.cro` example, are the same as ones that have been introduced later in the example for a `codesystem.file`.

Example for a `parents.cro`:

```

123456789012345678901234567890123456789012345678901234567 <- column number
xx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+ <- shows length of each field
 1 <- file starts from this line
  3 10 3 1 3 3 10 3 3 1 3
  1 3 10 10 10 1 3
 2
  1 3 3 3 1 1 3 3 3 3 3
  3 3 3 3 3 3 3 3 3 3 3 <- file ends at this line

```

Example for a `parents.hap`:

```

123456789012345678901234567890123456789012345678901234567 <- column number
xx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+xxx+ <- shows length of each field
 1 <- file starts from this line
  0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0
 2
  0 0 0 0 0 0 0 0 1 0 0
  0 1 0 0 0 0 0 0 0 0 1 <- file ends at this line

```

Multimapper/OUTBRED directive files

Four directive files and two preprocessing output files are needed in your working directory to be able to run the Multimapper/OUTBRED program (`outb_main`). These files are :

- | | |
|-----------------------------|-----------------------------|
| (1) randomwalk.file | (2) bg_controls.file |
| (3) priorlimits.file | (4) codesystem.file |
| (5) geno_dedu.out | (6) geno_pair.file |

Contents and formats of the directive files 1 to 4 are presented in the following four sections.

Directive files 5 and 6 are generated by preprocessor.

1 randomwalk.file

In this file, the user defines the chromosome and the trait of interest; the number of MCMC rounds, and the chromosomal starting points. Proposal ranges which directly affect the rejection rates are also specified here as well as the design of the data.

The randomwalk.file has a following format:

1. a chromosome number (controls which chromosome is being mapped).
2. a trait number (controls which trait is being mapped).
3. the number of MCMC rounds to be run.
4. initial locations of the three QTLs on the chromosome in centiMorgans (cM). Only restriction is that they must be separate points on the prior range (\leq length of the chromosome).
5. $\frac{1}{p_a}$ ($= \frac{1}{p_d}$), where p_a (p_d) is a proposal probability to add (delete) a QTL.
6. a proposal range (common) of the QTL location parameters.
7. a proposal range of the regression mean (a) and of environmental covariates.
8. a proposal range of the residual standard deviation (σ).
9. a proposal range of the regression coefficients of QTL genotypes.
10. a proposal range of the regression coefficients of the background control genotypes.
11. {1 or 0 }, where 1 indicates that regression coefficients are printed to the output file.
12. the data design { 1= backcross / fixation assumed (F1-parent is a mother)
2= backcross / fixation assumed (F1-parent is a father)
3= backcross / no fixation assumed
4= F2 / fixation assumed (grandfathers are from same line)
5= F2 / no fixation assumed
6= F2 / fixation assumed (grandfathers are from different lines) }
13. The number of covariates (e.g., age, sex or treatment) in the model and, if any, their indices (starting from zero) among the traits. After indices, the number of classes (if classification variable, 0=regression variable) of each covariate is given. (format xxxx xxx xxx ..)

14. Code used for missing phenotype data (e.g., -99999.0)
15. Variance (var) of the proposal distribution $N(o,var)$ for the regression coefficients of the QTL genotypes (the effects). (Needed in case of adding/deleting a QTL).
16. Documentation line that is printed out to the **o_MH_output.logi** file.
17. Thinning (x) of the chain. Only every x th iteration is printed out to the file.

An example file looks like this :

```

12345678901234567890123456789 <- column number
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx <- indicates length of the fields
1 <- file starts from this line
1
500000
20.0      50.0      80.0
3.0
  1.0
  0.01
  0.01
  0.10
  0.10
1
1
2   1   2   0   0 <- 2 regression covariates
-99999.0
0.5
QTL-ANALYSIS OF CERTAIN CROSS
10 <- file ends at this line

```

2 bg_control.file

The background controls for your data are assumed to be chosen based on some preliminary analyses (e.g., regression at the marker points) and they are introduced in this file as known entities.

The file must have as many rows as there are background controls, representing one line for each background control. There are two parameters in each line: the first parameter is the chromosome number and second parameter is the marker number. The first parameter (column) indicates on which chromosome the current background control locates. Second parameter is a

marker number (bc) representing the closest location of a particular QTL on a given chromosome. Numbering of the chromosomes and markers are expected to start from one.

At the very beginning the program asks how many background controls are needed and reads given number of bc from the start of **bg_control.file**.

An example of **bg_controls.file** containing 5 background controls:

```
1234567890
xxxx xxxx
  1   2   < - file starts from this line
  2   3
  2   5
  4   4
  5   2   < - file ends at this line
```

3 priorlimits.file

The prior limits for the regression parameters are specified in this file. The priorlimits specify the parameter space where the parameter values are acceptable. Common priorlimits for all genotypic regression coefficients are assumed. Also common limits for all background control coefficients are assumed.

