Dimensional reduction near deconfinement transition

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Oulu 27.6.2008

arXiv:0704.1416,
arXiv:0801.1566 with Philippe de Forcrand and Aleksi Vuorinen

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- If T much larger than other scales, non-zero Matsubara modes decouple from long distance physics:
  - Requirement in QCD (and YM): $T \gg m_D$, m_{mag},
  - Perturbatively $m_D \sim gT$, $m_{mag} \sim g^2T$
  - Non-pert: $m_D(T_c) \sim 3T_c \overset{?}{\ll} 2\pi T_c$ hep-ph/0303042

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- $\longrightarrow$ Construct effective 3d theory for $n = 0$ modes by taking all the superrenormalizable operators allowed by the symmetries.

The dimensionally reduced theories come with technical advantages

- Fermions come with odd Matsubara modes $\rightarrow$ no fermions in 3d
  - Major numerical advantage
- Lattice-continuum relations can be calculated exactly (for super-renormalizable)

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and some challenges

- Renormalization leads to linearly diverging additive counter-terms → lat-cont *must* be calculated exactly.
- → subtraction of the counter-terms leads to high significance losses.

Full symmetry group of Yang-Mills:

- $A_\mu \longrightarrow s(A_\mu + i\partial_\mu)s^{-1}, \qquad s \in \mathrm{SU(N_c)}$
- $s(\tau + \beta, \mathbf{x}) = zs(\tau, \mathbf{x}), \qquad z \in Z_{N_c}$

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Center symmetry broken in deconfined phase

- Order parameter: Wilson line $\Omega(\mathbf{x}) = \mathrm{P}\exp\left[i\int_0^\beta d\tau A_0\right]$
- Transforms in fundamental representation of Z_{N_c}

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- $\Omega = e^{i2\pi Tn/N_\mathrm{c}}\mathbb{1}$

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Standard dimensional reduction (EQCD) by expanding in A_0 around $\Omega = \mathbb{1}$

- $\mathcal{L}_\mathrm{EQCD} = \frac{1}{2}\mathrm{Tr}[F_{kl}^2] + \mathrm{Tr}D_iA_0D_iA_0 + m_E\mathrm{Tr}A_0^2 + \lambda_E\mathrm{Tr}A_0^4 + \ldots$
- Explicitly breaks the center symmetry.
- Does not give a good description when $A_\mu \sim 1$, near T_c.

Center-symmetric effective theories

- Goal: Want to construct an effective theory that
  - Preserves the Z_{N_c} center symmetry
  - Reduces to EQCD at high T
  - Is superrenormalizable
- Effective theory of Wilson lines not (super)renormalizable

(Pisarsky hep-ph/0608242)

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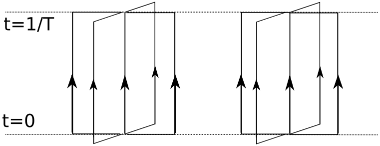
Idea: Construct effective theory for *coarse grained* Wilson loop

(Yaffe+Vuorinen hep-ph/0604100)

$$\mathcal{Z}(\mathbf{x}) = \frac{T}{V_{\text{Block}}} \int_V \mathrm{d}^3 y U(\mathbf{x}, \mathbf{y}) W(\mathbf{y}) U(\mathbf{y}, \mathbf{x}), \notin SU(N_c)$$



Dimensionally reduced theory for SU(2) Yang-Mills:

- For SU(2), sum of matrices proportional to SU(2)

$$\begin{aligned} \mathcal{Z} &= \lambda\Omega, \quad \Omega \in \mathrm{SU}(2), \quad \lambda > 0 \\ \mathcal{Z} &= \frac{1}{2}\Big\{ \underbrace{\Sigma}_{\sim \mathrm{Tr}\Omega} \mathbb{1} + i \underbrace{\Pi_a}_{\sim A_0} \sigma_a \Big\} \end{aligned}$$

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- Transformations under gauge and center transformations:

$$Z_2 \;:\; \mathcal{Z} \longrightarrow -\mathcal{Z}$$
$$\mathrm{SU}(2) \;:\; \mathcal{Z}(\mathbf{x}) \longrightarrow s(\mathbf{x})\mathcal{Z}(\mathbf{x})s^{-1}(\mathbf{x}),$$

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- Most general superrenormalizable Lagrangian from invariants Σ^2 and Π_a^2:

$$V(\mathcal{Z}) = -p_{\mathrm{Z}} + b_1\Sigma^2 + b_2\Pi_a^2 + c_1\Sigma^4 + c_2(\Pi_a^2)^2 + c_3\Sigma^2\Pi_a^2$$
$$\mathcal{L}_{\mathrm{Z}(2)} = {g_3}^{-2}\left[\frac{1}{2}\mathrm{Tr}\,F_{ij}^2 + \mathrm{Tr}\,\Big(D_i\mathcal{Z}^\dagger D_i\mathcal{Z}\Big) + V(\mathcal{Z})\right]$$

