REGULAR AND RANDOM COMPONENTS OF SUNSPOT ACTIVITY DURING ACTIVE SUN AND GREAT MINIMA: MODEL SIMULATION

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Abstract. We model sunspot production during the two different modes of sunspot activity (the normal activity level and great minima), using the idea of a threshold-like mechanism [Ruzmaikin, 1997]. The model includes a dynamo field, a constant relic field of the Sun and a random field. This model describes the main features of sunspot activity both during normal activity times (dominant 11-year cycle and weak 22-year cycle) and during the Maunder minimum (sparse sunspot occurrence with 22-year cycle) with the same model parameters, only varying the dynamo amplitude. The relic field must be about 3-10 % of the dynamo field in normal activity times.

INTRODUCTION

Time evolution of sunspot activity (SA) is of great interest for solar physics, since it reflects processes in the solar convection zone. The main feature of SA is its 11-year cycle due to the action of the dynamo mechanism. This 11-year cyclicity is modulated by the long-term secular Gleissberg cycle. (For a review see, e.g., Wilson [1994]; Vitinsky [1965].) Recently, a weak persistent 22-year cyclicity, associated with a dipole relic solar magnetic field [Cowling, 1945], has been found in SA [Mursula et al., 2000]. Sometimes SA is dramatically suppressed, forming great minima. The most recent was the Maunder minimum in 1645-1715 [Eddy, 1976]. SA series contains also a random component which is larger than observational uncertainties. Earlier it was common to describe SA as a multiharmonic process with some fundamental harmonics (see, e.g., [Sonett, 1983, Vitinsky, 1965] and references therein. Since early 1990's, SA was considered as low-dimensional deterministic chaos due to a strange attractor, (e.g., [Ostryakov & Usoskin, 1990, Mundt et al., 1991]). However, this approach was criticized because the analyzed data set is too short [Carbonell et al., 1993] and disturbed by filtering [Price et al., 1992]. While the majority of earlier papers were concentrated on either regular or random components, some papers studied both components [Sonett, 1982, Ruzmaikin, 1997, 1998]. They studied SA only during normal SA times. However, it has been suggested that the dynamo can be in a quite different mode during great minima (e.g., [Sokoloff & Nesme-Ribes, 1994, Schmitt et al., 1996]). Correspondingly, the relation between regular and random component can be different for great minima and normal SA.

Here we present a unified model of sunspot production, which describes both modes of SA. The magnetic field in the bottom of the convection zone is considered to be a superposition of a regular and random components, and sunspots occur if this total field exceeds a buoyancy threshold [Ruzmaikin, 1997, 1998]. This model includes also a solar relic magnetic field [Cowling, 1945, Sonett, 1982, Pudovkin & Benevolenskaya, 1984], whose signature was found recently in SA [Mursula et al., 2000]. This relic field can, due to amplification by the dynamo mechanism, play a significant role in sunspot occurrence [Boyer & Levy, 1984, Boruta, 1996].

PROPERTIES OF SA

As index of SA we used the group sunspot number (GSN) series [Hoyt & Schatten, 1998] which covers the period since 1610 including the period of the Maunder minimum (MM) in 1645-1715, and is

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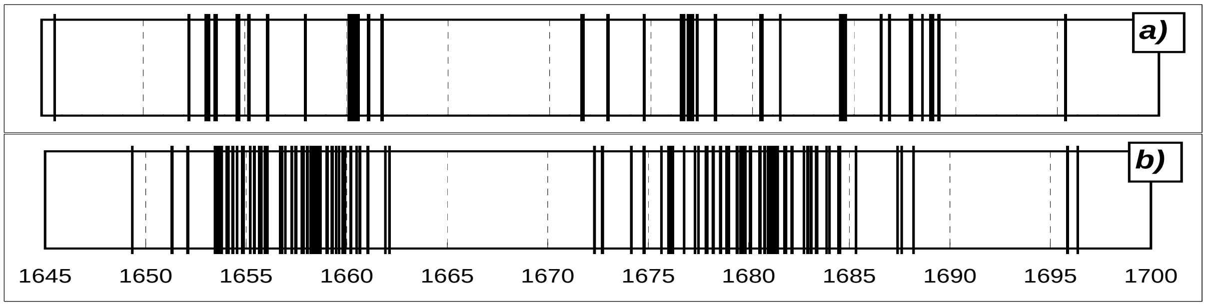

