Multislot mPPM and amPPM modulation for UWB applications

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Abstract— **PPM-modulation has widely been considered for the use in UWB systems. We propose spectrally efficient modifications of this well known method. In mPPM modulation format one transmits signals in m slots out of a window of M slots in order to achieve higher spectral efficiency than with standard PPM modulation. To achieve even greater spectral efficiency one could assign different amplitudes to the signals transmitted in m timeslots. This approach is denoted by amPPM. The paper considers both coherent and noncoherent versions of the proposed mPPM and amPPM modulation. We derive the analytical bit error rate results which are verified though computer simulations. We also derive the BER-results for pulse integration in noncoherent combining of mPPM-symbols.**

Keywords— **UWB, PPM**

I. Introduction

Growing pressure on spectrum has lead to a general need to increase efficiency of transmission. Recently [1], the Federal Communications Commission (FCC) accepted ultra wideband (UWB) signaling to be used in different applications. In UWB-proposals, e.g [2], [3], pulse position modulation (PPM) has often been considered as the data modulation format. In this paper we examine spectrally efficient mPPM and propose amPPM modulation format which are modifications of the well known PPM-principle. Different versions of PPM modulation format also including differential and combinatorial PPM are described e.g. in [4], [5], [6], [7], [8]. mPPM was introduced for the use in optical communications in [9] under the name MPMM. MPMM has been considered e.g in [10], [11], [12], [13] and the performance is assessed in the noisy photon counting channel. In this paper we consider it for UWB-communications and derive exact bit error rate (BER) results for one-shot transmission as well as in the case of pulse repetition used in UWB-communications. One can use a standard PPM detector in signal detection of mPPM and amPPM and hence the receivers are practically of the same complexity as PPM receivers. The difference when compared to PPM is in the signal processing after detection has occurred. To enhance the spectral efficiency of the signalling set, multislot signals are considered. A standard PPM modulation uses one out of M timeslots each T_s/M seconds to transmit a block of $n = \log_2 M$ bits in a time window of T_s. The optimum coherent receiver has a bank of M matched filters or correlator integrator processors and every T_s seconds makes decision based on the largest output filter sample. The coherent orthogonal PPM in AWGN channel has the symbol error rate performance [14] $P_M = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left\{ 1 - \left[1 - \frac{1}{2} \operatorname{erf c}\left(\frac{y}{\sqrt{2}} \right) \right]^{M-1} \right\} \cdot e^{-\left(y-\sqrt{2\gamma}\right)^2/2} dy$, where $\gamma = \frac{E_s}{N_0} = \frac{kE_b}{N_0}$. The noncoherent (orthogonal) receiver uses M envelope detectors to produce the decision and the corresponding symbol error rate in AWGN channel is [14] $P_M = \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{1}{n+1} e^{\frac{-nkE_b}{(n+1)N_0}}$.

Assume instead of sending one of M timeslots the transmitter sends in m of M equal amplitude timeslots in a window of T_s. The optimum receiver will now be required to find the m largest signals at the output of the M detectors. Since the amplitudes are the same one can form $M_m = \binom{M}{m}$ different combinations. The number of transmitted bits per T_s is now increased to $n_m = \log_2 \binom{M}{m}$bits. This modulation format is denoted as mPPM modulation.

If the amplitudes of the transmitted signals in different timeslots are all different the number of the possible signalling combinations is further increased to $M_m(a) = M(M-1)(M-2)...(M-m+1)$ or equivalently $M_m(a) = \frac{M!}{(M-m)!}$ and the number of transmitted bits is $n_m(a) = \sum_{i=0}^{m-1} \log_2(M-i)$ or equivalently $n_m(a) = \log_2 \frac{M!}{(M-m)!}$. The name for this type of modulation is amPPM where M is the number of timeslots available, m the number of timeslots used for transmission in a PPM window and a is to point out that all the m signals in different timeslots have different amplitudes.

In these cases the signal energy is divided into separate timeslots making it more vulnerable to noise and fading but still the overall flow of useful information will be increased under large range of signal, channel and interference parameters thus offering better spectral efficiency. To support this statement the bit error probability results are presented in section III following the notations of [15].

