# ON THE CLAIMED 5.5-YEAR PERIODICITY IN SOLAR ACTIVITY

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**Abstract.** Recently, Djurovic and Pâquet (1996) claimed to have found an oscillation with a period of about 5.5 years in several solar and solar-terrestrial parameters, in particular in solar activity as indicated by sunspot numbers. Since the temporal evolution of the solar activity and solar-terrestrial environment is of great interest in many fields, we have examined their claim in detail. We show here that their conclusion is based on an artefact due a questionable method applied, and due to the asymmetric form of the solar cycle. Accordingly, there is no reasonable evidence for the existence of a fundamental 5.5-year periodicity in solar activity.

## 1. Introduction

During recent years, numerous papers have been published where the long-term temporal behaviour of various solar and solar-terrestrial parameters are studied. Sunspot numbers form the longest and most uniform series of solar activity (SA) indices and are therefore of particular interest. Fundamental periodicities are of great importance when trying to understand the nature of SA. The main long-term periodicities found in solar activity are the 22-year magnetic Hale cycle, and the 11-year Schwabe cycle (see, e.g., Zirin, 1988; Kontor, 1993; Rozelot, 1994). Also other periodicities, e.g., the 80–100 year Gleissberg cycle, have been suggested.

In a recent paper, Djurovic and Pâquet (1996) claim to have identified an oscillation with a period of 5.5 years (called FYO, five-year oscillation) in several parameters describing solar activity and the solar-terrestrial relationship. In particular, one of their main conclusions is that there is a real 5.5-year oscillation in SA as indicated by the sunspot (Wolf) number series. Although peaks at the period of about 5.5 years have been found in the sunspot number power spectra (see, e.g., Kontor, 1993, and references therein), they have generally been interpreted to be the second harmonic of the fundamental 11-year cycle (see, e.g., Vitinskii, 1973; Currie, 1976; Sugiura, 1980). In the present paper we reanalyze the claim by Djurovic and Pâquet about the existence of a 5.5-year periodicity in sunspot numbers and show that, rather than being a real periodicity, the 5.5-year periodicity is indeed due to the enhanced power of the second harmonic which arises from the asymmetric form of the solar cycle. We show explicitly how the amplitude and phase of the 5.5-year harmonic component are located with respect to the phase of the average solar cycle, and study their long-term distribution over the 22 most recent solar cycles.

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## 2. Asymmetry of the Sunspot Cycle and 5.5-Year Periodicity

Let us first review the procedure adopted by Djurovic and Pâquet (1996) in their analysis. Using 55-day averaged data, they tried to eliminate the main 11-year cycle (and longer trends) by a least squares adjustment of data to the expression

$$F = P_2 + S \; , \tag{1}$$

where $P_2$ is a second-order polynomial form and $S$ is the best fitting sinusoid whose period was varied from 8 to 15 years in steps of 0.2 years. Such a fit was performed to each subsequent 11-year fraction of data separately. After removing the fit $F$ from the data, the residual was analyzed by calculating the Fourier spectrum, where a dominant peak was found at about 5.5 years. Assuming the zero hypothesis that the residual series has a white-noise distribution, Djurovic and Pâquet found that this spectral peak is significant at a high (99%) probability. This is one of their main arguments in favour of the existence of a new 5.5-year periodicity in solar activity.

Let us first illustrate in simple terms how the asymmetric form of the sunspot cycle can produce a strong superficial FYO periodicity in the above residual. As known since long, the rise time of a typical solar cycle from minimum to maximum is shorter than the decline time from maximum to the next minimum, leading to an asymmetric cycle shape. This feature is well reproduced, e.g., by the one-parametric nonlinear RLC oscillator model for the solar cycle presented by Polygiannakis, Moussas, and Sonnet (1996). We have depicted a series of a few solar cycles of this model in Figure 1(a) using the parameter value which corresponds to the best fit to the average shape of the first 21 sunspot cycles (Polygiannakis, Moussas, and Sonnet, 1996). By construction, this model sunspot series (Figure 1(a)) is strictly periodic with a period of 11 years and thus no other fundamental periodicities exist in this time series. However, since it is not purely sinusoidal, higher harmonic components are included. (Actually, when the value of the model parameter is increased, the shape of the cycle becomes more asymmetric.)

