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Monte-Carlo approach to Galactic Cosmic Ray propagation in the Heliosphere

M.Gervasi[a]* , P.G.Rancoita[a] , I.G.Usoskin[a,b]† and G.A.Kovaltsov[b]

[a]INFN sezione di Milano, via G.Celoria 16, I-20133 Milano, Italy

[b]A.F.Ioffe Physical-Technical Institute, 194021 St.Petersburg, Russia

In the present paper we consider a possibility of using stochastic simulation (Monte-Carlo) technique approach to the study of Galactic Cosmic Ray propagation in the Heliosphere. We developed a technique for calculation of the cosmic ray propagation in a spherically symmetric steady state approximation of the Heliosphere. A comparison of the calculation results with those obtained by other methods as well as with an analytical approximation shows a good agreement. Besides, in the frameworks of the approximation used, we calculated the solar modulation of monoenergetic fluxes of Galactic Cosmic Rays entering the Heliosphere, in the particle's energy range 0.1 - 15 GeV. We studied the details of the modulation in their dependence of the initial particle's energy. In particular, a linear scaling of particle's energy losses *vs.* diffusion time is shown.

1. Introduction

Study of transport of Galactic Cosmic Rays (GCR) in the Heliosphere is of great interest and importance. At present, there is no opportunity to measure GCR *in situ*, outside the Heliosphere. Therefore, it is important to know the details of solar modulation of GCR. On the other hand, knowledge of GCR transport processes might help in study of the heliospheric properties.

During last decades, the study of GCR transport in the Heliosphere has been improved and many models have been developed. Simple spherically- symmetric steady state ones (*e.g.* [1] and references therein) are good enough for a study of global modulation processes, while very sofisticated 2D and 3D time-dependent models (*e.g.* [2–4]) are used for study of fine short-time scale processes. All the models developed so far use various kind of finite differences numerical techniques.

Since the equation of GCR transport in the Heliosphere takes a form of Fokker-Plank equation (see eq.(1) below), one can apply a very flexible Monte-Carlo techniques to solve it (*i.g.* [5–7]). Recently, since the power of computers increased significantly, Monte-Carlo techniques have been applied to a number of astrophysical problems: transport of solar particles in the solar atmosphere (*e.g.* [8–10]), interplanetary space [11–14], particles' stochastic acceleration [5,15]. In the present study we use, at the first time, Monte-Carlo approach to GCR transport in the Heliosphere. The method is described in Section 2.

In Section 3 we show that the stochastic simulation techniques reproduces adequatelly a spherically symmetric steady state model of the Heliosphere.

An important advantage of Monte-Carlo techniques is that one can use a monoenergetic flux as the initial spectrum of GCR protons. This allows one to obtain Green functions of the process, making it easy to obtain the modulated spectrum for any kind of assumed local interstellar spectrum (LIS). Besides, the use of monoenergetic fluxes allows us to study the details of modulation (such as time spent by a particle inside the Heliosphere or average energy loss) in dependence of the galactic proton's energy (see Section 4).

In the present paper we present the first results of application of Monte-Carlo approach to the problem of GCR transport in the Heliosphere. We apply the method to a spherically symmetric steady state model and demonstrated that our

*and Dept.of Physics - University of Milano
†on leave from

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approach works adequatelly for this approximation.

2. Cosmic ray transport in the Heliosphere

Transport of GCR in the Heliosphere is described by the Fokker-Plank equation which can be written in the spherically symmetric case as [16,17]:

$$\frac{\partial U}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\kappa\frac{\partial U}{dr}) - \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 VU) + \\ +\frac{1}{3}\cdot\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V)\right)\cdot\left(\frac{\partial}{\partial T}(\alpha T U)\right) \quad (1)$$

where $U(r, T, t)$ is the cosmic ray number density per unit interval of kinetic energy T per nucleon, r - distance from the Sun, V - velocity of the radially directed solar wind, T - particle's kinetic energy per nucleon, κ - diffusion coefficient, $\alpha = \frac{T+2\cdot T_r}{T+T_r}$, T_r - proton's rest energy. This equation includes three major processes of GCR propagation in the Heliosphere : diffusion, convection by the outgoing solar wind and adiabatic energy losses. For a steady state approximation we can set $\frac{\partial U}{\partial t} = 0$.

