Inversio-ongelmien laskennallinen peruskurssi 2012

Final Assignment

1 Datasets

The dataset consists of three noisy measurements of three different unknown signals which have been convoluted with a given convolution kernel. The dataset can be downloaded from `http://cc.oulu.fi/~morispaa/inv/data2012/htyo_data.mat`. It is a MATLAB file and it contains the following data

- `a` is a vector of length 100 and it contains the convolution kernel
- `m1` is a noisy measurement of length 599. The norm of the error is 50.50 and the noise variance is 4.26. It is also known that the unknown signal is continuous and smooth.
- `m2` is a noisy measurement of length 599. The norm of the error is 113.85 and the noise variance is 21.66. It is also known that the unknown signal is continuous but it has cusps at points 150 and 350.
- `m3` is a noisy measurement of length 599. The norm of the error is 120.47 and the noise variance is 24.24. It is also known that the unknown signal is piecewise continuous and the discontinuity points are at point 200 and 400.

2 Compare different solution methods

Construct the convolution matrix and try to solve the three deconvolution problems using different methods:

1. Truncated singular value decomposition
2. Tikhonov regularization
3. CGLS
4. statistical inverse problem (with prior/priors of your own choosing!)

Use Morozov discrepancy principle (if applicable).

3 Structural priors

Sometimes we might have some information about the structure of the unknown. For example, we might know that the unknown has jumps (discontinuities) or other structural features at certain given points or intervals. Here in Problem 2 we know that the unknown has cusps and in Problem 2 there are discontinuities. This information can be inserted into a smoothness prior by giving those point a much higher variance than for other points.

Try this with Problems 2 and 3, i.e. try to solve these problems as statistical inverse problems using the first order difference prior. Modify the prior variance where necessary.

4 Final report

Write a final report explaining what you have done. Include plots of different solutions. Also, include the MATLAB code used to solve these problems. Send everything to me at `mikko.orispaa@oulu.fi`.

The deadline is 15.5.2012. If you get stuck or have some questions, come to see me. My "official" reception time is on Mondays at 10–12, but you can come also at other time, although it is better to e-mail me in advance.

Mikko Orispää
Dept. of Mathematical Sciences, room M223
`mikko.orispaa@oulu.fi`