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GLOBAL HELIOSPHERIC PARAMETERS AND COSMIC-RAY MODULATION: AN EMPIRICAL RELATION FOR THE LAST DECADES

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Abstract. We study empirical relations between the modulation of galactic cosmic rays quantified in terms of the modulation potential and the following global heliospheric parameters: the open solar magnetic flux, the tilt angle of the heliospheric current sheet, and the polarity of the heliospheric magnetic field. We show that a combination of these parameters explains the majority of the modulation potential variations during the neutron monitor era 1951 – 2005. Two empirical models are discussed: a quasi-linear model and a model assuming a power-law relation between the modulation potential and the magnetic flux. Both models describe the data fairly well. These empirical models provide a simple tool for evaluating various cosmic-ray related effects on different time scales. The models can be extended backwards in time or used for predictions, if the corresponding global heliospheric variables can be independently estimated.

1. Introduction

The theory of galactic-cosmic-ray (GCR) modulation in the heliosphere is quite developed, including very sophisticated 3D models. These models have been successful when studying different aspects of cosmic-ray modulation (*e.g.*, Potgieter, Burger, and Ferreira, 2001). However, they depend on several parameters, whose values cannot be directly obtained from observations. Accordingly, it is a complicated task to fit a theoretical model to the actual cosmic-ray intensity variations measured by ground-based stations or satellites. Therefore, it is also reasonable to study more general empirical relations between heliospheric variables and cosmic-ray modulation. There are a number of empirical models relating cosmic-ray variations to different solar/heliospheric indices (*e.g.*, Bazilevskaya and Svirzhevskaya, 1998; Belov *et al.*, 2001; Stozhkov, Okhlopkov, and Svirzhevsky, 2004). Such models typically deal with the intensity of cosmic rays of a fixed energy. For example, Belov *et al.* (2001) studied variations of cosmic-ray protons with energy of 10 GeV. Their model employs twelve free parameters and gives the best correlation (of 0.95) between the observed and modeled variations of GCR of the fixed energy. However, such a model has a limited application, *e.g.*, because it cannot be directly applied to study variations of cosmic rays in different energy ranges. Often count rates of a single neutron monitor (NM) are taken to represent variations of cosmic-ray intensity (*e.g.*, Usoskin *et al.*, 1998; Sabbah and Rybanský, 2006). Although this is a reasonable approach for qualitative correlation studies, it cannot be used for a quantitative analysis because a NM is an energy-integrating device and does not measure the energy spectrum of cosmic rays. Since the effective energy varies greatly between different NMs (Alanko *et al.*, 2003), NMs at different locations have different responses to cosmic-ray modulation. Therefore, the whole range of cosmic-ray variations cannot be represented by a time series of a single NM (even worse for a fixed GCR energy). Here, we use another approach: instead of cosmic-ray variations at a given energy or measured by a single NM, we study the global modulation parameter (ϕ) which can parameterize the shape of the GCR spectrum at one AU (Caballero-Lopez and Moraal, 2004; Usoskin *et al.*, 2005). We present here empirical models which describe the observed variations of the modulation potential for the last three decades. We use data of three variables related to the global heliospheric magnetic field: the open magnetic flux, the heliospheric current sheet tilt angle, and the global field polarity. We discuss two models, a quasi-linear model and a power-law model that depend on four or five free parameters, respectively, and succeed in reproducing the observed cosmic-ray variations quite well. We also discuss possible applications and extensions of the models used.

2. Heliospheric Modulation of GCR

It is common to describe the differential-energy spectrum of galactic cosmic rays at the Earth's orbit by the so-called force-field approximation where the energy spectrum of the ith GCR species (with the charge number Z_i and the mass number A_i) at Earth's orbit (J_i) is related to the unmodulated local interstellar spectrum (LIS) of the same species, $J_{\mathrm{LIS},i}$, via the modulation potential ϕ as:

$$J_i(T,\phi) = J_{\mathrm{LIS},i}(T+\Phi_i)\frac{(T)(T+2T_{\mathrm{r}})}{(T+\Phi_i)(T+\Phi_i+2T_{\mathrm{r}})}, \tag{1}$$