Let's consider $F = 4\pi r^2 U$ and assume that $V = const(r)$. Using this one can write eq.(1) in the form:

$$\frac{\partial F}{\partial t} = \frac{\partial^2}{\partial r^2}(\kappa F) - \frac{\partial}{\partial r}\left((\frac{1}{r^2}\cdot\frac{d(\kappa r^2)}{dr} + V)F\right) + \\ +\frac{\partial}{\partial T}\left(\frac{2}{3}\frac{V\alpha T}{r}\cdot F\right) \quad (2)$$

The Fokker-Plank equation can be solved by various numerical methods. In our study, we make use of the stochastic simulation method based on the equivalence between Fokker-Plank equations and stochastic differential equations [6,18] which can be solved numerically. The realisation of the numerical techniques we use here is similar to that applied recently for a study of solar particles' interplanetary transport [14]. Note that the problem of GCR transport differs significantly from a problem of solar particle transport as the source of GCR particles is outside the Heliosphere.

According to the stochastic simulation techniques, changes of a test particle's coordinate and energy during a small discrete time step Δt can be described, for the case of eq.(2), as [14] :

$$\Delta T = -\frac{2}{3}\cdot\frac{V\alpha T}{r}\cdot\Delta t \quad (3)$$

$$\Delta r = V\Delta t + \frac{1}{r^2}\cdot\frac{d(\kappa r^2)}{dr}\cdot\Delta t + G\sqrt{2\kappa\cdot\Delta t} \quad (4)$$

where G is Gaussian distributed random number with unit variance.

Thus, using eqs.(3,4) and small Δt we can trace "particles" step by step, fixing their radial distance and energy, as well as time spent inside the Heliosphere. Note that what we call as "particles" in the techniques are not really traced particles but rather some quasi-particles which, when averaged over the ensemble and time, reproduce the distribution of real particles.

Following the standard quasi-linear theory and taking into account the observations of the mean free path of solar particles, we adopt for the diffusion coefficient the form (*e.g.* [19]) :

$$\kappa = \begin{cases} \kappa_o\cdot\beta\cdot P & \text{if } P > P_c \\ \kappa_o\cdot\beta\cdot P_c & \text{if } P < P_c \end{cases} \quad (5)$$

where rigidity P_c=1 GV.

For the calculations we made use of the following parameters. The Heliosphere has the size of R_h=100 au; the solar wind velocity is taken to be a constant V = 400 km/s inside the Heliosphere. We considered as proton spectrum outside the Heliosphere both a model LIS (see Fig.1 and Section 3) and monoenergetic flux (Section 4). Particles are fixed in vicinity of the Earth's orbit (0.9 au < r < 1.1 au), similar to [14]. At the distance of 0.1 au there is a "mirror" which "reflectes" particles from the Sun. Being averaged over time, the number of particles fixed at the vicinity of the Earth's orbit corresponds to steady state solution of eq.(1).

Note, that though Monte-Carlo techniques requires long calculations, one can collect statistics for a set of smaller calculations.

3. Testing of the techniques

In order to test the techniques we performed calculations for a heliospheric model which allows

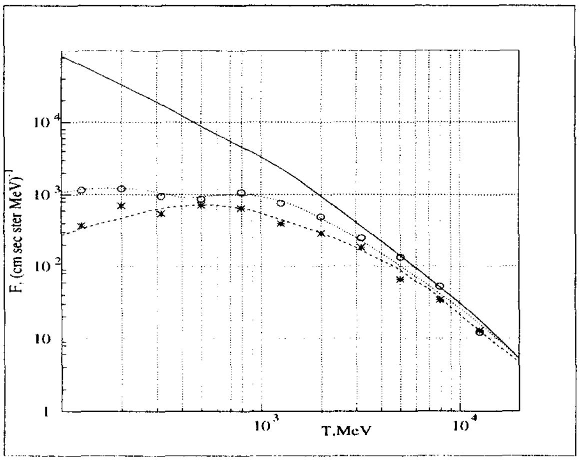

Figure 1. Testing the techniques. Solid line - proton LIS [20]; dotted line - analytical solution (eq.(7)), circles - our corresponding calculations; dashed line - calculations of medium modulation by [21], stars - our calculations for the same parameters.