II. System Model

In Fig. 1, a coherent mPPM-receiver is depicted. The receiver has switches $S_1, ...S_M$ which pass one out of M timeslots to subsequent correlator-integrator processing. The switch is open in each receiver branch only for the duration of the time slot being processed by the given receiver branch. It is easily seen that using this type of a receiver, the orthogonality between the time slots is maintained given correct synchronization. This receiver is a generalisation of a PPM-receiver. In Fig. 1, one notices that the waveforms $u_{1u}^*(t), ..., u_{Mu}^*(t)$ in each receiver branch can be different. The more often used approach, however, is to use only one signaling (pulse) waveform since the timeslot structure (given synchronization and no pulse ringing) ensures orthogonality. It is easily seen, however, that using this type of a receiver, the orthogonality may be lost given any nonlinearities in the system or that we have a severe multipath channel. Fig. 2 depicts the noncoherent amPPM receiver. As previously, the orthogonality of the decision variables is assured by allowing only one timeslot out of M to be processed in one receiver branch through the use of switches $S_1, ...S_M$. The signals used in Figs. 1, 2 are presented in the next chapter.

III. Bit Error Probability of Uncoded systems

A. Error probability for coherent mPPM

The transmitted signals are represented as $u_k(t-kT)$, $k = 1, 2, ..., M$ with

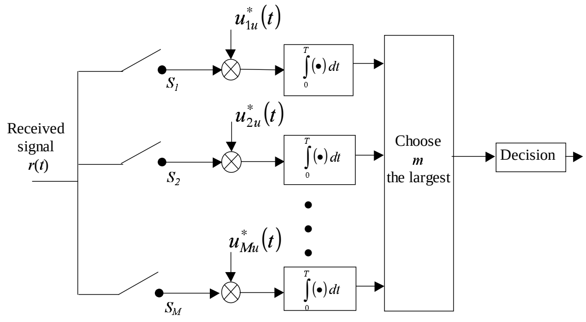

Fig. 1. A coherent mPPM-receiver.

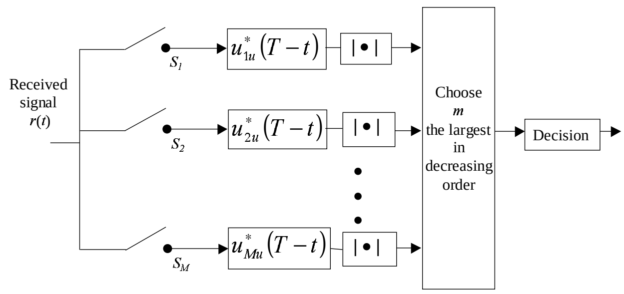

Fig. 2. A noncoherent amPPM-receiver

$$\int_0^T u_k\left(t-kT\right)u_m\left(t-mT\right)dt=\delta_{km} \quad (1)$$

where $u_k(t)$ is the complex envelope of the signal and δ_{km} is the Kronecker delta function. The energy of these signals is

$$\begin{aligned} E_k &= \frac{1}{2}\int_0^T \left|u_k\left(t\right)\right|^2 dt \quad (2) \\ &= A^2E_u = E,\ k=1,...,M \end{aligned}$$

The received signal envelope is then of the following form $r(t)=\alpha\, e^{-j\phi}u_k(t-kT)+z(t)$, $0\le t\le T$,where α is due to the channel attenuation , ϕ is the phase difference between the input and local signal, and $z(t)$ is Gausian noise. For simplicity, and without loosing on generality let us assume that signals in the first m timeslots are transmitted i.e. $u_l(t-lT)$, $l=1,2,...,m$. The received low pass equivalent of the signal becomes

$$r(t)=\alpha\, e^{-j\phi}\left(u_1(t-T)+\cdots+u_m(t-mT)\right)+z(t), 0\le t\le T \quad (3)$$

The decision variables are now given as

$$U_k=\mathrm{Re}\left\{e^{j\phi}\int_0^T r(t)u_k^*(t-kT)\,dt\right\}, k=1,\ 2, ...,\ M \quad (4)$$

An optimum receiver will choose the largest m. Parameter U_k can be represented as

$$\begin{aligned} U_l &= 2\alpha E+N_{lr},\ l=1,2,..,m \quad (5) \\ U_p &= N_{pr},\ p=m+1,...,M \end{aligned}$$

were N_{kr} are Gaussian zero mean variables with variance $\sigma^2 = 2EN_0$, and N_0 is the noise spectral density. Probability density functions (pdf) for U_k can be represented as [14]

$$\begin{aligned} p(U_l) &= \frac{1}{\sqrt{2\pi}\sigma}\, e^{-(U_l-2\alpha E)^2/2\sigma^2} \qquad l=1,2,...,m \\ p(U_p) &= \frac{1}{\sqrt{2\pi}\sigma}\, e^{-U_p^2/2\sigma^2} \qquad p=m+1,...,\ M \quad (6) \end{aligned}$$