where T is the particle's kinetic energy per nucleon, $\Phi_i = (eZ_i/A_i)\phi$, and $T_{\mathrm{r}} = 0.938$ GeV/nucleon is the proton's rest mass energy. The only temporally changing variable in the force-field approximation is the modulation potential (ϕ) which is related to the solar-activity variations. The force-field description of the energy spectrum (Equation (1)) provides a good single-parameter approximation of the actual shape of the GCR spectrum near Earth. We note that the force-field approach has been widely used since the 1960s to study cosmic-ray modulation (*e.g.*, Freier and Waddington, 1965; Lezniak and Webber, 1971; Urch and Gleeson, 1972; Boella *et al.*, 1998; Boezio *et al.*, 1999; Usoskin *et al.*, 2002a). On the other hand, the concept of the modulation potential is commonly used in various applications, *e.g.*, cosmic-ray-induced ionization (Pallé, Butler, and O'Brien, 2004; Usoskin, Gladysheva, and Kovaltsov, 2004), production of cosmogenic isotopes in the Earth's atmosphere (O'Brien, 1979; Webber and Higbie, 2003; McCracken, 2004; Usoskin and Kovaltsov, 2004) and in meteorites (Michel, Leya, and Borges, 1996). While the physical motivation for the force-field approximation has been given elsewhere (Gleeson and Axford, 1968; Caballero-Lopez and Moraal, 2004), for the purpose of the present study it is important that the modulation theory predicts that $\phi \propto 1/\kappa$, where κ is the effective radial diffusion coefficient of GCR transport in the heliosphere.

It should be noted that, in addition to the temporally-changing parameter ϕ, the force-field approximation (Equation (1)) includes also a fixed parameter, the local interstellar spectrum (J_{LIS}), whose exact shape is not known and may vary in different approaches (*cf.* Burger, Potgieter, and Heber, 2000; Moskalenko *et al.*, 2002; Webber and Higbie, 2003). Therefore, the exact value of the modulation potential ϕ makes sense only for a fixed J_{LIS} (see details in Usoskin *et al.*, 2005). In the present work, we use the proton's local interstellar spectrum given by Burger, Potgieter, and Heber (2000):

$$J_{\mathrm{LIS}}(T) = \frac{1.9 \times 10^4 \cdot P(T)^{-2.78}}{1 + 0.4866\, P(T)^{-2.51}}, \tag{2}$$

where $P(T) = \sqrt{T(T + 2T_r)}$. J and T are expressed in units of particles/($\mathrm{m}^2\,\mathrm{sr\,s\,GeV/nucleon}$) and in GeV/nucleon, respectively. The LIS of α particles has been taken to be of the same shape with the intensity being scaled (see Usoskin *et al.*, 2005 for details).

We note that the force-field model cannot physically explain all features of GCR modulation. In particular, it overlooks such important mechanisms as propagating diffusive barriers, cosmic-ray acceleration at the termination shock, or the effect of the heliospheric geometry, propagation and deformation of the heliospheric current sheet, *etc.* (Wibberenz *et al.*, 1998; Fisk *et al.*, 1998; Ferreira and Scherer, 2006). On the other hand, the force-field model presents a useful way to parameterize the CR spectrum at one AU (Usoskin *et al.*, 2005). Here we adopt this latter approach. We also note that this approach can be applied only to the NM energy range (above about 1 GV) but becomes invalid at lower energies.

The monthly series of the modulation potential was recently calculated for the period 1951 – 2004 by Usoskin *et al.* (2005), fitting the GCR spectra of the force-field model (Equation (1)) to the data from the world-wide neutron