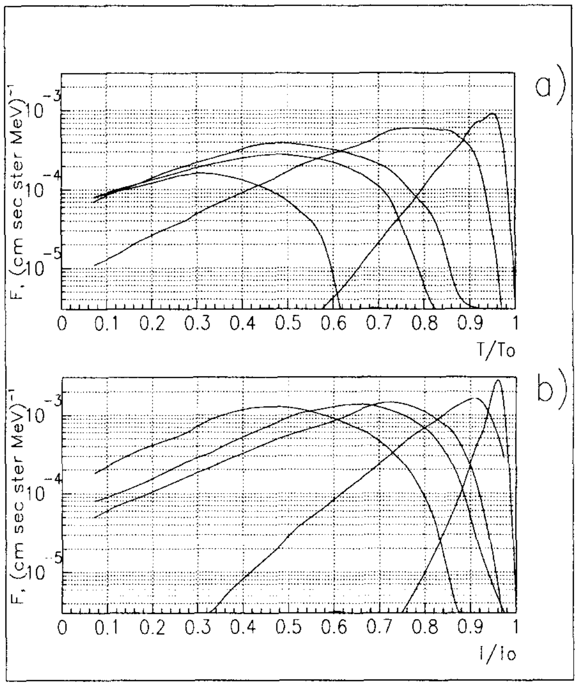

Figure 2. Modulated monoenergetic protons fluxes for medium (a) and weak (b) modulation. T_o=0.3, 0.7, 1, 3, 10 GeV.

an analytical solution. This model with only diffusion and convection terms is described with the following equation (comp. to (1)):

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\kappa\frac{\partial U}{dr}) - \frac{1}{r^2}\frac{\partial}{\partial r}(r^2VU) = 0 \quad (6)$$

and takes the following solution inside the Heliosphere:

$$U(r,P) = f_0(P)\cdot exp\Big(-\frac{V}{\kappa(P)}(R_h - r)\Big) \quad (7)$$

where U_0 corresponds to LIS. The dotted line in Fig.1 shows the analytical solution of eq.(6) for proton LIS as given in [20]. Circles denote the results of the stochastic simulation solution of the same equation. One can see that the simulation precisely reproduces the analytics.

Then we compared our results for spherically-symmetric modulation with those obtained by other methods, using the same model parameters and the same proton LIS. For the comparison, we took a recent study of GCR propagation in the Heliosphere in a spherically-symmetric steady state approximationthrough a numerical solution by means of finite differences of eq.(1) [21]. Following this paper we took the LIS of GCR as given by Webber and Potgieter [20] and the solar modulation strength (*e.g.* [22]) $\Phi = \frac{V(R_h - 1au)}{3\kappa_o} =$ 350 MV (weak modulation), and Φ = 750 MV (medium modulation). Besides, in order to reproduce exactly their calculation we took the diffusion coefficient to be $\kappa = \kappa_o\beta P$ instead of eq.(5). The results are also shown in Fig.1 (dashed line denotes the results of [21] and starts - our calculations). One can see a good agreement though we did not expect the exact coincidence as different approaches have been used.

Thus, we have shown that the stochastic simulation techniques adequately simulates GCR

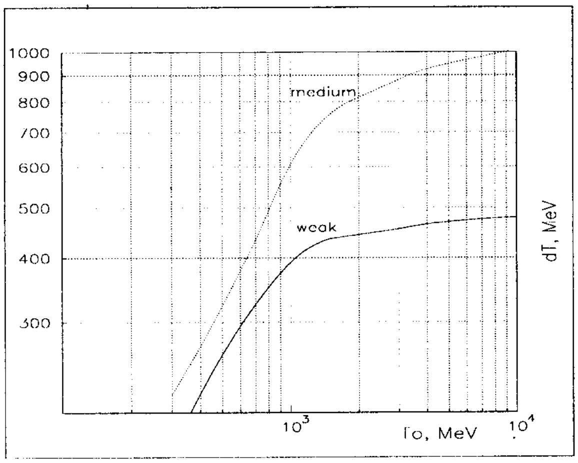

Figure 3. Energy loss of protons with initial energy T_o inside the Heliosphere for medium and weak modulation.