The probability of correct decision is

$$\begin{aligned} P_c|U_1,...,U_m = P\left(U_1>U_{m+1},...,U_1>U_M\right) \quad (7) \\ \cdot ...\cdot P\left(U_m>U_{m+1},...,U_m>U_M\right) \end{aligned}$$

which after some calculus becomes

$$\begin{aligned} P_c = \Bigg[\frac{1}{2^{M-m}\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-x^2}\ \cdot \\ \left[1+\mathrm{erf}\left(x+\sqrt{\frac{\alpha^2E}{N_0}}\right)\right]^{M-m} dx\Bigg]^m \quad (8) \end{aligned}$$

and the symbol error probability as $P_s=1-P_c$.

B. Error probability for coherent amPPM

With similar assumptions and reasoning as in previous chapter we can find the error probability for coherent amPPM as

$$\begin{aligned} P_c = \frac{1}{2^{2M-3}\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{x+\frac{a_{c1}}{\sigma}-\frac{a_{c2}}{\sigma}} \left[1+\mathrm{erf}\left(x+\frac{a_{c1}}{\sigma}-\frac{a_{c2}}{\sigma}\right)\right] \\ \cdot\left[1+\mathrm{erf}\left(x+\frac{a_{c1}}{\sigma}\right)\right]^{M-2}\left[1+\mathrm{erf}\left(y+\frac{a_{c2}}{\sigma}\right)\right]^{M-2} \quad (9) \\ \cdot e^{-x^2}e^{-y^2}dydx \end{aligned}$$

where $\frac{a_{c1}}{\sigma} = \frac{\alpha A_1\sqrt{E_u}}{\sqrt{2N_0}}$and $\frac{a_{c2}}{\sigma} = \frac{\alpha A_2\sqrt{E_u}}{\sqrt{2N_0}}$. The symbol error probability is again $P_s=1-P_c$. The details of the derivation can be found in [15].

C. Error probability for noncoherent mPPM with envelope detector

The receiver will create decision variables

$$U_k=\left|\int_0^T r\left(t\right)u_k^*\left(t\right)dt\right|,\ k=1,2,...,M \quad (10)$$

and choose the largest m. Bearing in mind that signals $u_k(t-kT)$ are orthogonal , one can show that pdf's for U_k can be expressed as [14]

$$p(U_l)=\frac{U_l}{2EN_0}\exp\left(-\frac{U_l^2+4\alpha^2E^2}{4EN_0}\right)I_0\left(\frac{\alpha U_l}{N_0}\right), \quad (11)$$

when $l=1,...,m$ and for $p=m+1,...,\ M$

$$p(U_p)=\frac{U_p}{2EN_0}\exp\left(-\frac{U_p^2}{4EN_0}\right), \quad (12)$$

where $I_0\left(\cdot\right)$ is the zero order modified Bessel function of the first kind. The probability of correct decision is defined in general form by Eq. 7 and now becomes

$$P_c=\left[\sum_{n=0}^{M-m}(-1)^n\binom{M-m}{n}\frac{1}{n+1}e^{-\frac{\alpha^2E}{N_0}\frac{n}{n+1}}\right]^m \quad (13)$$

Finally the symbol error probability is again given as $P_s=1-P_c$

D. Error probability for noncoherent amPPM with envelope detector

With similar assumptions and reasoning as in previous chapter we can find the error probability for noncoherent amPPM as

$$P_c = \exp\left(-\frac{a_{c1}^2}{\sigma^2} - \frac{a_{c2}^2}{\sigma^2}\right) \int_0^{+\infty}\int_0^{y} F_1\left(\frac{a_{c2}^2}{\sigma^2}, y\right) \cdot \left[F_2(y)\right]^{M-2}\left[F_2(z)\right]^{M-2}\exp(-y-z)\cdot I_0\left(2\sqrt{\frac{a_{c1}^2}{\sigma^2}y}\right) I_0\left(2\sqrt{\frac{a_{c1}^2}{\sigma^2}y}\right) dzdy \quad (14)$$

where $\frac{a_{c1}}{\sigma} = \frac{\alpha A_1\sqrt{E_u}}{\sqrt{2N_0}}$ and $\frac{a_{c2}}{\sigma} = \frac{\alpha A_2\sqrt{E_u}}{\sqrt{2N_0}}$ and

$$\begin{aligned} F_1(x,y) &= \exp[-x]\int_0^y \exp[-t] I_0\left(2\sqrt{xt}\right) dt \\ F_2(x) &= 1-\exp[-x] \end{aligned} \quad (15)$$

The symbol error probability is $P_s = 1 - P_c$. The details of the derivation can again be found in [15].