Next, we have fitted the model sunspot cycle series with one sinusoid. (A constant was added as another parameter in order to raise the sinusoid to the appropriate level. This is thus a $P_0 + S$ fit.) The best fitting sinusoid is shown in Figure 1(a) as a dotted line. Moreover, the residual (difference between the model and the sinusoid) is depicted in Figure 1(b). It is evident that the residual is cyclic with a period of half of the fundamental 11-year cycle of the model. The location of the extrema of the residuals with respect to the phase of the sunspot cycle clearly shows how the 5.5-year periodicity originates from the asymmetric shape of the cycle. The maxima (minima) of the residual curve are found just before (after) sunspot maxima and minima since, due to the asymmetry, the maxima (minima) of the sunspot cycle are slightly shifted with respect to the sinusoid.

We have depicted the power spectra of the Fourier transforms of the original model cycle and the residual in Figures 2(a) and 2(b), respectively. In the power

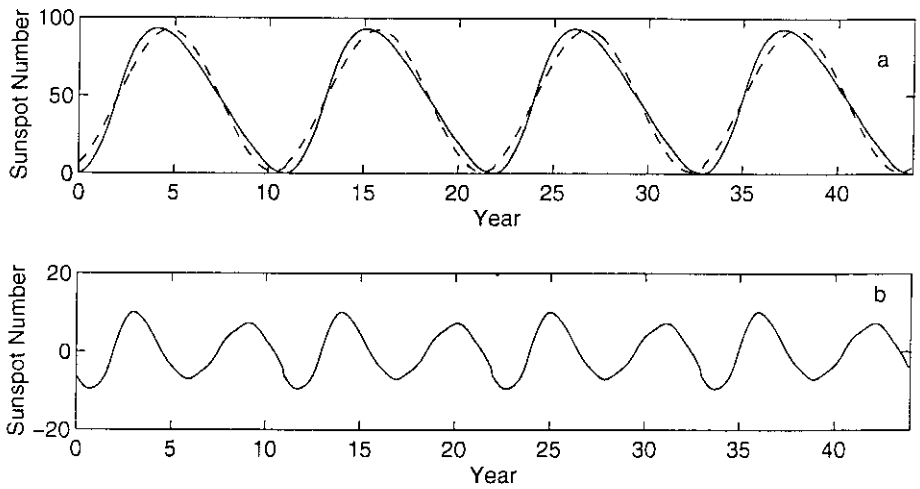


*Figure 1.* The time series of (a) a few cycles of the nonlinear one-parameter sunspot cycle model (Polygiannakis, Moussas, and Sonnet, 1996) corresponding to an average of the solar cycles 1−21 (solid line) and the best fitting sinusoid (dashed line); (b) the residual, i.e., the difference between the model curve and the best fitting sinusoid shown in (a).

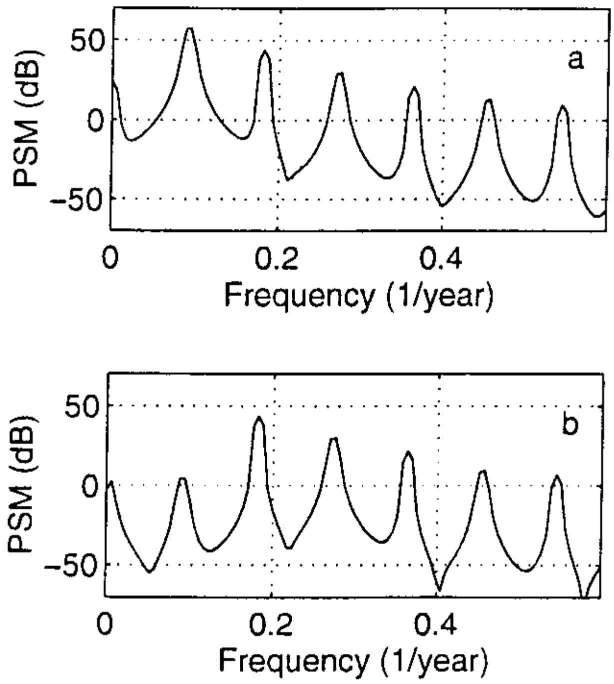


*Figure 2.* The power spectrum of (a) the model cycle of Figure 1(a); (b) the residual of Figure 1(b).

