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Advances in Space Research 52 (2013) 22–29

ADVANCES IN SPACE RESEARCH (a COSPAR publication)

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Eccentric dipole approximation of the geomagnetic field: Application to cosmic ray computations

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Received 9 October 2012; received in revised form 19 February 2013; accepted 27 February 2013
Available online 14 March 2013

Abstract

A comparison of the full IGRF model of the geomagnetic field with two simplified models, the truncated IGRF and the eccentric dipole model, is performed. The simplified models were found to provide a reasonable approximation for the large scale geomagnetic field distribution. In the application of the simplified geomagnetic models to the shielding of cosmic rays in the magnetosphere as quantified via the geomagnetic cut-off rigidity, the eccentric dipole and the truncated IGRF provide a good large scale view. The use of the simplified model does not introduce any additional systematic errors at the global scale but may be a source of moderate uncertainty at the regional scale in the tropical Atlantic region. This study quantitatively validates the use of such simplified geomagnetic models when describing the shielding of cosmic rays in the magnetosphere.

Keywords: Cosmic rays; Geomagnetic field; Paleomagnetism

1. Introduction

The geomagnetic field effectively shields the Earth from incoming cosmic rays – highly energetic nuclei of extra-terrestrial origin. Since cosmic rays are charged particles, their trajectories are bent in the geomagnetic field, leading to shielding, so that energetic particles need to possess minimal energy to be able to penetrate through the field towards Earth. The shielding depends on the direction of the geomagnetic field so that it is stronger in the equatorial region, where the magnetic field lines are tangential to the Earth's surface, and absent in the polar regions where the magnetic lines are vertical. Thus, the shielding is unevenly distributed over the globe.

In order to study the cosmic-ray induced effects in the Earth's atmosphere, such as cosmic ray induced ionization (e.g., Bazilevskaya et al., 2008) or production of cosmogenic radionuclides (e.g. Beer, 2000), one has to account properly for the geomagnetic shielding. This can be done straightforwardly for the recent epoch, when the geomagnetic field is well measured and known. This is normally done via the concept of the geomagnetic cutoff rigidity (Cooke et al., 1991), viz. the minimal rigidity (momentum over charge) a charged particle must possess to be able to reach the ground in the absence of the atmosphere. For the recent times, last century or so, covered by extensive geomagnetic measurements, the cutoff rigidity can be calculated (Smart et al., 2000; Shea and Smart, 2001) using the IGRF (International Geomagnetic Reference Field – see Section 2.1) with the full information on multipole components of the geomagnetic field. However, for more distant past, when direct geomagnetic measurements were not performed, one has rely upon paleo- or archeo-magnetic reconstructions (Genevey et al., 2008; Donadini et al., 2010), which provide less information on the higher harmonics of the field. In such a condition a simplified approach is used to assess the geomagnetic shielding of cosmic rays. It is typical to represent the geomagnetic field

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http://dx.doi.org/10.1016/j.asr.2013.02.020

only by its dipole components, which are known much better than the regional field structure in the past (Korte and Constable, 2005; Korte et al., 2011). Reconstructions of the global geomagnetic field at the millennial scale can resolve only the large-scale (Korte and Constable, 2008; Genevey et al., 2008). When spherical harmonic models are available, the contributions from dipole and quadrupole, or of an eccentric tilted dipole can be considered as described in Section 2.2. Although this is considered a reasonable approximation (Bartels, 1936; Elsasser et al., 1956; Fraser-Smith, 1987; Lowes, 1994; Olson and Deguen, 2012), a question of quantitative assessment of the possible uncertainties related to the use of such a simplified approach is still open.

In this paper we compare the geomagnetic cutoff rigidity calculated using different models of the geomagnetic field and assess their uncertainties and validity.

2. Geomagnetic models

The geomagnetic field has a complicated structure, which also depicts slow temporal variability. There are different ways to describe it mathematically. Here we review two ways: the IGRF and the eccentric dipole model.