E. Approximate error probability analysis for noncoherent mPPM with nonorthogonal signals

In UWB-applications it is often observed that the antennas introduce distortion to the pulse shape due to the differentiation observed both in the transmit as well as receive antenna (sometimes referred to as pulse ringing). This may be considered in terms of correlation between the antenna filter on the received signal and may lead to performance degradation but in some favorable cases in coherent processing also performance improvement given that the pulse waveforms are appropriately designed. The receiver will create decision variables

$$U_k = \left|\int_0^T r(t)\, u_k^*(t)\, dt\right|, \; k = 1, 2, ..., M \quad (16)$$

and choose m the largest ones. One can show that pdf's for U_k can be expressed as:

$$p(U_l) = \frac{U_l}{2EN_0}\exp\left(-\frac{U_l^2 + 4\alpha^2\rho_2^2E^2}{4EN_0}\right) I_0\left(\frac{\alpha\rho_2 U_l}{N_0}\right), \quad (17)$$

when $l = 1, ..., m$ and for $p = m + 1, ..., M$

$$p(U_p) = \frac{U_p}{2EN_0}\exp\left(-\frac{U_p^2 + 4\alpha^2\rho_1^2E^2}{4EN_0}\right) I_0\left(\frac{\alpha\rho_1 U_l}{N_0}\right), \quad (18)$$

where $I_0(\cdot)$ is the zero order modified Bessel function of the first kind and $\rho_1 = m\left|\rho_{i,j}\right|$ and $\rho_2 = 1-(m-1)\left|\rho_{i,j}\right|$. $\rho_{i,j}$ $(i \neq j)$ is the crosscorrelation value between PPM timeslots. Probability of correct decision is defined in general form by Eq. 7 and now becomes ($\sigma^2 = 2EN_0$, $\beta_1 = 4\alpha^2\rho_1^2E^2$, $\beta_2 = 4\alpha^2\rho_2^2E^2$)

$$\begin{aligned} P_c &= \left[\int_0^\infty \left[\int_0^x \frac{y}{\sigma^2}\exp\left(-\frac{y^2+\beta_1}{2\sigma^2}\right) I_0\left(\frac{\alpha\rho_1 y}{N_0}\right) dy\right]^{M-m} \right. \\ &\left. \cdot\frac{x}{\sigma^2}\exp\left(-\frac{x^2+\beta_2}{2\sigma^2}\right) I_0\left(\frac{\alpha\rho_2 x}{N_0}\right) dx\right]^m \end{aligned} \quad (19)$$

Finally the symbol error probability is again given as $P_s = 1-P_c$. This analysis applies to the cases where $\left|\rho_{i,j}\right|$ is the same between all timeslots within the PPM time window. Hence we can use the above analysis to obtain the best case and worst case performance given the crosscorrelation level. From the above results one can draw the conclusion that in noncoherent system the best possible performance is obtained if $\left|\rho_{i,j}\right| = 0$. The worst case performance is obtained if we assume that all timeslots experience the same value $\left|\rho_{i,j}\right|$. In practice $\left|\rho_{i,j}\right|$ is a function of the system design and hence must be measured. Also the values in different timeslots for $\left|\rho_{i,j}\right|$ are not necessarily the same.

