A 2D stochastic simulation of galactic cosmic rays transport in the heliosphere

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Abstract

We present a new code to numerically simulate the transport of galactic cosmic rays in a 2D axisymmetric heliosphere. The model solves the transport equation by stochastic simulation techniques, tracing trajectories of test particles in a guiding center approximation. It includes such basic modulation mechanisms as diffusion, convection and adiabatic cooling. We present the first results from this model and compare the effects of the different modulation mechanisms in the model.

1. Introduction

Galactic cosmic rays suffer from modulation in the heliosphere. Basic modulation mechanisms are diffusion, convection, adiabatic cooling and drifts. We have earlier presented a basic, spherically symmetric 1D-model of the transport of galactic cosmic rays in the heliosphere [4]. Since recent experimental results imply the importance of heliolatitudinal effects, we have developed the 1D-model into a more sophisticated 2D-model. The present model includes diffusion in both radial and latitudinal direction. Drifts will be included in the model in the near future.

2. Methods

Transport of cosmic rays in the heliosphere is described in 2-dimensional case as [1]:

\[
\frac{\partial U}{\partial r} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 K_{rr} \frac{\partial U}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( K_{\theta \theta} \sin \theta \frac{\partial U}{\partial \theta} \right) + \\
+ \frac{1}{3r^2} \frac{\partial}{\partial r} \left( r^2 V \right) \frac{\partial}{\partial T} (\alpha TU) - \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 VU \right)
\]

where \( \theta \) is the heliolatitude, \( U \) is the cosmic ray number density per unit interval of kinetic energy, \( T \) is the particle’s kinetic energy, \( \alpha = (T + 2T_R)/(T + T_R), T_R \) is...
the particle’s rest energy, and \( V \) is the solar wind velocity. Diffusion coefficients in radial and latitudinal direction are:

\[
K_{rr} = K_{\parallel}(\cos \Psi)^2 + K_{\perp}(\sin \Psi)^2, \quad K_{\theta\theta} = K_{\perp}
\]

(2)

where \( \Psi \) is the angle between the solar direction and the interplanetary magnetic field (IMF). The parallel and perpendicular (to the IMF direction) diffusion coefficients \( K_{\parallel} \) and \( K_{\perp} \) are defined as [3]

\[
K_{\parallel} = K_0\beta K_p(P) \frac{B_E}{B}, \quad K_{\perp} = (K_{\perp})_0 K_{\parallel}
\]

(3)

where \( K_0 \) is the diffusion coefficient, \( \beta = v/c \), \( B \) and \( B_E \) are the standard IMF strengths at a given heliodistance and at 1 AU, respectively. The \( K_p(P) \) is given by [2]:

\[
K_p(P) = \begin{cases} 
\beta \cdot P_C, & P \leq P_C \\
\beta \cdot P, & P > P_C
\end{cases}
\]

where \( P_C = 1000 \text{MV} \).

Modulated galactic cosmic ray spectra at the Earth’s orbit can be presented as

\[
G(T, t) = \int_T^\infty G_{\text{LIS}}(T_B) \cdot M(T_B, T, t) \cdot dT_B
\]

(4)

where \( G_{\text{LIS}}(T_B) \) is the local interstellar spectrum, \( M(T_B, T, t) \) is the modulation function, which gives the probability of a particle with initial energy \( T_B \) to be found at the Earth’s orbit with energy \( T \). Here we assume the standard Parker spiral magnetic field and radially constant solar wind velocity and solved the transport equation (Eq. 1) numerically by stochastic simulation techniques. This method allows to study different modulation effects and galactic cosmic ray spectra by tracing individual test particle’s path through the heliosphere. We traced each test particle from the heliospheric boundary to the Earth’s orbit. Particles were ‘killed’ if their energy diminished below 400 MeV. At the distance of 0.1 AU, there is a mirror reflecting particles back. The size of the heliosphere was taken to be 100 AU.

### 3. Results

Some test particle paths for injection energies \( T_B = 10 \text{ GeV} \) and \( T_B = 1 \text{ GeV} \) are shown in Fig. 1 for low modulation and medium modulation conditions corresponding roughly to modulation strengths \( \Phi = 350 \text{ MV} \) and \( 700 \text{ MV} \) in 1D case, respectively). For 1 GeV particles, every 1000th steps is drawn, for 10 GeV particles every 200th step. One can see that the path of a low energy particle is very random, particle travels a long time in the heliosphere before reaching the
Earth. For an energetic particle the path is more straightforward and particle may reach the Earth quite fast. We note also that particles first diffuse for the long time in the outer heliosphere ($R > 10\,\text{AU}$) and then quickly reach 1 AU.

We have also calculated the distribution of the time spent by GCR in the heliosphere (Fig. 2). We calculated about $10^5$ particles to estimate the average behavior. Since the CPU time increases considerably with increasing modulations the statistics is lower for medium modulation. Each dot in Fig. 2 corresponds to one registered particle. Expectedly, the time spent in the heliosphere increases by increasing modulation. Still, the shape of the cloud of particles seems to remain the same but shifts upwards.

We studied the energy losses of an initially monoenergetic flux of particles with $T_B = 10\,\text{GeV}$ and $T_B = 3\,\text{GeV}$ also at low and medium modulation conditions (Fig. 3.). Note that these curves exactly correspond to the modulation function $M$ in Eq. 4.

4. Discussion

We have developed our earlier stochastic simulation 1D model of galactic cosmic ray transport in the heliosphere into a 2 dimensional model. As first results of the model, we have presented some test particles paths in the heliosphere at different energies and modulation conditions. We have also estimated the time that particles spend in the heliosphere and the modulation function of a galactic cosmic ray particle. As the next step we include drifts into the model. This work is already under development.
Fig. 2. Time spent in the heliosphere

Fig. 3. Modulation function

References
1. Parker 1965, Planetary Space Sci. 13, 9