2.1. IGRF

The International Geomagnetic Reference Field (IGRF) is a reliable standard model which represents the large scale internal part of the geomagnetic field on and above Earth's surface (Finlay et al., 2010). The IGRF model parameters are added periodically for a next epoch of five years so that the parameters are interpolated/extrapolated between the five-year epoches. The preceding IGRF parameters can be updated and become DGRF parameters, and parameters for the extrapolation over the next five years are published by the IAGA Working Group V-MOD (`http://www.ngdc.noaa.gov/IAGA/vmod/`). The model is derived from observations collected by satellites, at observatories at land and during magnetic surveys. The parameters for the IGRF model are available since 1900 AD, and the current IGRF model is eleventh generation dated on 2010 and is valid until 2015, when the next generation is to be released.

The IGRF model uses the multipole representation of the geomagnetic field based on an assumption that the density of current between the surface and ionosphere is negligible near the surface, so that the field can be taken to be curl-free. This allows the field **B** to be presented as the gradient of a scalar potential V (Jacobs, 1991)

$$\mathbf{B} = -\nabla V \tag{1}$$

The scalar potential V is represented through a finite series of numerical Gauss (spherical harmonic) coefficients g_n^m and h_n^m of degree n and order m, which represent multipole (dipole, quadrupole, etc.) components, centered at the Earth center and aligned with the geographical axis.

$$V(r,\theta,\phi,t) = a\sum_{n=1}^{N}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}\left(g_n^m(t)\cos(m\phi) + h_n^m(t)\sin(m\phi)\right) \cdot P_n^m(\cos(\theta)) \tag{2}$$

where r, θ, ϕ, t are the geocentric distance, geographic colatitude and east longitude of the given location, and time, respectively. P_n^m are the associated Legendre polynomials.

The full IGRF model uses about 200 Gauss coefficients, corresponding to multipoles up to degree and order 13, before 2000 ten multipoles were used. We will henceforth refer to the results based on this full model as IGRF. However, in the past paleomagnetic reconstructions are able to provide less detailed information, the most reliably resolving dipole and quadrupole components, corresponding to an approximation based on the centered aligned dipole and quadrupole. We will refer to the results based on this truncated IGRF model as t-IGRF.

2.2. Eccentric dipole model

The Eccentric dipole approximation (Fraser-Smith, 1987; Olson and Deguen, 2012) also uses the first eight Gauss coefficients of the geomagnetic field representation but arranges them differently. It considers only a magnetic dipole which is however displaced from the Earth center and tilted with respect to the geographical axis. The magnetic dipole moment is defined using the first three Gauss coefficients g_1^0, g_1^1 and h_1^1, while five higher order coefficients define the displacement and the tilt of the dipole (see formalism in the Appendix of Usoskin et al., 2010). We will refer to the results based on this model as ED.

3. Comparison of geomagnetic models

In this section we compare the three different geomagnetic models, viz. IGRF, t-IGRF and ED, at different distances from the Earth's surface.

3.1. Total field

The magnetic field representations by the three considered models for the epoch of 2010 are shown in Figs. 1 and 2, for the Earth's surface and 10 Earth radii away, respectively. Panels A through C stand for the IGRF, t-IGRF and ED models, respectively.

While all the models correctly reproduce the main pattern of the surface large scale field, including the South Atlantic Anomaly and the sigmoid shape of the geomagnetic equator, there are some regional features that truncated models cannot catch. However, the discrepancy quickly fades away as the distance from the surface increases. All the three plots are nearly identical at 10 Earth radii (Fig. 2). Already at a few radii above the surface, hardly any essential difference exists between the models. This is quantified in Table 1, which shows the difference between the IGRF and ED models as a function of the