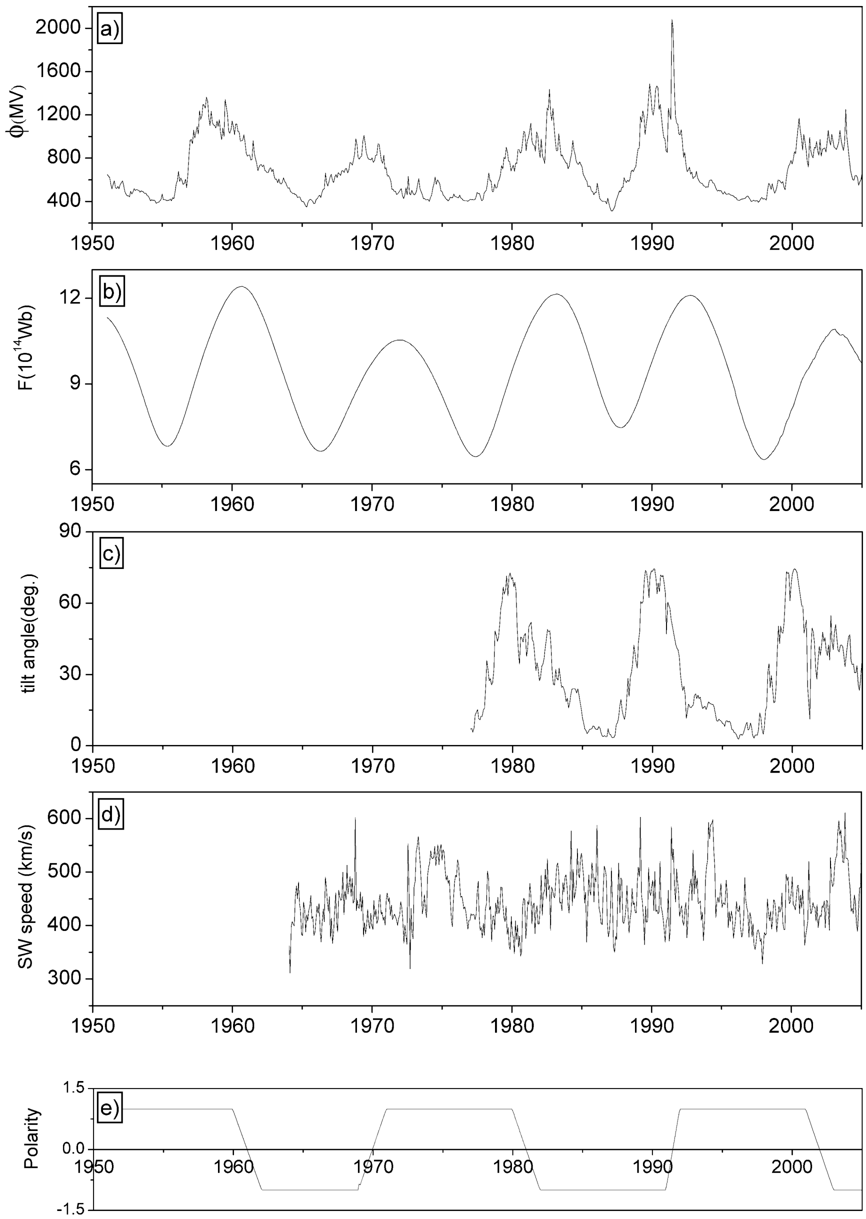

Figure 1. Monthly values of the heliospheric indices. (a) Heliospheric modulation potential (ϕ). (b) Open solar magnetic flux (F). (c) Heliospheric current sheet tilt angle (α). (d) Solar wind speed. (e) Polarity of the heliospheric magnetic field (p).

monitor network. Here, we use values of ϕ extended until the end of 2005 (see *http://cosmicrays.oulu.fi/phi*). This series is shown in Figure 1a and is called here the experimental series (ϕ_{exp}).

3. Global Heliospheric Parameters

The modulation of GCRs in the heliosphere is related to the following processes: particle diffusion along and perpendicular to the heliospheric magnetic field (HMF); gradient, curvature, and heliospheric current sheet (HCS) drift effects; and convection and adiabatic energy losses in the expanding solar wind. These processes are defined by the geometrical structure, polarity, strength, and turbulence level of the HMF and the solar-wind speed. Since these heliospheric characteristics vary in time depending on the phase of the solar magnetic cycle and on the solar activity level, the resultant modulation of GCRs is also varying in time. Unfortunately, not all of these heliospheric variables can be directly measured or even reliably evaluated. When constructing our empirical models, we put the following requirements on the heliospheric variables to be included in the model. First, the values of the variables used in the model should have been monitored for a relatively long time, at least during the last few decades. Second, the variables should represent the global heliosphere rather than a very local state of the interplanetary medium.

Based on these requirements, we refrain from using the solar-wind speed data measured in the Earth's vicinity. Although the solar wind is responsible for important processes for the cosmic-ray modulation (convection and adiabatic deceleration), the direct correlation between cosmic-ray variations and the solar-wind speed as measured near the Earth is quite weak (*e.g.*, Belov, 2000; Sabbah and Rybanský, 2006). Similarly, we have found only an insignificant correlation between the modulation potential and the solar-wind speed. This is due to the fact that the solar-wind speed at one AU in the ecliptic plane varies rather weakly over the solar cycle (see Figure 1d), while the solar-wind speed is highly variable at higher heliolatitudes (*e.g.*, McComas, Bame, and Barraclough, 1998, 2002). This indicates that the locally-measured solar wind does not well represent the global properties of the solar wind. Therefore, we do not include the solar-wind speed in our model.