Perturbative matching

Match the parameters $\{p_z, g_3, b_1, b_2, c_1, c_2, c_3\}$ perturbatively:

- Demand: In deconfined phase, 2 degenerate minima at $\langle \mathcal{Z} \rangle = \pm\frac{1}{2}v\mathbb{1}$
  - Reparametrize: $\mathcal{Z} = \pm\left\{\frac{1}{2}v\mathbb{1} + g_3\left[\frac{1}{2}\phi + i\chi\right]\right\}$
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- Rearrange potential:
  - hard potential ($\sim T$): $V_h = h_1 \mathrm{Tr}\,(\mathcal{Z}^\dagger\mathcal{Z}) + h_2(\mathrm{Tr}\,\mathcal{Z}^\dagger\mathcal{Z})^2$
  - soft potential: $V_s = g_3^2\Big[\underbrace{s_1 \mathrm{Tr}\,\Pi^2 + s_2\left(\mathrm{Tr}\,\Pi^2\right)^2}_{\mathrm{EQCD}} + \underbrace{s_3\Sigma^4}_{\mathrm{Dom.Wall}}\Big]$

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Note difference to EQCD:

- EQCD: Integrate out T-scales
- Here: *Replace* T-scales

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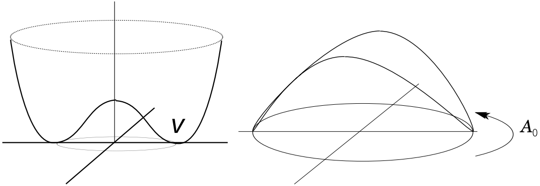

- All $\sim gT$ couplings set by PT
- $m_\phi \equiv rT$ is the mass of coarse-grained Wilson line
  Magnitude = details of cg = $\sim T$ physics
  $\rightarrow$ Any $r = \mathcal{O}(1)$ should give same IR

Perturbatively matched effective theory:

$$
\begin{aligned}
V(\mathcal{Z}) &= b_1\Sigma^2 + b_2\Pi_a^2 + c_1\Sigma^4 + c_2(\Pi_a^2)^2 + c_3\Sigma^2\Pi_a^2 \\
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\begin{aligned}
b_1 &= -\frac{1}{4}r^2T^2, \qquad b_2 = -\frac{1}{4}r^2T^2 + 0.441841g^2T^2, \\
c_1 &= 0.0311994r^2 + 0.0135415g^2, \\
c_2 &= 0.0311994r^2 + 0.008443432g^2, \\
c_3 &= 0.0623987r^2, \\
g_3^2 &= g^2T
\end{aligned}
$$

Theory parametrized by (g, T) of full theory and r.

Beyond the valley of perturbation theory

- Non-perturbative physics needs non-perturbative treatment
- Even though the matching was perturbative, the long distance physics is highly non-perturbative.
- Combined strategy: Perturbation theory for short distance (matching), lattice for long distance (simulations).

Beyond the valley of perturbation theory

$$S_a = S_W + S_{\mathcal{Z}} + V(\hat{\Sigma}, \hat{\Pi}),$$

$$\boxed{\beta = \frac{4}{a g_3^2}}$$

$$S_W = \beta \sum_{x, i<j} \left[1 - \frac{1}{2} \mathrm{Tr}\,[U_{ij}] \right],$$

$$S_{\mathcal{Z}} = 2 \left(\frac{4}{\beta} \right) \sum_{x,i} \mathrm{Tr} \left[\hat{\Pi}^2 - \hat{\Pi}(x) U_i(x) \hat{\Pi}(x+\hat{i}) U_i^\dagger(x) \right]$$
$$+ \left(\frac{4}{\beta} \right) \sum_{x,i} \left(\hat{\Sigma}^2(x) - \hat{\Sigma}(x) \hat{\Sigma}(x+\hat{i}) \right),$$

$$V = \left(\frac{4}{\beta} \right)^3 \sum_x \left[\hat{b}_1 \hat{\Sigma}^2 + \hat{b}_2 \hat{\Pi}_a^2 + \hat{c}_1 \hat{\Sigma}^4 + \hat{c}_2 \left(\hat{\Pi}_a^2 \right)^2 + \hat{c}_3 \hat{\Sigma}^2 \hat{\Pi}_a^2 \right],$$

The UV-sectors of the lat and $\overline{\text{MS}}$ differ ⇒ non-trivial matching (two-loop lattice perturbation theory [AK 0704.1416]):

$$\Sigma = g_3\hat{\Sigma} + \mathcal{O}(\beta^{-1}), \quad \Pi = g_3\hat{\Pi} + \mathcal{O}(\beta^{-1}), \quad c_i \quad = \hat{c}_i + \mathcal{O}(\beta^{-1}),$$

