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A Test of the Active-Day Fraction Method of Sunspot Group Number Calibration: Dependence on the Level of Solar Activity

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Abstract The method of active-day fraction (ADF) was proposed recently to calibrate different solar observers to standard observational conditions. The result of the calibration may depend on the overall level of solar activity during the observational period. This dependency is studied quantitatively using data of the Royal Greenwich Observatory by formally calibrating synthetic pseudo-observers to the full reference dataset. It is shown that the sunspot group number is precisely estimated by the ADF method for periods of moderate activity, may be slightly underestimated by 0.5 – 1.5 groups ($\leq 10\%$) for strong and very strong activity, and is strongly overestimated by up to 2.5 groups ($\leq 30\%$) for weak-to-moderate activity. The ADF method becomes inapplicable for the periods of grand minima of activity. In general, the ADF method tends to overestimate the overall level of activity and to reduce the long-term trends.

Keywords Solar activity · Sunspots · Solar observations · Solar cycle

1. Introduction

Calibration of different solar observers to compile a long, homogeneous sunspot number series is of great importance now, when the "classical" sunspot-number series have been found to be partly erroneous (Clette *et al.*, 2014; Usoskin, 2017). Different methods of recalibration of the sunspot number series were proposed (Clette *et al.*, 2014; Lockwood, Owens, and Barnard, 2014; Svalgaard and Schatten, 2016; Clette and Lefèvre, 2016; Usoskin *et al.*, 2016; Chatzistergos *et al.*, 2017). While most of the new recalibrations were based on the direct pairwise or "backbone" comparison of individual observers, forming a daisy-chain

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of calibrations with error accumulation in time, an alternative active-day fraction (ADF) method was proposed by Usoskin *et al.* (2016) (refined by Willamo, Usoskin, and Kovaltsov, 2017). The ADF method is based on a comparison of the statistic of active day (days when at least one sunspot group was observed, as opposed to quiet days with no spots) occurrence for each observer to that of the reference dataset, which is the Royal Greenwich Observatory (RGO) series for the period 1900 – 1976 (see details given by Usoskin *et al.*, 2016). As such, the method is free of daisy-chaining and error accumulation since each observer is compared directly to the reference dataset. As a drawback, however, the method may be somewhat sensitive to the overall level of solar activity during the period when the calibrated observer was making observations. This was qualitatively mentioned in our earlier works (Usoskin *et al.*, 2016; Willamo, Usoskin, and Kovaltsov, 2017) but never investigated in sufficient detail. This issue has been actively discussed during several in-person meetings and teleconferences of a group working on the sunspot-number series recalibration, and it has also led to an unpublished critique (Svalgaard and Schatten, 2017).

Accordingly, here we perform a full quantitative test of the dependence of the ADF method on the average solar-activity level during the period of observations of a calibrated observer.

2. Method

2.1. RGO Pseudo-observers

The ADF method was tested using pseudo-observers based on the reference RGO dataset (available at solarscience.msfc.nasa.gov/greenwch.shtml). From the entire RGO dataset we have selected several subsets, each covering three solar cycle max-to-max intervals, with start times separated by one solar cycle, as listed in Table 1. The periods covered by these pseudo-observers correspond to different levels of solar activity, ranging from an average of 3.4 sunspot groups during the activity minimum around the boundary between the nineteenth and twentieth century to 6.15 groups during the modern grand maximum. Each of these subsets was considered as a pseudo-observer and calibrated to the full reference dataset following the procedure described by Willamo, Usoskin, and Kovaltsov (2017). This approach enables directly assessing possible biases of the method. Since the pseudo-observers are simply subsets of the reference dataset, the formal calibration should be (consistent with) zero: any non-zero difference between the "calibrated" and the reference data indicates a bias in the calibration procedure.

Table 1 Three-cycle pseudo-observers used in this study to "reconstruct" the RGO-based group number [G]. Columns present the time intervals, the mean value of G during the interval, and the obtained observational threshold [S] with 68% confidence interval.