In theoretical considerations, it is often assumed that the cosmic-ray diffusion coefficient, κ, depends on the HMF strength as $\kappa \propto B^{-n}$ (Wibberenz, Richardson, and Cane, 2002; Caballero-Lopez *et al.*, 2004). Keeping in mind that $\phi \propto 1/\kappa$, one can expect that the modulation potential depends on the HMF strength as $\phi \propto B^n$, where the HMF strength should be global. Similar to the solar wind, the values of B measured in the Earth's vicinity are quite local. As a global proxy for the HMF, one can use the solar magnetic flux through the source surface (Lockwood, Stamper, and Wild, 1999; Solanki, Schüssler, and Fligge, 2000). The magnetic flux, thus defined, is related to the global dipole component of the solar magnetic field (Mackay, Priest, and Lockwood, 2002). Therefore, following the approach developed by Usoskin *et al.* (2002b), we use the open solar magnetic flux (F), averaged over the source surface (Solanki, Schüssler, and Fligge, 2000, 2002), as the index of the global HMF (see Figure 1b). Since the open flux is roughly proportional to the HMF strength, we expect the following dependence of the modulation potential

$$\phi \propto F^n. \tag{3}$$

Another variable related to the global heliospheric structure is the tilt angle of the heliospheric current sheet, α, which affects the cosmic-ray modulation (Jokipii and Thomas, 1981; Kóta and Jokipii, 1983). The structure of the HCS strongly depends on the phase of the solar cycle. The magnetic axis of the Sun is tilted with respect to rotational axis, and the tilt varies over a solar cycle. Together with the Sun's rotation and radially expanding solar wind, the tilt produces a complicated 3D structure of the HCS, the so-called ballerina skirt. In an axisymmetric approximation, which can be obtained by using 27-day averaging, the global waviness of the HCS is defined by the HCS tilt angle, which roughly corresponds to the tilt of the solar dipole axis with respect to the rotational axis. During periods of strong solar activity, *i.e.*, around the polarity reversal of the solar magnetic field, the dipole axis is strongly tilted and the current sheet is disrupted. During periods of minimum solar activity, the tilt angle is small and structure of the sheet is more regular. The values of the HCS tilt angle (α) used in this study (shown in Figure 1c) have been obtained from the Wilcox Solar Observatory since 1976 (newer model, obtained using the radial boundary condition). Note that the tilt angle values above 70° cannot be reliably measured due to the observation technique, and $\alpha \approx 70^\circ$ actually indicates only a lower bound.

One more global variable related to the structure of the heliosphere and the GCR modulation is the overall solar-magnetic polarity which alternates every solar cycle. The polarity defines the direction of drifts affecting cosmic-ray transport in the heliosphere and leading to the well-known 22-year cycle in GCR intensity (Jokipii, Levy, and Hubbard, 1977). For instance, during the negative-polarity periods, cosmic rays enter the Earth's orbit both from the polar regions and along the current sheet. During positive-polarity periods, CR particles are effectively driven away from the inner heliosphere along the HCS. These drifts are not included in the force-field (or any other spherically symmetric) model. Accordingly, we have included the polarity (drift) effects explicitly. Since the drifts effectively facilitate/hamper the access of GCRs into the inner heliosphere, we include them as a modification of the effective relation between F and ϕ, so that it is slightly different for positive and negative magnetic cycles. The HMF polarity is known for the last century and is parameterized by means of the variable p: $p = 1$ (-1) for positive (negative) polarity periods (see Figure 1e). Around the polarity reversal epoch, p is linearly interpolated within 20 months between 1 and -1, with $p = 0$ corresponding to the center of the reversal period (taken as mid-1959, mid-1970, mid-1980, mid-1991, late 2001).

4. Regression Models

In this section, we introduce two regression models relating the modulation potential ϕ to the above discussed global heliospheric variables (see Figure 1): HMF flux

(F), HCS tilt angle (α), and the global HMF polarity (p). We have determined such a set of parameter values that minimizes, for the period 1976 – 2005, the RMS log-discrepancy between the experimental and model ϕ values:

$$\epsilon = \sqrt{\frac{1}{n} \sum_{n} \left(\ln(\phi_{\text{exp}}) - \ln(\phi_{\text{model}})\right)^2}. \tag{4}$$

We note that the log-discrepancy corresponds to the mean relative error $\delta\phi \equiv \frac{\Delta\phi}{\phi} \approx \exp\epsilon - 1$. Therefore, the least log-discrepancy implies the least relative error. Two periods of very strong Forbush decreases (June – July 1991 and October 2003) were removed from the data sets used for model fitting because Forbush decreases are caused by transient interplanetary shocks and are not directly related to the global heliospheric indices.