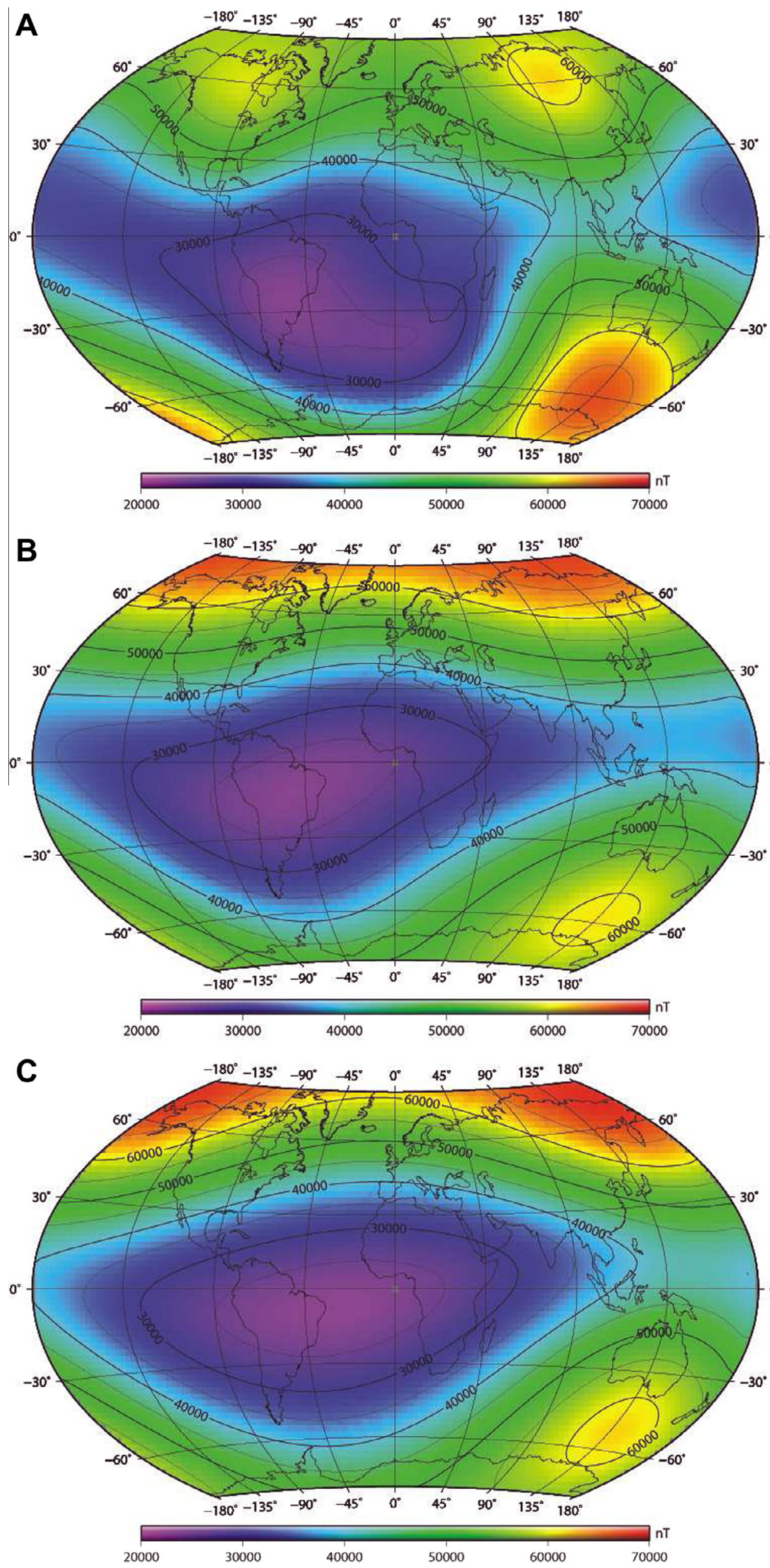

Fig. 1. The total geomagnetic field (see color scale below each panel) at the Earth surface for the epoch 2010 as calculated using the full IGRF (panel A), t-IGRF (panel B) and ED (panel C) models.

geocentric distance (the difference between IGRF and t-IGRF models is similar to this and not shown here). Shown are the maximum difference between the modeled total field values in a grid of $2.5^\circ \times 2.5^\circ$ as the absolute value (in nT) and in percent to the IGRF field in the concerned grid point, as well as the standard deviation between the models, also in absolute values and in percent. One can see that the difference is essential at the Earth's surface ($R = 1$),