Pseudo-observer	Time [years]	$\langle G\rangle$	S [msd]
1	1883.9 – 1917.6	3.43	$46(^{52}_{41})$
2	1894.1 – 1928.4	3.65	$30(^{34}_{24})$
3	1907.0 – 1937.4	4.01	$14(^{17}_{10})$
4	1917.6 – 1947.5	4.65	$-15(^{-11}_{-18})$
5	1928.4 – 1957.9	5.4	$-14(^{-10}_{-17})$
6	1937.4 – 1968.9	6.15	$-38(^{-29}_{-44})$

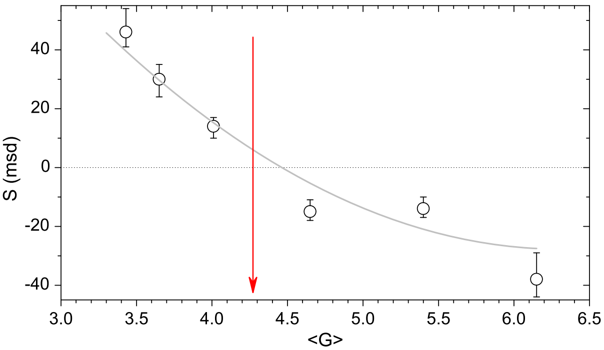

Figure 1 Observational threshold S derived for the pseudo-observers (see text) for the six intervals given in Table 1 with the 68% confidence intervals. A best-fit parabolic curve to guide the eye is plotted *in gray* to emphasize the nonlinearity of the relation. The *red arrow* denotes the mean $\langle G\rangle = 4.27$ value for the period 1900 – 1976.

2.2. Observational Thresholds

In the framework of the ADF method, each observer is characterized by their observational threshold $[S]$, which is the minimum observed (*i.e.* uncorrected for foreshortening) area (in millionths of the solar disk: msd) of a sunspot group resolvable by the observer. In other words, the observer is assumed to record all groups larger than, and miss all groups smaller than, S. The value of S, along with its uncertainties, is defined by matching the statistics (cumulative probability distribution function; cpdf) of the ADF occurrence recorded by the observer with that of the reference dataset by applying to the latter the observational threshold (see full details in Willamo, Usoskin, and Kovaltsov, 2017). This assumes that the calibrated observer is "poorer" than the reference observer, meaning that the positive threshold is applied to the reference dataset.

The formal application of the ADF calibration procedure to the pseudo-observers 1, 2, and 3 (see Table 1) yields thresholds of 15 to 45 msd, implying that these pseudo-observers appear poorer than the reference set. This can be understood as follows. Of course, all pseudo-observers must have the same quality (*i.e.* $S = 0$) by construction, as subsets of the reference dataset. However, because of the low level of solar activity, there are fewer active days (lower ADF) during time interval 1, which is interpreted by the ADF method as a lower quality of the observer, or in other words, a positive threshold S. This formal threshold is a bias of the method, related to the low level of activity, which is below the mean group number $\langle G\rangle = 4.27$ for the reference dataset. It appears large for pseudo-observers 1 and 2, but small, almost consistent with no threshold, for pseudo-observer 3.

The situation is inverted for pseudo-observers 4 through 6, which correspond to time intervals with enhanced solar activity $\langle G\rangle > 4.27$. Because of that, there are more active days during these periods, which is interpreted by the method as a quality of the pseudo-observers higher than that of the reference dataset. This would lead to formally negative thresholds, which does not make sense, however, since the sunspot group area cannot be negative. In order to deal with this, we defined the negative threshold as follows. The threshold S was considered negative when it was applied to the calibrated observer data while keeping the reference dataset as it is (no threshold).

Values of S defined in this way are given in Table 1 and presented in Figure 1. The values of S are positive for low-activity cycles (points left of the red arrow in the figure) and negative for high-activity cycles, as expected. An interesting feature is that the relation between S and $\langle G\rangle$ is strongly nonlinear, suggesting that the method is more accurate for moderate and high cycles than for low cycles. A line to guide the eye (best weighted-fit parabola) is shown for illustration.