We note that contrary to the theoretical expectations (see Equation (3)) discussed earlier, there is no simple relation between the modulation potential (ϕ) and the open magnetic flux (F). A scatter plot of annual values of ϕ vs. F is shown in Figure 2 separately for the periods with nearly flat and highly tilted HCS, corresponding to low and high solar activity. While it is apparent that the relation depends on the tilt angle, the experimental data do not allow us to distinguish between the two alternative models to be discussed here. The exponent n in Equation (3) is often supposed to be 1 (*e.g.*, Le Roux and Potgieter, 1995; Kóta and Jokipii, 2001), thus leading to a linear relation between ϕ and F. This assumption is considered in our first model, called the quasi-linear model, with the proportionality coefficient

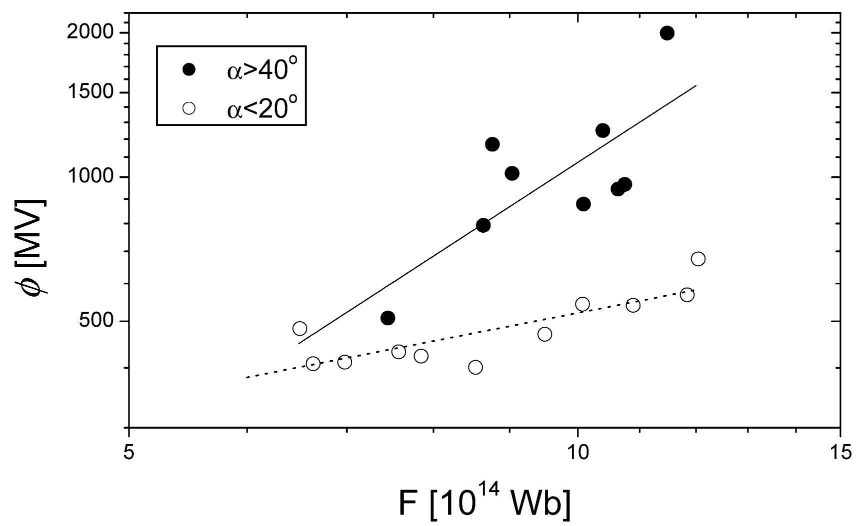

Figure 2. Scatter plot of the annual ϕ vs. F values. *Filled* and *open dots* correspond to the periods with high (>40°) and low (<20°) values of the tilt angle (α), respectively. *Lines* depict the best-fit power-law relation.

depending on α. On the other hand, it has been shown that n may differ significantly from one (Caballero-Lopez *et al.*, 2004) and may vary in time with the phase of the solar cycle (Wibberenz *et al.*, 2001; Ferreira and Potgieter, 2004). Ferreira and Potgieter (2004) suggested that n may be a function of α. Accordingly, our second model, called the power-law model, assumes the relation between ϕ and F in the form of Equation (3) with $n = n(\alpha)$. We note that both quasi-linear (*e.g.*, Belov, 2000) and non-linear (*e.g.*, Bazilevskaya and Svirzhevskaya, 1998) empirical models have been used in earlier studies.

First, we consider the *quasi-linear model*, which linearly sums up different effects:

$$\phi = \phi_0 \left[1 + \frac{F}{F_0} \left(1 + \frac{\alpha}{\alpha_0} + \beta p \right) \right]. \tag{5}$$

The first term in the innermost parentheses is proportional to F and corresponds to the diffusion modulation, purely determined by the HMF intensity. The second and third terms, depending on the product of the HMF intensity with the tilt angle and the HMF polarity, roughly correspond to tilt-dependent part of modulation and drifts, respectively. We note that this model has four free formal parameters: ϕ_0, F_0, α_0, and β. The best set of formal parameter values, minimizing the ϵ discrepancy, is $\phi_0 = 160\,\mathrm{MV}$, $F_0 = 5.25 \times 10^{14}\,\mathrm{Wb}$, $\alpha_0 = 35^\circ$, $\beta = -0.09$. The least log-discrepancy is $\epsilon = 0.151$. The model ϕ calculated using these parameters is shown in Figure 3a.

The *power-law model* is taken to have the following form:

$$\phi = \phi_0 + \phi_1 \left(\frac{F}{F_0} \right)^{1+\alpha/\alpha_0} (1 + \beta p). \tag{6}$$

This model has five parameters, whose best values are $\phi_0 = 150\,\mathrm{MV}$, $\phi_1 = 86\,\mathrm{MV}$, $\alpha_0 = 91^\circ$, $F_0 = 2.5 \times 10^{14}\,\mathrm{Wb}$, $\beta = -0.03$. The modeled ϕ series is shown in Figure 3b. The log-discrepancy is $\epsilon = 0.154$.