Figure 1: *Days with sunspots during the deep Maunder minimum. a) actual recorded days with sunspots (GSN series); b) a sample of simulation for* $B_o = 0.05$, $\sigma_o = 3$, $A_{11} = 0.03$.

more correct and homogeneous than the Wolf series [Hoyt & Schatten, 1998, Letfus, 1999]. Time behaviour of SA during MM was significantly different from normal SA. Therefore, we studied the great minimum separately from normal SA.

Main features of sunspot activity during the *Maunder minimum* in 1645-1715 are as follows:

1. Sunspots occurred seldom ($\approx$ 2% of days) [Hoyt & Schatten, 1996].
2. Daily sunspot occurrence was clustered in two major groups in 1652-1662 and 1672-1689, implying for a dominant 22-year cyclicity during MM [Usoskin et al., 2000].

The main features of SA during *normal solar activity* periods are:

1. The 11-year cyclicity is the most significant feature. The ratio between 12-month smoothed sunspot maxima and minima is about 10-200.
2. Monthly GSN values fluctuate randomly around the running average SA profile. The normalized fluctuations have Gaussian distribution implying for a correlated noise (e.g., [Oliver & Ballester, 1996].
3. There is a persistent, roughly constant 22-year cycle in sunspot activity at about 20% level of present sunspot cycle intensity level [Mursula et al., 2000].

THE SIMULATION MODEL

Following [Ruzmaikin, 1997, 1998], we suggest that if the total magnetic field in the dynamo layer of the convection zone exceeds the buoyancy threshold, sunspots occur. The total field consists of a regular field and randomly fluctuating field generated by random motions [Ruzmaikin, 1998]:

$$B_{tot} = B_{reg} + b, \tag{1}$$

The regular field is below the threshold in the framework of the mean-field $\alpha - \Omega$ dynamo theories, and therefore the random $b-$field is important to exceed the threshold ([Ruzmaikin, 1998] and references therein).

In our model, B_{reg} contains a constant relic field B_o and the dynamo field in the form of a 22-year sinusoid (Hale cycle) with amplitude A_{11}:

$$B_{reg}(t) = B_o + A_{11} \cdot sin(\pi \cdot t/T_{11}), \tag{2}$$

Since the random component of SA is correlated noise, we assume that the momentary variance of the random field, $\sigma(t)$, is proportional to the regular component of SA at the moment [Ostryakov & Usoskin, 1990a], $\sigma(t) = \sigma_o \cdot |B_{reg}(t)|$. We assumed the exponential probability distribution function of the random field, $p(b) \propto exp(-|b|/\sigma)$ [Ruzmaikin, 1998]. We have studied also the Gaussian distribution $p(b) \propto exp(-b^2/\sigma^2)$ Here we show results only for the exponential case and discuss both cases later.

SIMULATION RESULTS

We numerically simulated SA separately for normal activity and great minimum. For each day t, the value of b was generated by a pseudo-random number generator. If the simulated $|B_{tot}|$ (Eq. 1) exceeds the threshold ($|B_{tot}| > B_{th}$), sunspots occurred. The number of sunspots was proportional to ($|B_{tot}| - B_{th}$). Values of the field are in arbitrary units, with the value of the threshold, B_{th}, chosen to be unity.

