

Function spaces, Spring 2008
Problem set # 6, due April 29th

- (1) Show that $\|\cdot\|_{\text{Tr}W^{1,p(\cdot)}(\Omega)}$ is a norm.
- (2) Show that $\|\cdot\|_{\text{Tr}W^{1,p(\cdot)}(\mathbb{H})}$ is a norm.
- (3) Can the norm $\|\cdot\|_{\text{Tr}W^{1,p(\cdot)}(\Omega)}$ be defined for other sets $\Omega \subset \mathbb{R}^{n+1}$?
- (4) In Lemma 12.7, show that $|F(x, t)| \leq c_2 \langle |f| \rangle_{x,t}^n$.
- (5) In Lemma 12.7, show that $|\nabla F(x, t)| \leq \frac{c_2}{t} M_{B^n(x,r)}^\# f$.
- (6) Why is it possible to assume that $F(x, t) = 0$ for $t > 1$ in the proof of Theorem 12.5?
- (7) Give an example of a function in $C_0^\infty(\overline{\mathbb{H}}) \setminus C_0^\infty(\mathbb{H})$.