Both modeled time series of ϕ follow the experimental ϕ fairly well for most of the time with the overall correlation coefficient of about 0.90 and the average deviation of $\Delta\phi \approx 90\,\mathrm{MV}$. However, there are several relatively-short periods when a significant difference between the model and the experimental ϕ values is observed: around 1980, 1991, 1992 – 1994, and around 2004. Most of these periods correspond to very strong solar transient phenomena (*e.g.*, Forbush decreases), which cannot be modeled using global heliospheric indices. We have also tested that introducing time lags between the variables does not yield statistically better results, but makes the models more complicated. Therefore, we do not include time lags in the present models.

Of special interest is the offset ϕ_0, which seems to attain a very similar value for both models. It does not have a physical meaning but includes an uncertainty related to the local interstellar spectrum of GCRs, J_{LIS}, used in the force-field approximation in Equation (1) (see details in Usoskin *et al.*, 2005).

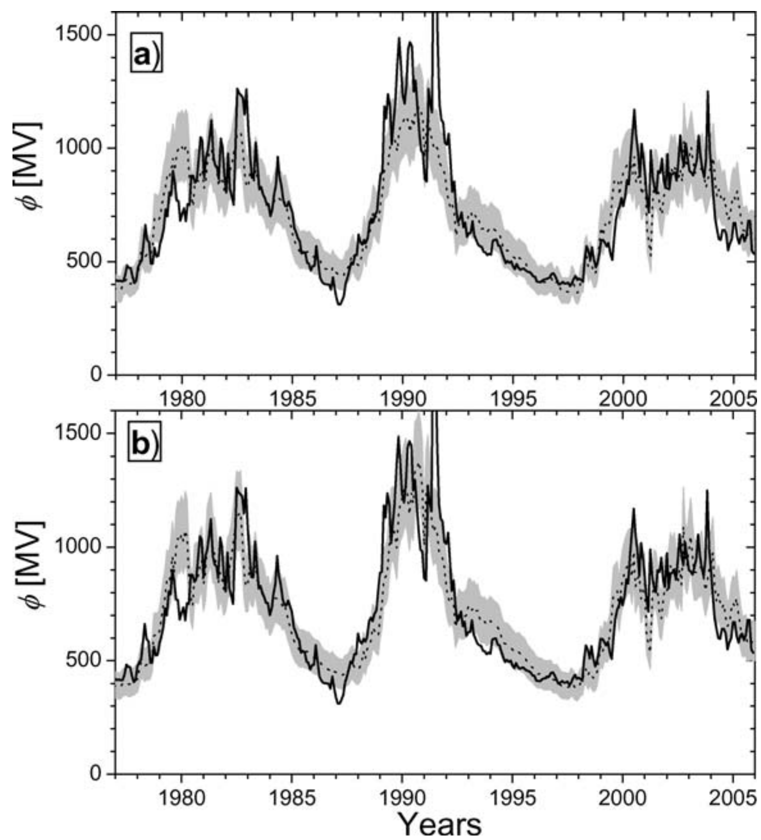

Figure 3. Monthly values of the heliospheric modulation potential ϕ. *Panels* (a) and (b) correspond to the quasi-linear and power-law models, respectively. *Solid line* represents the experimental ϕ values, while *dotted lines* with *grey shading* depict the reconstructed modulation potential with the 68% confidence intervals.

5. Neutron Monitor Count Rate

As a test of the applicability of the proposed approach, we have computed, using the modeled modulation potential (ϕ), the count rates of a neutron monitor and compared them with the actual measurements. The neutron monitor count rate (N) can be presented as a sum of count rates N_i due to different species i of GCR:

$$N = \sum_i N_i = \sum_i \int_{T_{ci}}^{\infty} J_i(T, \phi) Y_i(T) \mathrm{d}T, \tag{7}$$

where Y_i is the specific yield function of the NM for the species i of GCR, T_{ci} is the kinetic energy corresponding to the local geomagnetic cutoff rigidity. The following simple empirical relation has been found between the NM count rates and the modulation potential (Usoskin *et al.*, 2005):

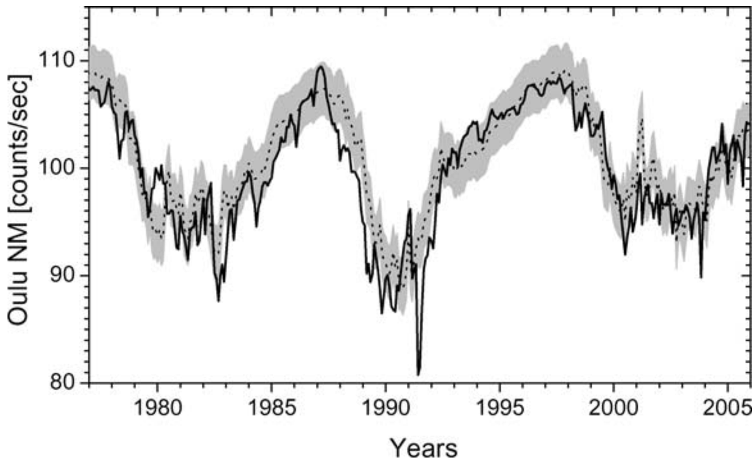

Figure 4. Monthly values of the Oulu NM count rate, actual (*solid line*) and reconstructed by the power-law regression model (*dotted lines* with *grey shading*, corresponding to the 68% confidence intervals). The result for the quasi-linear law model (not shown) is very similar.