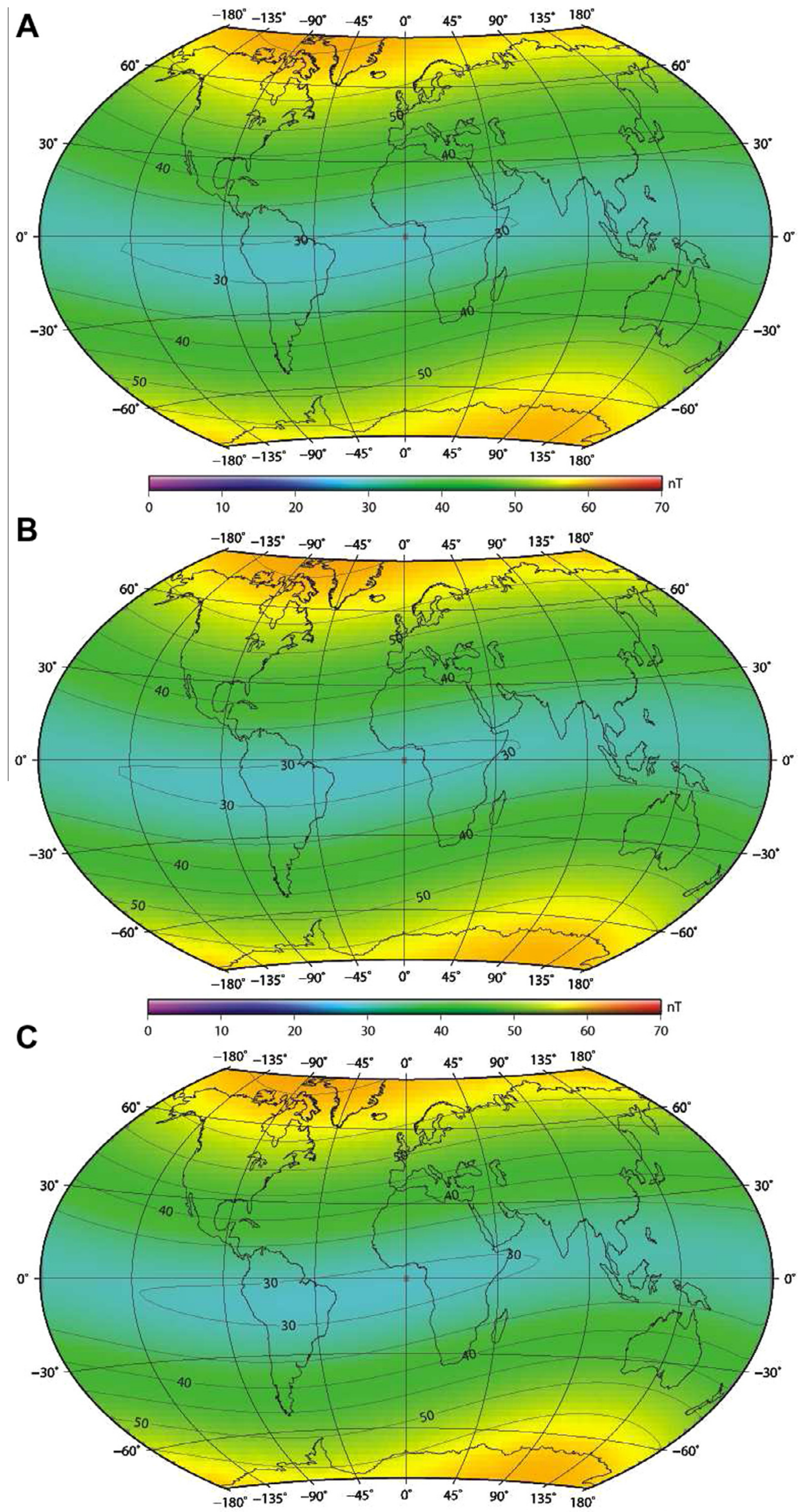

Fig. 2. The same as Fig. 1 but for the geocentric distance of 10 Earth radii.

where it may be up to 16 μT (42%) in the Central Atlantic region. The globally averaged difference, in the form of the standard deviation between the series is about 6 μT (16%). This suggests that the simplified model (eccentric dipole or a truncated IGRF) cannot adequately describe regional magnetic field distribution at the surface. However, the difference fades away quickly and the standard deviation is a few percent already at two radii distance.