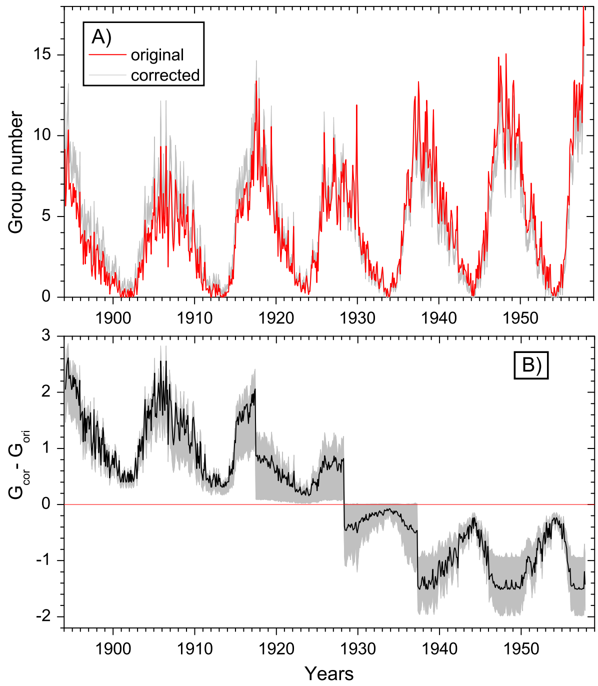

Figure 2 Comparison between original [G_{ori}, *red*] and "corrected" [G_{cor}, *gray*, see text] monthly group numbers for the RGO dataset. Panel **A**: Temporal variability of the two series. Panel **B**: The difference between the two series. *Gray shading* denotes the 68% (1σ) confidence intervals.

2.3. Data "Correction"

Next we "corrected" the data of the pseudo-observers using the values of the threshold obtained above (Table 1) and applying the procedure described earlier (Usoskin *et al.*, 2016; Willamo, Usoskin, and Kovaltsov, 2017). For negative values of S, the correction corresponding to $-S$ was applied by inverting its sign (not added to, but subtracted from the reported G-values). Corrections were applied to the RGO data for the corresponding pseudo-observers from Table 1. Since they overlap in time, we applied the average over them for each day, as described by Willamo, Usoskin, and Kovaltsov (2017). The resulting corrected series is shown (with the 68% confidence interval) in Figure 2A in gray. The corrected series is slightly higher than the original series in the early part and slightly lower in the later part of the plot. The difference between the corrected and the original series is shown in panel B. Jumps are caused by the boundaries between pseudo-observers.

The group number [G] is significantly overestimated for low solar cycles (1894 – 1917), with the difference reaching 2.5 sunspot groups. Corrections are small and consistent with zero within the 68% confidence interval for moderate cycles (1917 – 1937). The group number appears to be slightly underestimated, up to 1.5 groups, for high cycles (1937 – 1957), which also include the highest known solar cycle: Cycle 19.

3. Discussion and Conclusions

We have analyzed the dependence of the ADF method of sunspot-group number recalibration on the level of solar activity during the observational period of an observer. This dependence was discussed earlier (Usoskin *et al.*, 2016) but has not been assessed quantitatively.

Using fragments of the reference dataset of the RGO, we formed several pseudo-observers who were formally calibrated against the full reference dataset. Since the pseudo-observers, by construction, are identical to the reference dataset, the true calibration must be zero or consistent with zero within the uncertainties. Accordingly, any non-zero formal "correction" of the pseudo-observers can be interpreted as a bias in the method. We found that indeed, as proposed earlier, the ADF method tends to overestimate weak solar activity and underestimate high activity. However, the bias introduced is nonlinear. While moderate cycles with peak values of $G = 10 - 12$ are reproduced correctly, high and very high cycles $G = 12 - 17$ may appear underestimated by 0.5 – 1.5 groups, or less than 10%. On the other hand, weak-to-moderate cycles with $G = 7 - 9$ appear overestimated by 2 – 2.5 groups, or about 30%. As discussed by Usoskin *et al.* (2016), the ADF method cannot work with a very weak cycle, *e.g.* during the Dalton minimum. It is important to note that this is related not to the height of individual cycles, but to the average level of solar activity during the entire time span of a given observer.

As a result, in the long run, the ADF method tends to overestimate the overall level of activity and to reduce the long-term trends. Therefore, the sunspot-activity level reconstructed by the ADF method can be considered as a conservative upper limit for estimates of the long-term trends of solar activity.

Our conclusions are listed below.

- The ADF method works accurately for a period of moderate solar activity.
- Application of the ADF method may lead to a slight underestimate, by 0.5 – 1.5 sunspot groups ($\leq 10\%$), of the solar activity during periods of high and very high activity.
- Application of the ADF method may lead to a significant overestimate, by up to 2.5 sunspot groups ($\leq 30\%$), of the solar activity during periods of low-to-moderate activity.
- Overall, the ADF method tends to overestimate the general level of activity and reduce the long-term trends.

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