$$N = N_0 \cdot \left(1 + \frac{1}{\mathcal{A}\phi + \mathcal{B}}\right), \tag{8}$$

where the coefficients N_0, $\mathcal{A}$, and $\mathcal{B}$ are defined individually for each neutron monitor, taking into account their different effective area, altitude, and geomagnetic cutoff rigidity. For example, the coefficients for the Oulu NM (polar, sea-level, 9-NM64 type) are $N_0 = 47.97$ counts/s, $\mathcal{A} = 3.99 \times 10^{-4}\,\mathrm{MV}^{-1}$, $\mathcal{B} = 0.6343$. The coefficients for the Climax NM (mid-latitude, altitude 3400 m above the sea level, IGY type) are $N_0 = 47$ counts/s, $\mathcal{A} = 3.704 \times 10^{-4}\,\mathrm{MV}^{-1}$, $\mathcal{B} = 0.503$. The count rates computed in this way for the Oulu NM are shown in Figure 4 together with the actual measurements for the period 1976 – 2005. The two curves are in good agreement with each other: the cross-correlation coefficient is 0.91, and the mean relative deviation is about 2%. Periods of notable difference correspond to the specific periods discussed in Section 4. We have checked that the agreement is equally good also for other NM stations (*e.g.*, Climax, Hermanus, Kerguelen, Kiel, Rome), thus confirming the validity of the approach. The agreement is equally good for the quasi-linear model (not shown here).

6. Extension for the Last 50 Years

The shortest time span is covered by the tilt angle (α), whose observed values are available only since 1976, while the experimental ϕ values have been reconstructed since 1951. Here we try to extend the above models for the period 1951 – 1976, using a simple approximation for the solar cycle dependence of the HCS tilt angle.