Table 1
Differences in the total field between ED and IGRF models as a function of the geocentric distance R [in Earth radii]: Maximum absolute and relative difference as well as the standard deviation (SD).

R	Max (nT)	Max (%)	SD (nT)	SD (%)
1	16,156	42.3	6127	16
2	628	11.1	200	3.5
3	101	6.1	31.4	1.9
4	28.5	4.2	8.9	1.3
5	10.8	3.2	3.43	1.01
7	2.6	2.1	0.84	0.7
10	0.573	1.4	0.2	0.5
15	0.106	0.9	0.04	0.3
20	0.03	0.7	0.01	0.2

3.2. Field direction

It is not only the total field, but rather its attitude, which is important for the shielding against cosmic rays. For example, in the region of the South Atlantic (SA) magnetic anomaly the total field is weakend by a factor of 3 in comparison with the maximum field at the South magnetic pole. However, the field lines in the SA anomaly region are parallel to the Earth's surface so that charged cosmic ray particles need to go across the magnetic field to imping on the atmosphere, leading to the effective shielding (the cutoff rigidity is about 13 GV). In the polar region, on the contrary, the field lines are vertical and open to the outer space thus being even focusing cosmic ray particles downwards. The corresponding geomagnetic cutoff is zero in the polar cap region for vertically impinging particles.

The orientation of the geomagnetic field with respect to the Earth surface is given by the inclination angle, which is the angle between the magnetic field direction and its horizontal component. Thus, the zero inclination implies that the field is parallel to the surface (maximum shielding against cosmic rays), and ±90° inclination implies vertical field with no shielding. The distribution of the geomagnetic field inclination over the Globe is shown in Fig. 3 for the surface. One can see that the ED and t-IGRF models adequately (within a few degrees) reproduce the surface field inclination everywhere except for the South tropical Atlantic region where the difference may be up to 30°. However, the agreement between the different models is improving with the radial distance – the inclination is accurately (within 1–2°) reproduced by the ED and t-IGRF models already at a few Earth radii distance, even in the SA anomaly region. Another directional parameter of the magnetic field – the declination, viz. the angle of the field with respect to the North direction, does not play a role in the cosmic ray shielding and is not discussed here.

3.3. Subconclusions

Here we summarize the results of the comparison of simplified models, ED and t-IGRF, with the full IGRF model. The simplified models describe the magnetic field distribution more or less adequately everywhere at the Earth's surface, except for a region (about 40° across) in the South tropical Atlantic, where the simplified models are unable to follow the fast spatial changes in the field strength and orientation. However, the accuracy of the simplified models quickly improves with the geocentric distance, and they provide a good approximation already at a few Earth radii.

4. Geomagnetic cutoff rigidity

Here we describe the impact of the use of simplified geomagnetic models for the calculated cutoff rigidities, which quantify the shielding ability of the geomagnetic field against cosmic rays (e.g., Shea and Smart, 2001). The effective vertical cutoff rigidity (Smart and Shea, 2009) is the minimum rigidity a charged particle must possess to reach the middle atmosphere (20 km altitude) in the vertical directions. All particles with higher rigidity are considered allowed to reach the given location while all the particles with lower rigidity are considered rejected. Although this concept is a simplification of the real situation, it provides a reasonable approximation for the purpose of cosmic ray shielding as the effective cutoff rigidity takes into account the penumbra structure (Smart et al., 2000; Kudela and Bobik, 2004)

Here we performed detailed computations of the effective vertical geomagnetic cutoff rigidities for the entire Globe using the standard back-tracing approach (Smart et al., 2000). Computations of the particle trajectories were done with the PLANETOCOSMICS numerical code (Desorgher et al., 2005) using the full IGRF and the eccentric dipole models of the internal geomagnetic field. (The results for the truncated IGRF model are essentially similar to those for the ED model and are not shown here). The external field was modeled in the same way for all the internal field models, by using the model by Tsyganenko (1989) and assuming it undisturbed (geomagnetic $Kp = 0$) for simplicity (cf. Desorgher et al., 2009). The external field dominates beyond 10 Earth radii.