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$$\begin{aligned}
\hat{b}_1 =&\ b_1/g_3^4 - \frac{2.38193365}{4\pi}(2\hat{c}_1 + \hat{c}_3)\beta \\
&+ \frac{1}{16\pi^2}\left\{(48\hat{c}_1^2 + 12\hat{c}_3^2 - 12\hat{c}_3)\left[\log 1.5\beta + 0.08849\right] - 6.9537\,\hat{c}_3\right\} + \mathcal{O}(\beta^{-1}), \\
\hat{b}_2 =&\ b_2/g_3^4 - \frac{0.7939779}{4\pi}(10\hat{c}_2 + \hat{c}_3 + 2)\beta \\
&+ \frac{1}{16\pi^2}\left\{(80\hat{c}_2^2 + 4\hat{c}_3^2 - 40\hat{c}_2)\left[\log 1.5\beta + 0.08849\right] - 23.17895\,\hat{c}_2 - 8.66687\right\} \\
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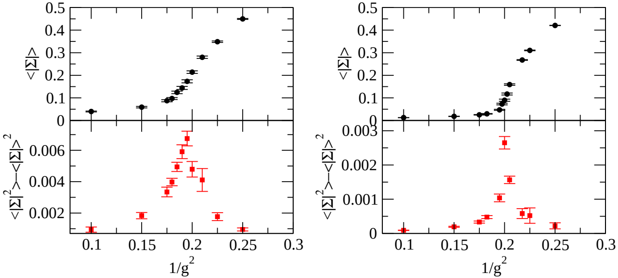
Matching exact in g, but $\mathcal{O}(a)$ errors $\rightarrow$ continuum extrapolation

Results from simulations:

Z_2-restoring phase transition



- left $(\beta = 12, n = 64, r^2 = 5)$
- right $(\beta = 6, n = 64, r^2 = 5)$

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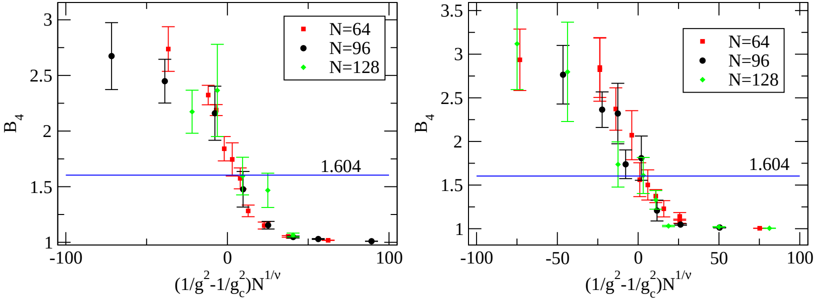

- left $r^2 = 5$, right $r^2 = 10$.
- 3d Ising universality class.

$$B_4 = \langle \bar{\Sigma}^4 \rangle / \langle \bar{\Sigma}^2 \rangle^2 = 1.604 \ldots \text{at criticality}$$
$$\nu = 0.63$$

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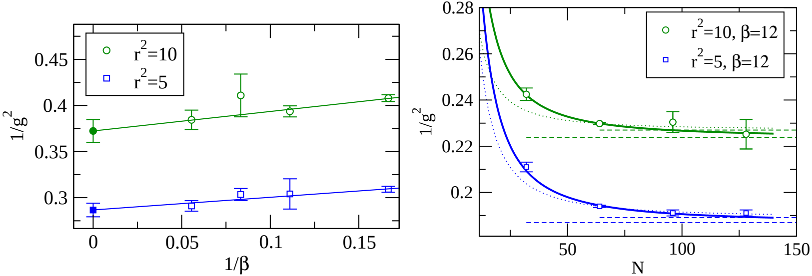

$V \to \infty$ $a \to 0$

- large r → short correlation length → fine lattice
- small r → long correlation length → large volume
- $r = 0$: Σ decouples → $\lambda\phi^4$ already done, X.P. Sun hep-lat/0209144

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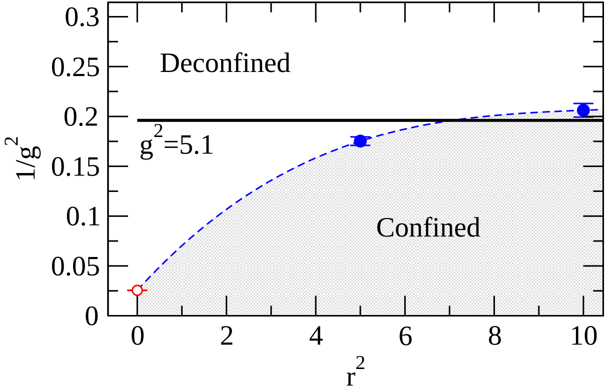

- Phase diagram resembles the full theory (unlike in EQCD).
- Insensitive to $r > 1$
- Phase transition at correct g!

Outlook

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  - Extension to large N_{c} apparent:
    - At $N_{\mathrm{c}} > 4$ no more N_{c}-dependent (super-renormalizable) operators.
- Can the theory accommodate "fuzzy bag"?
  - $p(T) = B_{\mathrm{MIT}} + B_{\mathrm{fuzzy}} T^2 + f_{\mathrm{pert}} T^4$