The Maunder minimum

Since the 11-year component of SA was very weak during MM, we assume that A_{11} was small during MM. A sample simulation shows (Fig. 1b) the time behaviour

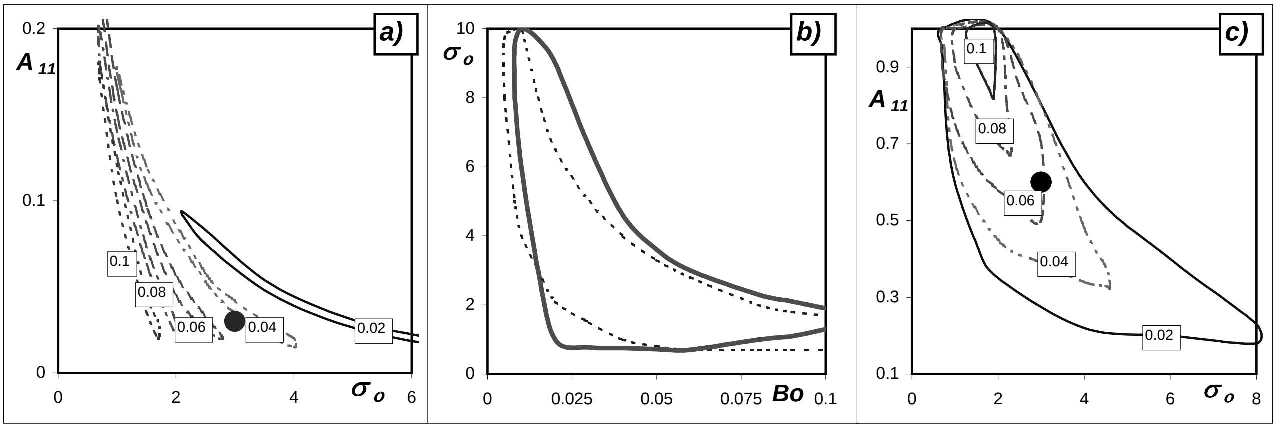

Figure 2: *Area of possible values of model parameters. a)* A_{11} *vs.*σ_o *for the great minimum. Value of* B_o *is fixed (as shown in boxes). The solid circle denotes values of parameters used for sample simulations shown in Fig. 1b. b)* σ_o *vs.* B_o *for all* A_{11}*. The allowed area is between the two solid curves for the normal SA times, and between the two dashed curves for the Maunder minimum. c)* A_{11} *vs.*σ_o *for the normal sunspot activity times. Value of* B_o *is fixed (as shown in boxes). The solid circle denotes values of parameters used for sample simulations shown in Fig. 3b.*

similar to that of the actual sunspot occurrence. We made 10^4 simulation sets for the 20088 days of the deep MM in 1645-1699. In order to study the range of possible values of model parameters we used two constraints.

Constraint I. Correspondingly to the 369 (out of 20088) days with reported sunspots during the deep MM, the number of simulated sunspot days was constrained be 369 ± 57.

Constraint II. There were long spotless periods in 1645-1652, 1662-1672 and 1690-1699 (Fig. 1a). We require that these statistically significant spotless periods should exist in the simulated series, i.e., not more than one sunspot day per year is allowed for these intervals.

Using these constraints we found areas of possible values of the model parameters for the great minimum mode (Fig. 2a). For a fixed B_o, the allowed area of A_{11} *vs.* σ_o is prolonged but narrow. The area of all possible values of σ_o and B_o (for all values of A_{11}) is shown in Figs. 2b.

Normal activity level

In order to study the range of possible parameter values for normal activity times, we also used two constraints:

Constraint I concerns the empirical G-O rule [Gnevyshev & Ohl, 1948], saying that the sum of sunspot numbers over an odd cycle exceeds that of the preceding even cycle. We require that "odd" cycles are 10-30 % more intense than "even" cycles.

Constraint II limits the ratio of 12-month averaged maximum to minimum intensities of a cycle to be 10-200.

The relation between A_{11} and σ_o for fixed B_o is shown in Fig. 2c. The area of possible values of σ_o and B_o for all values of A_{11} is shown in Fig. 2b. A sample of simulation is shown in Fig. 3b. There is a good similarity with the actual GSN data (Fig. 3a) for the period of fairly constant SA level (solar cycles 9-13). We simulated 1000 11-year solar cycles. The length of simulated cycles varied from 9.5 to 12.5 years, and the cycle amplitude varied within 100-200 arbitrary units. The G-O rule is valid throughout the entire simulated series. The normalized noise of the simulated series is Gaussian giving additional support to our approach.