F. Approximate error probability analysis for coherent mPPM with nonorthogonal signals

Parameter U_k can be represented in this coherent case as

$$\begin{aligned} U_l &= 2\alpha E + N_{lr}, \; l = 1, 2, .., m \\ U_p &= N_{pr}, \; p = m+1, ..., M \end{aligned} \quad (20)$$

were N_{kr} are Gaussian zero mean variables with variance $\sigma^2 = 2EN_0$,and N_0 is the noise spectral density. Probability density functions (pdf) for U_k can be represented as

$$\begin{aligned} p(U_l) &= \frac{1}{\sqrt{2\pi}\sigma}\, e^{-(U_l-2\alpha\rho_2E)^2/2\sigma^2} \quad l = 1, 2, ..., m \\ p(U_p) &= \frac{1}{\sqrt{2\pi}\sigma}\, e^{-\left(U_p-2\alpha\rho_1E\right)^2/2\sigma^2} \quad p = m+1, ..., M \end{aligned} \quad (21)$$

where $\rho_1 = m\rho_{i,j}$ and $\rho_2 = 1 + (m-1)\rho_{i,j}$. $\rho_{i,j}$ $(i \neq j)$ is the crosscorrelation value between PPM timeslots. The probability of a correct decision after some calculus becomes

$$P_c = \left[\frac{1}{2^{M-m}\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} \exp\left(-\frac{(x-2\alpha\rho_1E)^2}{2\sigma^2}\right) \cdot \left[1 + \operatorname{erf}\left(\frac{x-2\alpha\rho_2E}{\sqrt{2}\sigma}\right)\right]^{M-m} dx\right]^m \quad (22)$$

and the symbol error probability as $P_s = 1 - P_c$. This analysis applies to the cases where $\rho_{i,j}$ is the same between all timeslots within the PPM time window. Hence we can use the above analysis to obtain the best case and the worst case performance given the crosscorrelation level.

G. Noncoherent Multichannel Signaling with mPPM

In UWB-applications it is customary to send several pulses (up to thousands PPM-symbols) per datasymbol. In coherent configurations, MRC-combining is available and hence (ideally) no losses occur in pulse repetition. The pulse duration in practical applications, however, is often so short that coherent reception is not feasible and one resorts to noncoherent receivers. In noncoherent configurations, however, a combining loss is observed. Given that we send L replicas of the PPM-symbol to transmit one data symbol and that we have square law detector, the pdfs of the decision variables are ($l = 1, ..., m$ and $k = m+1, ..., M$) [14]

$$\begin{aligned} p(u_l) &= \frac{1}{4EN_0}\left(\frac{u_l}{s^2}\right)^{\frac{L-1}{2}}\exp\left(-\frac{s^2+u_l}{4EN_0}\right) I_{L-1}\left(\frac{s\sqrt{u_l}}{2EN_0}\right) \\ p(u_k) &= \frac{1}{(4EN_0)^L\,(L-1)!}\, u_k^{L-1}\exp\left(-\frac{u_k}{4EN_0}\right) \end{aligned} \quad (23)$$

with non-centrality parameter $s^2 = 4E^2 \sum_{i=1}^{L} \alpha_i^2$. The conditional probability that transmitted timeslot has energy higher than empty timeslot is

$$p(U_m < u_1 \mid U_1 = u_1) = 1 - \exp(-\frac{u_1}{4EN_0}) \sum_{k=1}^{L-1} \frac{1}{k!} \left(\frac{u_1}{4EN_0}\right)^k \tag{24}$$

The probability of correct symbol decision is $P_c = \left[\int_0^\infty [p(U_m < u_1 \mid U_1 = u_1)]^{M-m} p(u_l) du_l\right]^m$ which after some mathematical manipulations gives an expression for symbol error probability in equal gain combining as

$$P_M = 1 - \left[\int_0^\infty \left[1 - e^{-x} \sum_{k=0}^{L-1} \frac{x^k}{k!}\right]^{M-m} \cdot \left(\frac{x}{\gamma}\right)^{\frac{L-1}{2}} e^{-(\gamma+x)} I_{L-1}\left(2\sqrt{\gamma x}\right) dx\right]^m \tag{25}$$

where $\gamma = \frac{E}{N_0}$ is the SNR per symbol observed over L channels.

H. Diversity combining in impulsive interference with mPPM

Let us assume that ξ represents the relative interference hit probability, i.e. $\xi = 0.5$ denotes that on the average 50 % of the diversity replicas are interfered. Without considering the practical algorithms, let us assume that we can identify the interfered diversity replicas reliably and discard them (i.e. the receiver has perfect side information). Also assuming that the hit probability on each diversity replica is uniformly distributed, we can find an expression for the probability that j out of L diversity branches are combined

$$P_{comb}(j) = \binom{L}{j} \xi^j \left(1 - \xi\right)^{L-j}$$

and the symbol error probability given that we combine j out of L diversity branches ($j > 1$) is

$$P_M(j) = 1 - \left[\int_0^\infty \left[1 - e^{-x} \sum_{k=0}^{j-1} \frac{x^k}{k!}\right]^{M-m} \cdot \left(\frac{x}{\gamma}\right)^{\frac{j-1}{2}} e^{-(\gamma+x)} I_{j-1}\left(2\sqrt{\gamma x}\right) dx\right]^m \tag{26}$$