spectrum of the model series (Figure 2(a)), the fundamental 11-year periodicity dominates the higher harmonics by more than 15 dB. Still, the higher harmonics are clearly visible in the spectrum, with the power decreasing systematically with the order of the harmonic. Since, as noted above, the model series has only one periodicity of 11 years, the higher harmonics do not correspond to a real periodicity but arise from the asymmetric form or higher order momenta of the signal. It is seen in the power spectrum of the residual (Figure 2(b)) how effectively the subtraction of the best fitting sinusoid reduces the fundamental and leaves the second harmonic at 5.5 years as the dominant peak in the spectrum. Note also that the second harmonic peak is more than 10 dB stronger than the higher harmonics. The appearance of a similar peak in the spectrum of actual sunspot data led Djurovic and Pâquet ( 1996) to claim the existence of the FYO in sunspot number series. However, from the treatment presented above it is clear that such an argument can not serve as a proof for the existence of a fundamental 5.5-year periodicity.

## 3. Long-Term Evolution of the Asymmetry

After the above demonstration of the principle with the model sunspot cycle, let us now turn to study the actually observed sunspot cycles. We used monthly averaged sunspot numbers, and made a best fit to a sinusoid and a constant ($P_0 + S$) fit for each cycle separately as already described above. Each cycle was defined to extend from one official sunspot minimum to the next. Using actual minima as end points in the fit has the advantage of decreasing the height of steps that necessarily arise at the boundary of two fit intervals. Note that our procedure slightly differs from that adopted by Djurovic and Pâquet (1996), where 55-day averages and stricty 11-year fractions of data (independent of the actual cycle length) were used.

As above, the residual, i.e., the difference between the actual sunspot number series and the best fitting sinusoid was calculated. However in this case, due to the strong fluctuations of the monthly sunspot numbers, it is necessary to smooth the residual in order to demonstrate the long-period variation more clearly. Accordingly, the 11-month smoothed sunspot number series and the residual are plotted in Figures 3(a) and 3(b), respectively. It is clear that the residual is dominated by the second harmonic, and that the maximum values of the residual are located at the same phases of the sunspot cycles as found above for the model cycle (see Figure 1(b)).

The power spectrum of the (unsmoothed) sunspot numbers and the residual are depicted in Figures 4(a) and 4(b), respectively. It is obvious that the harmonic pattern of the power spectrum is less regular than in the case of the model cycle (Figure 2). For example, due to the varying length and amplitude of the real sunspot cycles, the power of the different spectral peaks in Figure 4 is spread to a larger range of periods than in Figure 2. This also applies to the second harmonic peak. The more irregular form of the second harmonic also enhances the power of the