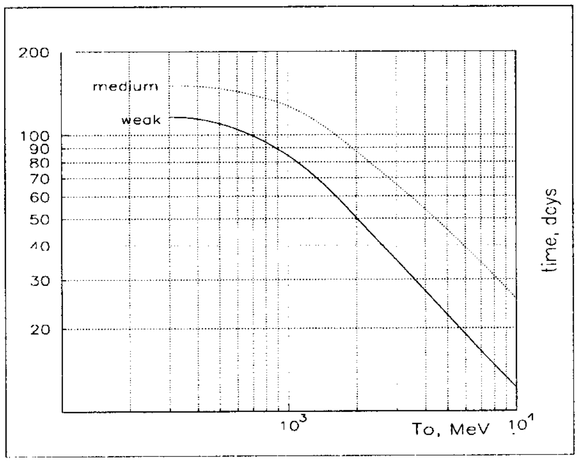

Figure 4. Time spent by protons with initial energy T_o inside the Heliosphere for medium and weak modulation.

propagation in the Heliosphere in a spherically symmetric steady state approximation.

4. Modulation of monoenergetic galactic cosmic ray fluxes

In contrast to other methods of numerical solution of the transport equation, the stochastic simulation techniques allows one to simulate Green functions of a process. If we trace GCR particles with initial energy T_o (the initial LIS is $\delta(T-T_o)$) we can study the effect of modulation for monoenergetic flux or, in other words, Green function of the solar modulation of GCR. Once calculating a set of Green functions for various T_o and presenting the initial interstellar spectrum as a superposition of δ-functions, we can easily obtain the corresponding modulated spectrum for any assumptions on the LIS without new calculations.

The results of the monoenergetic fluxes modulation are shown in Fig.2. The Figure shows the spread in energy of monoenergetic flux after modulation (at the Earth's orbit). The initial LIS is considered to be $\delta(T - T_o)$. The Figure shows Green functions for a set of five initial energies T_o = 0.3, 0.7, 1, 3, 10 GeV for medium (Φ=750 MV) and weak (Φ=350 MV) modulation conditions. As we are able to simulate the modulation of a monoenergetic flux, we can study energy dependence of the modulation. Fig.3 shows the averaged energy losses of particles (due to adiabatic deceleration) before they reach the Earth's orbit in the dependence on the initial energy T_o for medium and weak modulation conditions. The energy loss is connected to the time spent by a particle diffusing in the Heliosphere before it reaches the Earth's orbit. This time is shown in Fig.4 in dependence on the initial energy T_o for medium and weak modulation conditions. One can see that the time of diffusion varies from few days up to half an year though it takes only about half a day for a photon to pass the distance of 99 au. This gives the averaged diffusion radial velocity starting from about 1000 km/sec for medium modulation, and from 1500 km/sec for weak modulation, rapidly increasing with the initial particle's energy.

The time in Fig. 4 is in agreement with the observed delays between the solar activity and long-time variations of cosmic ray flux detected

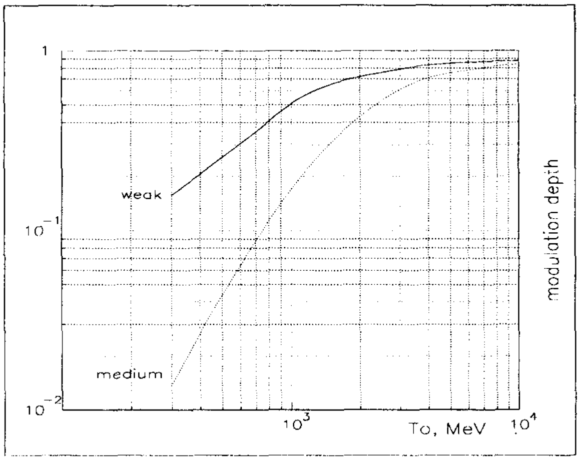

Figure 5. The modulation depth (see text) for protons with initial energy T_o for medium and weak modulation.