For $j = 0$ we get $P_{comb}(0) = \frac{\left[\binom{M}{m}-1\right]}{\binom{M}{m}}$ and for $P_M(1)$ we use equation 13. In the above equations $\gamma = \frac{jE}{LN_0}$ is the SNR per symbol observed over j combined channels. The final symbol error rate is of the following form [16]

$$P_M = \sum_{j=0}^{L} P_{comb}(j) P_M(j) \tag{27}$$

IV. Numerical Results

To show that the analytical results agree with reality, we simulated both coherent and noncoherent mPPM-modulated systems with $M = 8$ and $m = 1, 2$ and 3. Figs. 3 and 4 show that the analytical results argree well with the simulations. One also observes that the performance degradation is not severe with increasing m suggesting higher spectral efficiency.

Fig. 5 gives results for mPPM with noncoherent receiver and nonorthogonal case. The results show that the noncoherent receiver is susceptible to correlation between timeslots and that the phenomenon comes more pronounced with increasing m. It is concluded that noncoherent receiver should have orthogonality to operate optimally. Fig. 6 presents similar results for coherent receiver. Results indicate that at least in theory the performance can be increased when the pulse ringing affects the reception in such a manner that the correlation between the signals is negative. In Fig. 7 the performance of noncoherent mPPM with pulse repetition is presented as function of symbol error rate and E_s/N_0 (E_s is the symbol energy). The results indicate that the theoretical curves agree well with the simulated ones. The noncoherent combining loss is 2-3 dB with $L = 10$ and 5-7 dB for $L = 100$ depending on the SNR. Fig. 8 presents the result in impulsive interference as function of diversity order L. The results show that the analysis is useful in obtaining optimal diversity order given that we have knowledge of the interference hit probability.

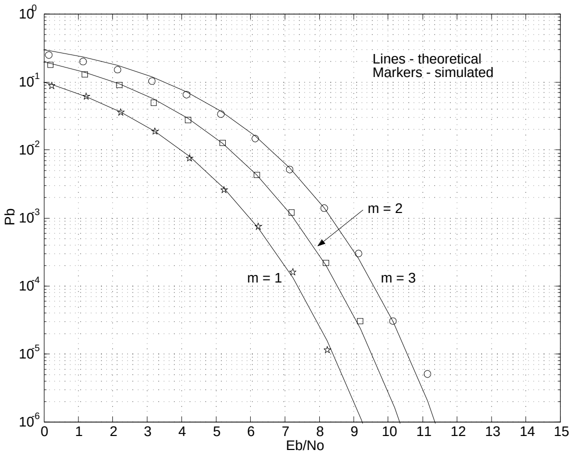

Fig. 3. Bit error rate performance of coherent mPPM detection in AWGN channel

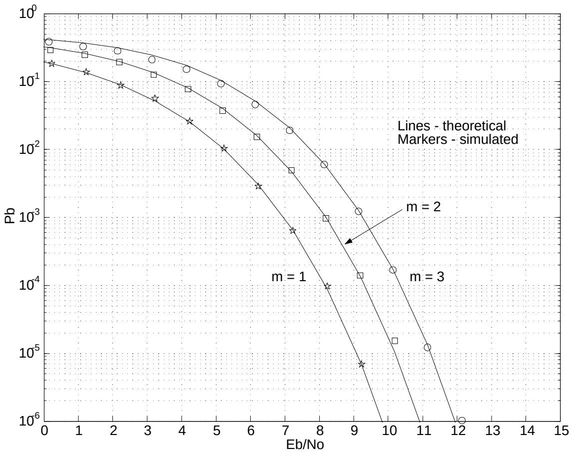

Fig. 4. Bit error rate performance of noncoherent mPPM detection in AWGN channel

V. Conclusions

We have considered two spectrally efficient new modulation formats mPPM and amPPM which are modifications of the well known PPM modulation. We have also derived the exact bit error rate formulas in AWGN channel for the proposed modulation methods. We have also analysed the performance