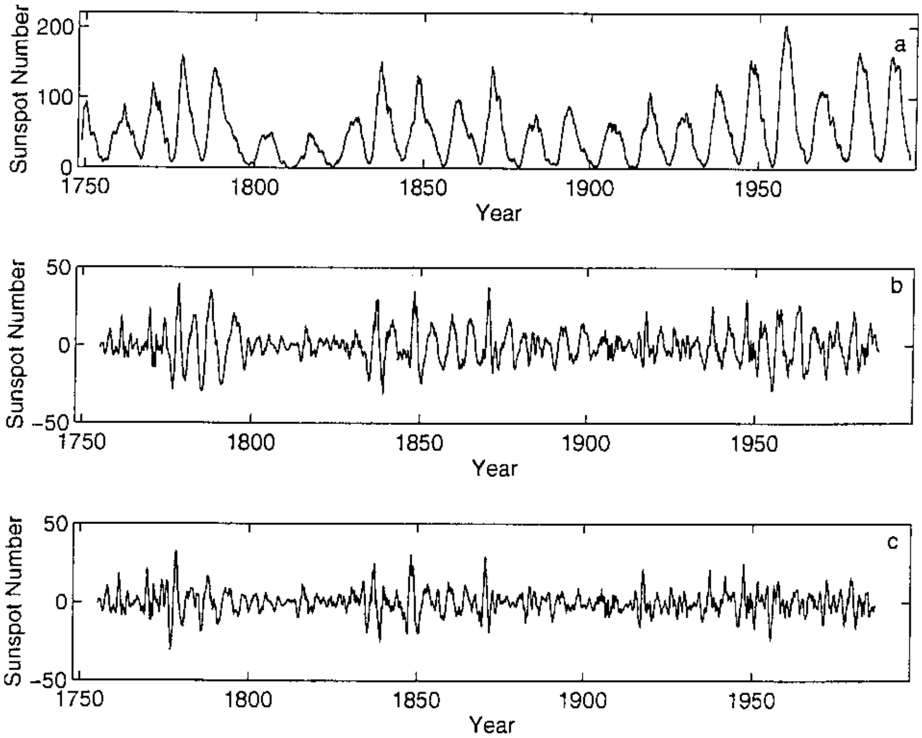


*Figure 3.* The 11-month smoothed time series of (a) sunspot numbers; (b) the residual obtained by fitting each full cycle separately with a sinusoid and a constant; (c) the residual obtained by fitting each full cycle separately with a sinusoid and a second-order polynomial form.

fourth harmonic relatively higher than in the case of the model cycle. However, the maximum power of the second harmonic is nearly 10 dB higher than that of the third harmonic. Accordingly, the ratio between the second and third harmonics for the real sunspot series is somewhat smaller than, but of the same order of magnitude as the same ratio for the model cycle. This ratio, and the location of the minima and maxima of the residual in the solar cycle as discussed above, give strong evidence that the second harmonic in the case of real sunspot cycles arises from the same effect as in the case of the model cycle. Thus, the power peak in the sunspot number residual at about 5.5 years originates from the asymmetry of the sunspot cycle and does not provide evidence for additional periodicities, contrary to the claims by Djurovic and Pâquet (1996).

Note that the largest values of the residual (Figure 3(b)) are about 40, i.e., much higher than in the case of the model residual (Figure 1(b)) where the maximum was about 10. This is understandable since the model corresponds to the parameter value adjusted to fit to the average of the first 21 cycles (Polygiannakis, Moussas, and Sonnet, 1996). Figures 3(a) and 3(b) also depict clearly that the amplitude of the residual closely follows the amplitude of the sunspot cycle. The largest values