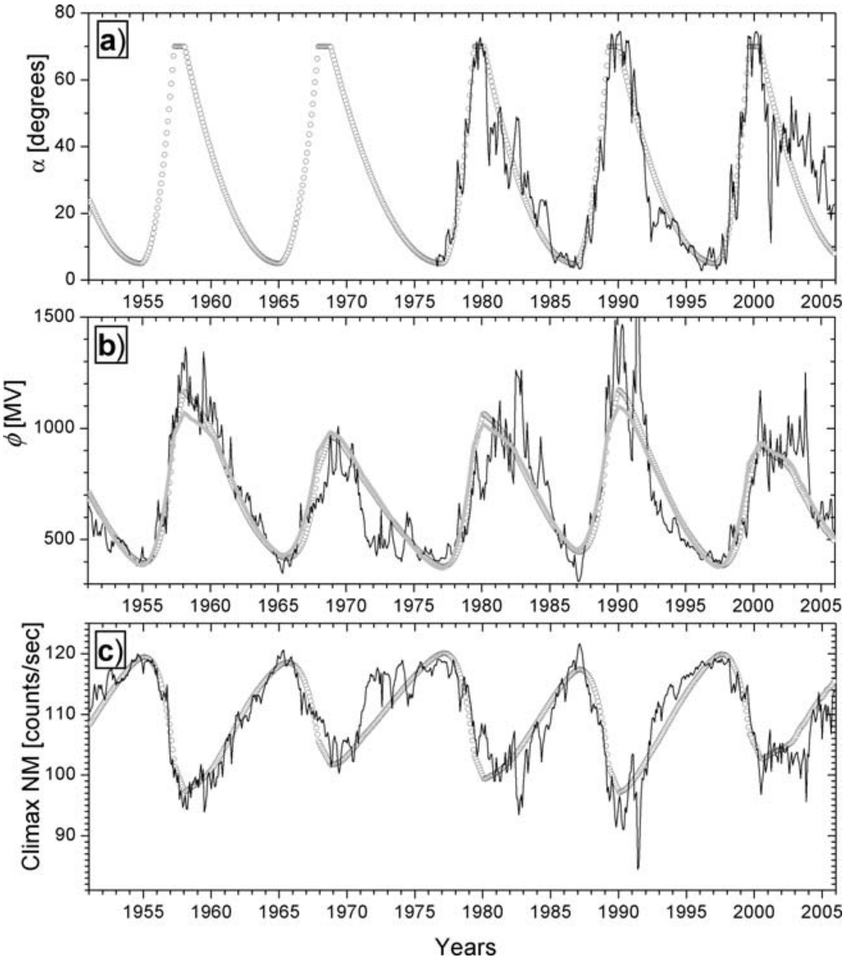

Figure 5. (a) Monthly values of the tilt angle (α): *solid line* depicts the actual data, while the *small circles* corresponds to the model α cycle (see Equation (9)). (b) Modulation potential (ϕ): actual values are depicted by the *solid line*, while the *small circles* and *grey lines* correspond to the reconstructions, based on the model α cycle, by means of the power-law and quasi-linear regression models, respectively. (c) Count rate of the Climax NM: actual is shown by the *solid line*, while the count rate computed from the power-law model (see *panel b*) is shown by the *small circles*.

First we note (*cf*. Cliver and Ling, 2001) that the solar-cycle variation of α can be roughly represented by a simple cyclic shape (see Figure 5a), which is defined solely by the cycle phase (x) and is approximated as follows:

$$\alpha = \begin{cases} 5^\circ + 1100\,x^2, & x \le 0.24, \\ 70^\circ, & 0.24 < x \le 0.32, \\ 5^\circ + 140\,(1-x)^2, & x > 0.32 \end{cases} \tag{9}$$

The phase (x) takes the values from 0 to 1 between two successive cosmic ray minima, which are defined here as being seven months delayed with respect to the dates of sunspot minima. This approximation ensures a smooth connection between the cycles. One can see that this approximation, shown by dots in Figure 5a reasonably describes the smooth behaviour of the observed α values, particularly for the ascending phase of a cycle. In the declining phase, especially after 2002, there are larger deviations due to the excursions of the tilt.

Using this simulated cyclic α series, we have applied both models to reconstruct the modulation potential for the period 1951 – 2005 (Figure 5b). The agreement between the modeled and the experimental ϕ is fairly good (cross-correlation of 0.86 and 0.87 for the quasi-linear and power-law models, respectively), keeping in mind the simple approximation for the HCS tilt-angle cycle. We have also compared the count rate of the Climax NM (which provides the longest series of cosmic-ray observations since 1951), computed from this reconstructed ϕ series, with the real data (Figure 5c). (Only power-law model is shown as the quasi-linear model is very similar.) Again, the agreement is quite good, except for a few specific periods discussed in Section 4, and the famous "mini-cycle" in the cosmic-ray modulation during 1972 – 1974 caused by an unusual heliospheric structure (*e.g.*, Usoskin *et al.*, 1998; Wibberenz and Cane, 2000; Wibberenz *et al.*, 2001). This further verifies the validity of the used concept of the HCS tilt-angle cycle.

7. Discussion and Conclusions

We have presented two empirical models, the quasi-linear model (Equation (5)) with four free parameters and the power-law model (Equation (6)) with five parameters, to describe the temporal behaviour of the modulation potential (ϕ) during the last three decades. The models studied here incorporate only three variables corresponding to the global heliosphere: the tilt angle (α) of HCS, the global HMF flux (F), and the global HMF polarity (p). Both models reproduce the observed variations of the modulation potential (ϕ) with a reasonable accuracy for the last three solar cycles. A few short periods when the modeled and observed values disagree correspond to the known very strong transient phenomena or unusual heliospheric structures, which cannot be adequately represented with the force-field approximation. An extension of the models with simulated (Equation (9)) HCS tilt angle confirms the validity of the present approach. A slightly better formal agreement could be achieved at the cost of an increased number of free parameters (*cf.* 12 free parameters used by Belov, 2000). However, the main aim of this study was to find a rather simple model, which would be stable and independent of the fitting period and be based on only the most important heliospheric variables. We have presented here two models, a quasi-linear model and a power-law model, which yield roughly equally good fits to the data.

The discussed empirical models allow for a quick, rough estimate of the cosmic-ray modulation on different time scales. Potentially, the model can be extended backwards in time or used for predictions, if the corresponding heliospheric variables can be independently estimated.

Concluding, the empirical models presented here provide a simple tool for evaluating various cosmic-ray-related effects for different applications on different time scales. Finally, we note that the empirical models presented here have their limitations as they do not take into account transient, non-stationary, or spherically asymmetric processes. The models are applicable only for the neutron monitor energy range (above 1 GV) and may become invalid for cosmic rays with lower energy.

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