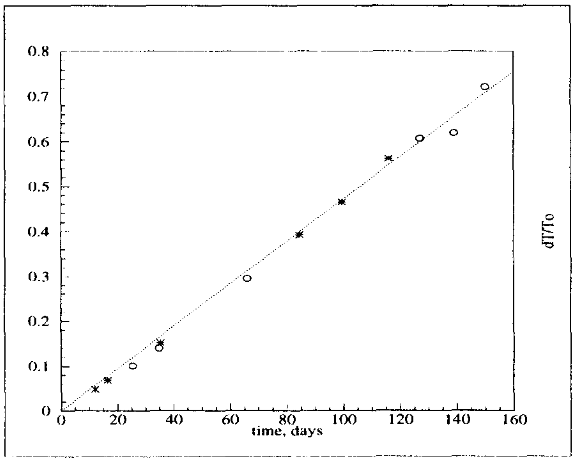

Figure 6. The part of energy dT/T_o lost by protons with initial energy T_o *vs.* time spent in the Heliosphere for medium (circles) and weak (stars) modulation. The line is the linear fit (see text).

by ground based neutron monitors (energy range: 100 MeV - few GeV) [23]. This delay is found to be several months (up to one year) for odd solar cycles when the modulation is considered to be diffusion-dominated (*e.g.* [24]) and a spherically symmetric approximation adequately reproduces the protons modulation. For even cycles with drift-dominated modulation, the observed time delay is shorter [23].

Fig.5 shows modulation depth in dependence of the initial proton's energy T_o for medium and weak modulation conditions. Here, as modulation depth we mean a part of particles with the initial energy T_o which can reach the Earth's orbit. In other words, the modulation depth is an integral of curves in Fig.2 over the energy. One can see that for the initial energy of few hundred MeV, the depressing of GCR flux varies from one (weak modulation) up to two orders of magnitude giving huge variations during a cycle of solar activity. For the initial energy of about 10 GeV, the modulation depth is of the order of magnitude of 10% though the variations of the GCR flux within a solar cycle are only few percent.

Fig.6 shows what part of the initial energy T_o of a particle is lost in dependence of the time spent by the particle inside of the Heliosphere. Calculated points (circles - for medium and stars - for weak modulation) correspond to the values of T_o = 10, 7, 3, 1, 0.7, 0.3 GeV (from the left to the right). One can see a linear scaling of the dT/T_o value *vs.* time spent in the Heliosphere. The linear fit is $dT/T_o = 4.72 \cdot 10^{-3} \cdot t$, where t is in days. The linear scaling does not depend on the modulation strength as points lie on the same line for medium and weak modulation conditions. This means that the time spent by a particle inside the Heliosphere is a very important parameter for modulation as it defines the adiabatic energy loss of a particle

5. Conclusions

In the present paper we, for the first time, introduced the stochastic simulation (Monte-Carlo) techniques for a study of GCR transport inside the Heliosphere and modelling of solar modulation of GCR. We have shown that the simulation results agree with analytical solution for a case without adiabatic energy losses, as well as with results obtained by other authors using different techniques.

We calculated modulation for monoenergetic fluxes of GCR which allows to study the energy dependence of solar modulation effect. We have shown that this is a powerful method for the study of solar modulation of GCR. Using these results one can easily calculate the modulation for any kind of LIS. In particular, we discussed how averaged energy losses, averaged time spent by GCR inside the Heliosphere, modulation depth depend on the initial particle's energy. We have shown that there is a linear scaling of the energy losses *vs.* time spent in the Heliosphere and this scaling doesn't depend on the modulation strength.

In the present paper we have done calculations in the framework of a spherically-symmetric steady state approximation of the Heliosphere. Our calculation doesn't include proton drift in the heliospheric neutral sheet. The drift might be important during cycles with $qA < 0$ when the neutral sheet tilt angle is below 30^o since protons can use the neutral sheet as a short cut to the Earth's orbit (*e.g.* [25,26]). In order to take into account the drift and consider more realistic models, our next steps will be towards a stochastic simulation technique for a two- (and later three-) dimensional model of the Heliosphere. Since the method allows us to fix time spent by a particle in the Heliosphere, it is natural to move towards time-dependent models as well.

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