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MATLAB EXERCISE 5

- (1) (a) Construct signals u_i and s_i as in Exercise 4, Problem 1.
(b) Construct also the convolution kernels (boxcar and Hann window) as in Exercise 4.
(c) Construct the convolution matrices C_1 and C_2 as in Exercise 4.
- (2) Construct noisy measurement data m_n by convolving signals with convolution kernels and by adding gaussian noise

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m = conv(kernel, signal);  
m_n = m + c * randn(size(m));
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where $c = 0.01 * \max(\text{data})$. Calculate the error norm

$$\|\varepsilon\| = \|m - m_n\|.$$

- (3) Try to solve deconvolution problems with noisy data.
- (4) Solve the deconvolution problems using truncated singular value decomposition and Morozov discrepancy principle:
 - (a) Calculate the SVD of the theory matrix
 $[u, s, v] = \text{svd}(\text{matrix});$
 - (b) Calculate the truncated SVD solution $x^{(k)}$ with different values of k , plot the results.
 - (c) Try to find the optimal k using the Morozov discrepancy principle.
- (5) Try to find optimal solution to the deconvolution problems using L-curve method:
 - (a) Start with $k = 1$, and calculate the truncated SVD solution $x^{(k)}$.
 - (b) Calculate the discrepancy $\|Cx^{(k)} - m_n\|$ and the norm of the solution $\|x^{(k)}\|$.
 - (c) Plot the point $(\log \|x^{(k)}\|, \log \|Cx^{(k)} - m_n\|)$
 - (d) Increase k by one, and go to (a)
- (6) Study how the measurement error affects to the optimal solution by solving problems with more and less noise.