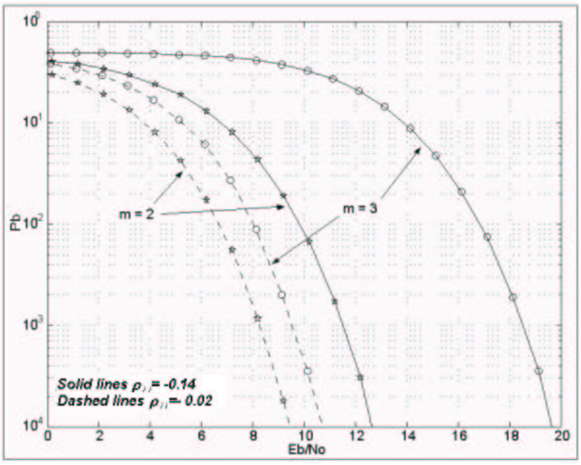

Fig. 5. Performance of noncoherent mPPM with cross-correlation between timeslots equal to -0.02 or -0.14, m=2,3 and M=8

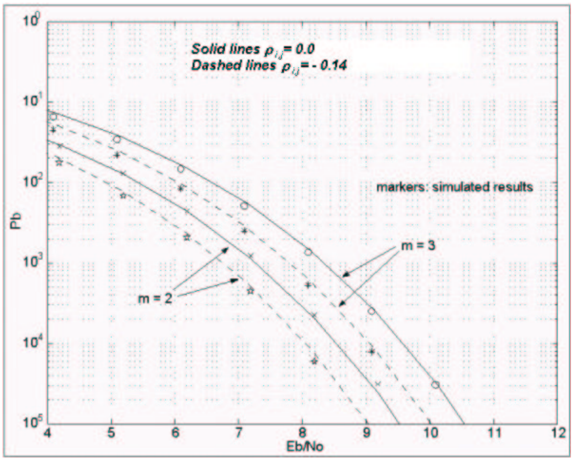

Fig. 6. Performance of coherent mPPM with cross-correlation between timeslots equal to 0 or -0.14, m=2,3 and M=8

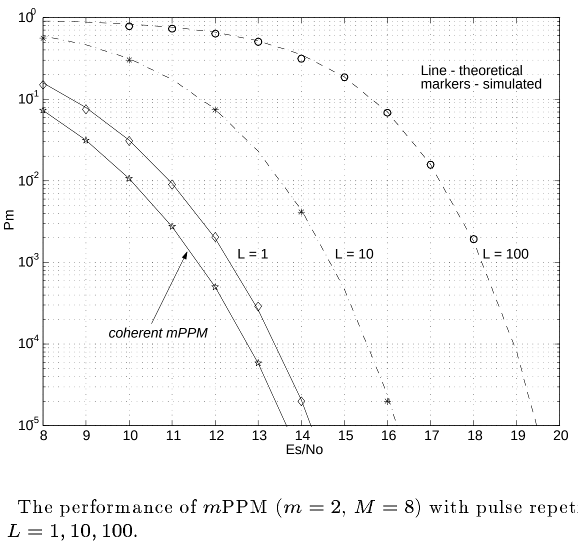

Fig. 7. The performance of mPPM ($m = 2$, $M = 8$) with pulse repetition of $L = 1, 10, 100$.

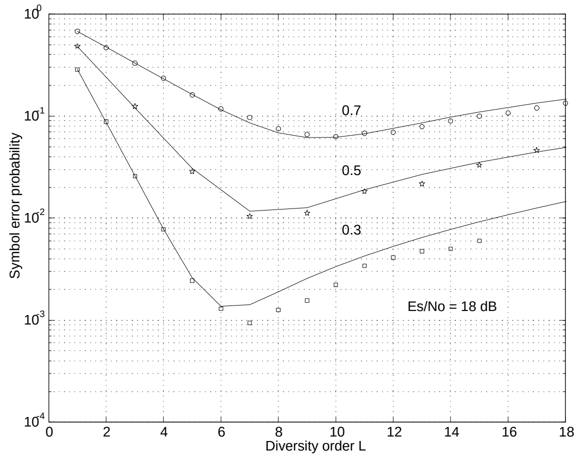

Fig. 8. Performance of mPPM in impulsive interference with $\xi = 0.3, 0.5, 0.7$ as a function of the diversity order L

in pulse repetition with equal gain combining. The results show that with small increase in transmit power a significant increase in data rates is achievable with mPPM.

VI. Acknowledgements

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