The **priorlimits.file** includes always just 5 lines specifying following information in following order:

1. Lower and upper limits for the regression intercept parameter (a).
2. Lower and upper limits for the residual S.D. (σ).
3. Mean (always 0) and variance σ_b^2 of the Normal prior $N(0, \sigma_b^2)$ for the regression coefficients of the QTL genotypes.
4. Lower and upper limits for the regression coefficients of the background control genotypes.
5. Lower and upper limits for the QTL location(s) specifying chromosomal segment to be mapped.

Example of priorlimits.file:

```
1234567890123456789
xxxxxxxxx xxxxxxxxx
  -1000.0    1000.0    < - file starts from this line
           0.0     170.0
           0.0     500.0
        -500.0     500.0
           0.0     100.0    < - and ends to this
```

Note that a natural lower bound for σ is zero and upper bound is the phenotypic standard deviation. Note also that if given upper bound for the QTL-location is bigger than the chromosomal length, the program automatically uses the chromosomal length as an upper bound.

4 codesystem.file

All possible genotypes of the design (BC or F2) must be presented in this file.

How the genotype are coded can be presented in the form :

homozygotes : 1, ..., N_{homoz}

Heterozygote (ij) 1000*i+j for all (ij) combinations, however in a such way that duplicates (ji)=(ij) are not included.

The first column indicates genotype, the second column indicates genotype-code in the <filename>.cro file, and the third column must be coded as presented above. Example file for a design with 4 possible homozygotes and total of 10 possible genotypes, is presented in the following format.

Note that the partial genotype info must appear after the complete genotype info.

An example of **codesystem.file** :

```
12345678901234
xx xxxx xxxxxx
AA  1    1    < - file starts from this line
BB  2    2
CC  3    3
DD  4    4
AB  5    1002
AC  6    1003
```

```

AD  7    1004
BC  8    2003
BD  9    2004
CD 10    3004
A-  -2   -1    < - partial genotype including A-allele
B-  -3   -2    < - partial genotype including B-allele
C-  -4   -3    < - partial genotype including C-allele
D-  -5   -4    < - partial genotype including D-allele
--  -1  -1000  < - list of genotypes always end to info from missing
                       genotype, which is also indicator for the end of file.

```

Example for a backcross :

```

AA  1    1
AB  2    1002
--  -2  -1000

```

and F2 full-sib family :

```

AA  1    1
BB  2    2
CC  8    3
DD  9    4
AB  3    1002
BC  4    2003
AC  5    1003
BD  6    2004
AD  7    1004
CD  10   3004
--  -1  -1000

```

Timestamps

When the execution of the Multimapper/OUTBRED is started, the program creates a new file named `o_MH_timestamp`. After finishing, you can check how much real computing time was required by calculating the difference between the last updating time and the creation time of the `o_MH_timestamp` file that can be found from a directory. Note that the time calculated in this way, is relative to the other load in the computer. However, when there is no other load in the computer, the execution time is accurate.

Program outputs

- 1) Technical notes of the run are collected into a file called `o_MH_output.logi`.
- 2) In each MCMC round, the program prints out a value of the variable N_{qtl} (number of QTLs) into a ASCII file named `o_MH_output__Nqtl`.
- 3) The program prints out QTL locations from MCMC rounds into three files: `o_MH_output__lx0`, `o_MH_output__lx1`, and `o_MH_output__lx2`. If the number of QTLs in the MCMC round is zero no files are updated, if it is one only first file is updated, if it is two two files are updated and so on.
- 4) Similarly, program prints out QTL genotypic coefficients into three files :`o_MH_output__reg0`, `o_MH_output__reg1`, and `o_MH_output__reg2`.
- 5) The program prints out a value of residual standard deviation (σ) and of intercept a (non-identifiable) into a file named `o_MH_output__reg_sigma_a`. First column is an intercept (a).
- 6) The program prints out parental haplotypes at only every 100 000 round to the file `o_phaplos`.

Estimating posterior QTL-intensity from MCMC output

Summarizing posterior density of QTL (or posterior QTL-intensity) based on information on large MCMC sample is important final task. See Hoti et al. (2002) for kernel density estimation of QTL-intensity and general discussion on this topic. Kernel density estimation outperforms histogram approximation and therefore should be used in general, especially when estimating mode (i.e. the best putative QTL position on the marker map). Different kernel implementations (Matlab-programs) to summarize QTL position information are available at Fabian Hoti's homepage (<http://www.rni.helsinki.fi/~fjh>).

Construction of simple histogram approximation of the posterior QTL-intensity from large MCMC

output files can be easily done with C-program named `MMfreq`. It produces output file (`output.txt`) which can be visualised with any visualisation program. This program does not require lot of memory for execution and it can be applied for compressed files.

Program `MMfreq` reads its directives from standard input and can be executed in following way :

```
cat o_MH_output_1x* | MMfreq LC NC output.txt
```

Alternatively, if the output-files are in a compressed form:

```
zcat o_MH_output_1x* | MMfreq LC NC output.txt
```

LC = Length of the analysed chromosome (linkage group)

NC = number of resulting histogram - classes (i.e., bins)

⇒ width of each class is then obtained as LC/NC.