The spatial distribution of the calculated cutoff rigidities are shown in Fig. 4A and B, for the IGRF and ED models, respectively. One can see that the main pattern is well reproduced by both models, with only hardly observable difference in the Atlantic region. In Fig. 4C we plot the difference between the cutoff rigidities as calculated in the two geomagnetic models. The global mean difference is zero, which means that the global field is reproduced correctly by the ED model. The difference between the model, quantified as the standard deviation, is smaller than 1 GV which is small and is comparable to the changes due to the geomagnetic disturbances. The only region with noticeable difference is the tropical Atlantic, where the ED model underestimates the cutoff rigidity in the northern part (red spot a few tens of degrees across) and overestimates in the southern mirrored imaged (blue spot) by several GV. However, for studies operating with large enough

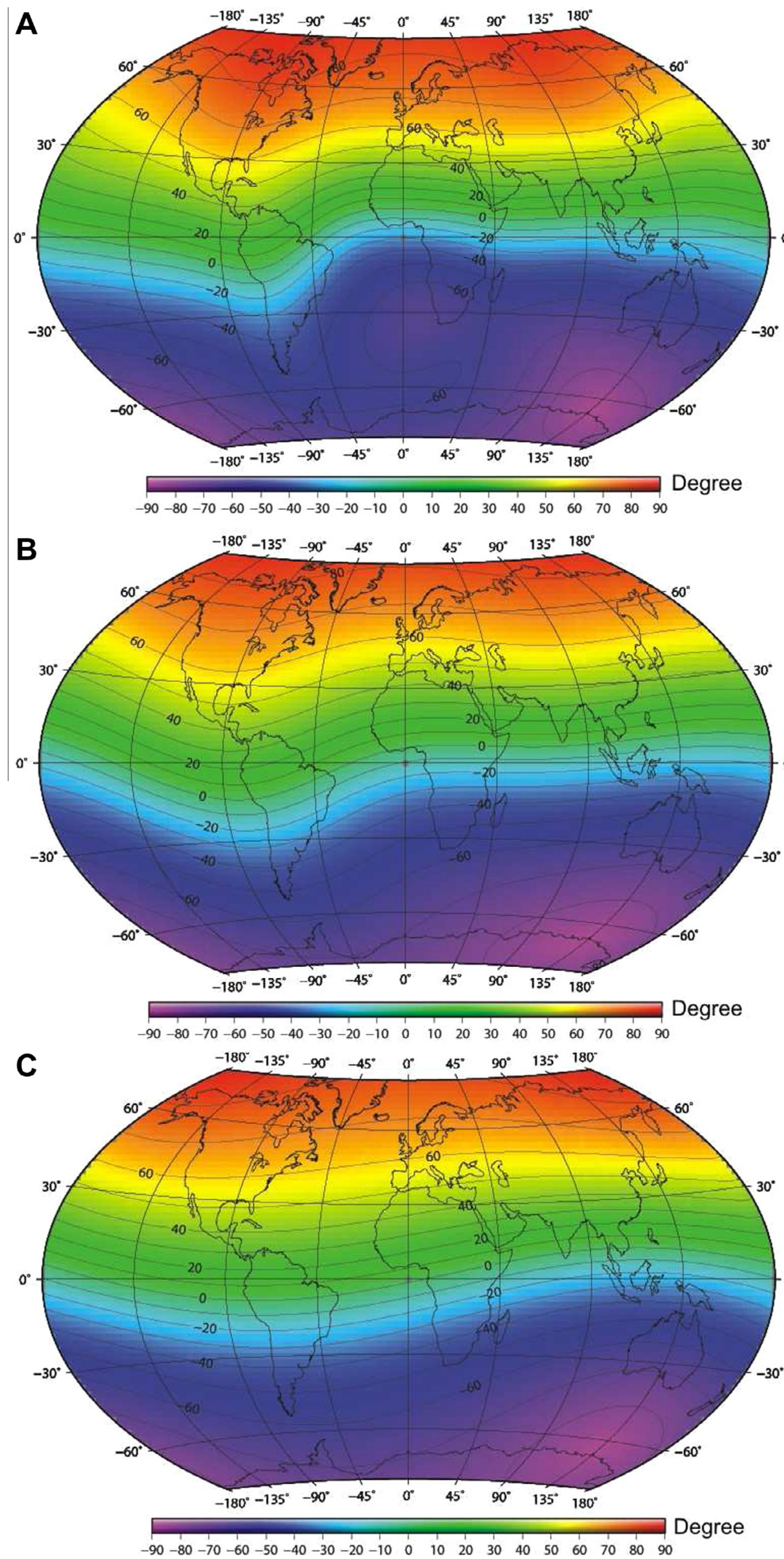

Fig. 3. The inclination angle of the geomagnetic field (see color scale below each panel) at the Earth surface for the epoch 2010 as calculated using full IGRF (panel A), t-IGRF (panel B) and ED (panel C) models.

spatial average (global or zonal mean scales), the difference between the models is not important.

Sometimes, when going further back in time on multi-millennial time scales, even this limited information on the geomagentic field can not be reliably reconstructed. Only the dipole moment and possibly the dipole's tilt can be evaluated. We note that such a model with a centered (tilted or co-axial with the geographical axis) dipole