DISCUSSION AND CONCLUSIONS

Our model can reproduce all the main features of SA during both great minima and normal activity times. The range of possible values of B_o and σ_o is essentially similar for these two different modes of SA (Fig. 2b). The model reproduces SA behaviour for the two modes of SA with the same values of B_o and σ_o only changing A_{11}. The dynamo can be significantly suppressed during great minima while both the relic field and random component remain unchanged. While the random component of SA plays a major role during MM, the regular component is more important during normal SA times, leading to the 11-year cyclic behaviour of SA. The presence of a fluctuating field is necessary to exceed the buoyancy threshold even in the latter mode.

In the framework of the model, the amplitude of the dynamo field, A_{11}, should be not less than 20% of the

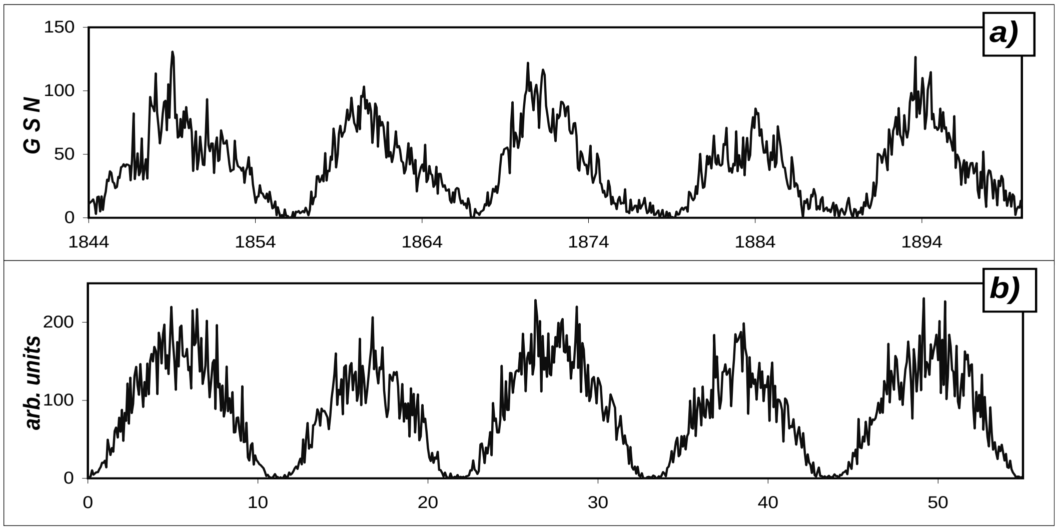

Figure 3: *Sunspot activity: a) actual monthly group sunspot numbers for the period of roughly constant SA level; b) a sample of the monthly simulated SA for* $A_{11} = 0.6$, $B_o = 0.05$, $\sigma_o = 3$.

threshold level during normal SA times, in agreement with the theoretical expectations [Ruzmaikin, 1998]. For MM, the value of A_{11} is much smaller, 0.03-0.1. This implies that the dynamo was greatly suppressed during MM but had to be non-zero.

The value of the relic field, B_o, is small but non-zero, varying from 0.01 to ≤ 0.1, which is about 2-10% of the dynamo field. This value of B_o leads to a dominant 22-year cycle in SA during MM and to a weak but persistent 22-year variation during normal SA times.

Concluding, we have shown that the main features of SA throughout the entire period of direct solar observations, including the two different different SA modes (normal and great minimum), can be reproduced by a unified model assuming a dynamo field, a weak constant relic field, and a randomly fluctuating field. This also supports the recent result [Mursula et al., 2000] that the 22-year cyclicity in SA is due to the action of the magnetic 22-year cycle of the dynamo field in the presence of a weak constant (relic) magnetic dipole in the convection zone.

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