`output.txt` = name of the output-file in where frequencies are stored in the format:

(bar center, frequency) separated with space.

Posterior prob. distribution for different number of QTLs

In the package, there is a C-program named `MMndist` which reads file `o_MH_output_Nqtl` and prints out four (approximative) posterior probabilities of (number of QTLs) N_{qtl} having value 0,1,2 or 3 and also (approximative) posterior expectation for N_{qtl} . Note that the prior distribution assumed for number of QTLs in the method (and the software) is actually an accelerated truncated Poisson distribution instead of ordinary Poisson distribution (see Sillanpää et al. 2004).

Matlab

Visualisation of and construction of the QTL intensity and the phenotypic effect graphs from MCMC output files, can easily be created with Matlab software. I have used version 4.2c. Some example scripts that can be run under Matlab are found below.

Plot of sample path for the number of QTLs:

```
load o_MH_output__Nqtl  
plot(o_MH_output__Nqtl)
```

plotting QTL-intensity histogram constructed by MMfreq :

```
load output.txt  
x=output(:,1);  
bar(x,output(:,2));
```

construction and plotting QTL-intensity histogram :

```
load o_MH_output__lx0  
load o_MH_output__lx1  
load o_MH_output__lx2  
  
k=[ o_MH_output__lx0  
    o_MH_output__lx1  
    o_MH_output__lx2];  
  
clear o_MH_output__lx0 o_MH_output__lx1 o_MH_output__lx2;  
bin=zeros(1,100);  
for s=1:100  
bin(s)=(s-0.5)/100;  
end  
[n,x]=hist(k,bin);  
  
save chromo.mat n x  
  
bar(x,n);
```

If one is only interested in a QTL-intensity histogram, for a shortcut, one may write after the clear-sentence just one line : `hist(k,100)`

and a graph of phenotypic effects:

```
load o_MH_output__lx0
load o_MH_output__lx1
load o_MH_output__lx2

load o_MH_output__reg0
load o_MH_output__reg1
load o_MH_output__reg2

k=[o_MH_output__lx0
   o_MH_output__lx1
   o_MH_output__lx2];

regM0=[(o_MH_output__reg0(:,1)+o_MH_output__reg0(:,2)+
        o_MH_output__reg0(:,3)+o_MH_output__reg0(:,4))/4];
regM1=[(o_MH_output__reg1(:,1)+o_MH_output__reg1(:,2)+
        o_MH_output__reg1(:,3)+o_MH_output__reg1(:,4))/4];
regM2=[(o_MH_output__reg2(:,1)+o_MH_output__reg2(:,2)+
        o_MH_output__reg2(:,3)+o_MH_output__reg2(:,4))/4];

kd=[o_MH_output__reg0(:,1)-regM0
    o_MH_output__reg1(:,1)-regM1
    o_MH_output__reg2(:,1)-regM2 ];

clear o_MH_output__lx0 o_MH_output__lx1 o_MH_output__lx2;
clear o_MH_output__reg0 o_MH_output__reg1 o_MH_output__reg2;

N=length(k);
load chromo.mat
lkm=length(n);
m=zeros(lkm,3);
ug=zeros(lkm,3);
dq=zeros(lkm,3);
pp=zeros(1,lkm);

xl=x;
p=zeros(1,lkm);

disp('classification...');
for s=1:N
if rem(s,1000) == 0
disp(s)
end

[dummy,l]=min(abs(xl-k(s)));
k(s) = l(1);

end
disp('..classification');

disp('median and quantiles..');
for l=1:lkm
pist=find(k==l);
p=length(pist);
pp(l)=p;
y=sort(kd(pist));
if ~isempty(y)
m(l)=median(y);
```



```

ind2=floor(0.025*p);
if ind2==0
ind2=1
end
uq(1)=y(ceil(0.975*p));
dq(1)=y(ind2);
else
m(1)=0.0;
uq(1)=0.0;
dq(1)=0.0;
end
end
disp('done');
end

axis([0,100,-2.0,1.5])

plot(xl,m);

hold on

plot(xl,uq,'w:');

hold on

plot(xl,dq,'w:');

```

Execution in UNIX/LINUX by using script file

It is possible to execute the program with a command: `runthis > output.out &`
 where `runthis` is a following executable script file :

```

#!/bin/sh

outb_main <<END

simul.map

simul.cro

5

END

```

`simul.map` (`simul.cro`) is a given filename for genetic linkage map (genotypic and phenotypic observations) and “5” is a given number of background controls to be read from the `bg_controls.file`.

Acknowledgement

M.S. is grateful for Matti Taskinen for providing the Makefile, MMfreq and executable script-file for the software, and for Dr. Cristopher J. Basten allowing to use QTL Cartografer *.map and *.cro fileformats in this software.

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