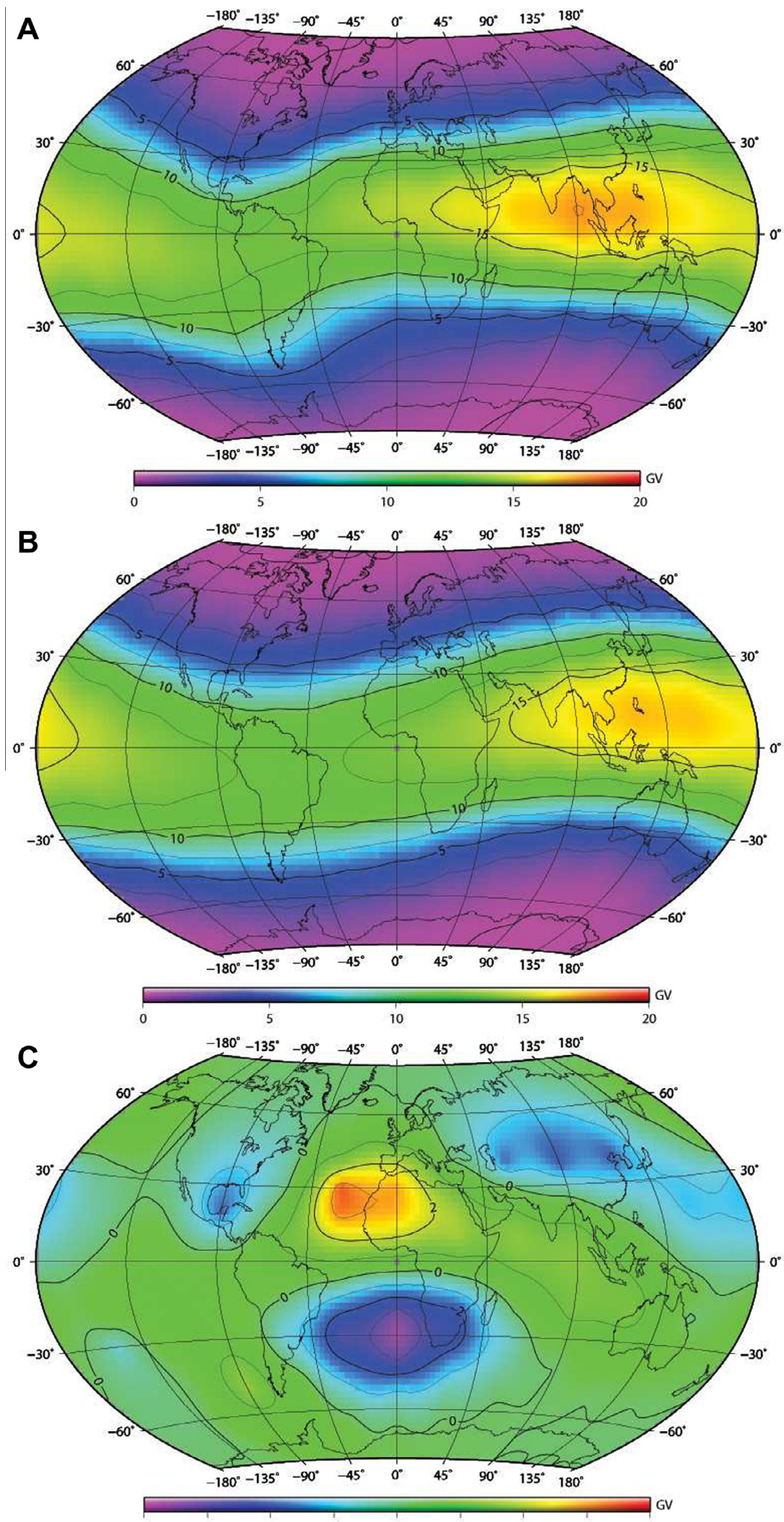

Fig. 4. The effective vertical geomagnetic cutoff rigidity R_c calculated for the epoch 2010 using the PLANETOCOSMICS code for the IGRF (panel A) and eccentric dipole (panel B) models of the internal geomagnetic field models. Panel C depicts the difference between panels A and B.

destroys information on regional scales, most importantly in the South Atlantic Anomaly region and in mid-latitude regions (Usoskin et al., 2010). Accordingly, only the global effects can be studied with such geomagnetic models.

5. Conclusions

We have performed a comparison between the full IGRF and two simplified numerical models of the geomagnetic field – the truncated IGRF (only dipole+quadrupole moments considered) and the eccentric dipole model. The external field was modeled in all the cases by the model of Tsyganenko (1989). We found that while both simplified models adequately describe the global scale magnetic field distribution, including the South Atlantic magnetic anomaly, some discrepancy exists in the Atlantic region at the Earth's surface. However, this discrepancy quickly fades away with the geocentric distance. Thus overall, the simplified models, both truncated IGRF and the eccentric dipole, do provide a reasonable approximation for the large scale geomagnetic field distribution.

We have also studied the applicability of the simplified geomagnetic models to the shielding of cosmic rays in the magnetosphere as quantified via the geomagnetic cutoff rigidity. Again, the eccentric dipole and the truncated IGRF provide a good large scale view but may disagree with the full model at smaller regional/local scales, particularly in the tropical Atlantic region. We note that for the practical applications of, e.g., production of cosmogenic isotopes in the atmosphere or cosmic ray induced ionization, the global-scale effects are not sensitive to the choice of the geomagnetic model. However, when studying local effects, in particular in the tropical region, one should keep in mind that the result may be slightly (within a few GV of the vertical cutoff rigidity) dependent on the chosen model. The largest difference is observed in the tropical Atlantic region (see Fig. 4C).

Considering that only the low order spherical harmonics of the geomagnetic field can be reliably reconstructed for the past times (Korte and Constable, 2005; Genevey et al., 2008), this study quantitatively validates the use of such simplified geomagnetic models when describing the shielding of cosmic rays in the magnetosphere. We confirm that the large-scale field contributions of dipole and quadrupole are sufficient to represent global features for cosmic ray shielding. The use of the simplified model does not introduce any additional systematic errors at the global scale but may be a source of moderate errors at the regional scale in the tropical Atlantic region.

The use of even simpler models (centered dipole only) is limited to a study of global effects since the results on the regional scale (especially in the South Atlantic Anomaly and mid-/high-latitude regions) can be significantly distorted.

Acknowledgement

This work was partly done in the framework of the EU FP7 Project SEPServer.

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