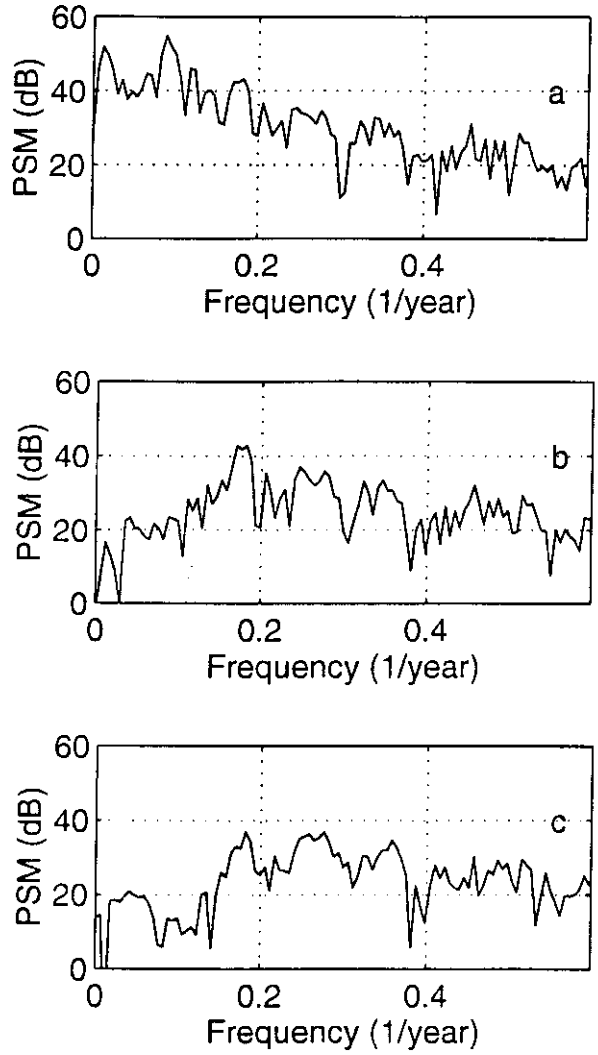


*Figure 4.* The power spectra of (a) sunspot number series of Figure 3(a); (b) the (unsmoothed) residual corresponding to Figure 3(b); (c) the (unsmoothed) residual corresponding to Figure 3(c).

of the residual are found during the active cycles in late 18th, mid-19th, and mid-20th centuries. This nicely demonstrates the well-known Waldmeier effect that the higher sunspot cycles are more asymmetric.

## 4. Additional Comments on the Method

While the above presentation demonstrates the essential features of our argument, there are a couple of important methodological details which need to be discussed.

As mentioned in Section 2, Djurovic and Pâquet (1996) fitted the data with a second-order polynomial form and a sinusoid ($P_2 + S$), although there is no *a priori* reason to include this particular higher-order polynomial form in the fit. Moreover, we will show below that the second-order polynomial may take a dominating role in the fit, thereby disturbing the harmonic pattern of the residual and the subsequent interpretation based on it. In order to clarify this point and to study the differences between the two fit methods ($P_0 + S$ vs $P_2 + S$), we have repeated our calculations for the first 21 sunspot cycles in a similar way as described above, except for using now the second-order polynomial (and a sinusoid) in the fit.

The (smoothed) sunspot residual obtained from the $P_2 + S$ fit and its power spectrum are depicted in Figures 3(c) and 4(c), respectively. It is clear that including the second-order polynomial term in the fit has changed the residual and its harmonic structure with respect to that obtained from the $P_0 + S$ fit (Figures 3(b) and 4(b)). For example, the long-period part of the power spectrum (Figure 4(c)), e.g., around the fundamental period of about 11 years, is now more effectively diminished. This also affects the second harmonic by reducing its peak power by several dB from that in Figure 4(b). (Note also that since the longer periods are particularly affected, the distribution of power around 5.5 years is changed such that the corresponding peak appears to be shifted to a slightly shorter period. A similar shift is found also for the third and fourth harmonics). Another effect of the $P_2 + S$ fit is that, since the second-order polynomial form is non-harmonic, the harmonic pattern of the power spectrum becomes less clear than in Figure 4(b), and higher harmonic components are enhanced. For example, the relative power of the third harmonic with respect to the second harmonic is larger than in the $P_0 + S$ fit.

However, despite the changes introduced by using the second-order polynomial fit, the most important results obtained earlier remain essentially the same. For example, although relatively much less significant, the second harmonic still remains as the dominant periodicity of the residual (Figure 4(c)). This is also seen in Figure 3(c) where, despite the smaller maxima and the more irregular form of the residual, the overall dominance of the second harmonic and the similarity with the residual of Figure 3(b) is notable. Thus, including the second-order polynomial form in the fit does not change the earlier conclusions based on the simpler and more appropriate $P_0 + S$ fit. On the contrary, the more complicated $P_2 + S$ fit used by Djurovic and Pâquet only tends to obscure their results.

We would like to note that our way of making the fit for each full sunspot cycle from minimum to minimum allows for a large, even a dominating contribution of the second-order polynomial in the fit. This is obvious since a parabola with a negative second-order term coefficient can fairly well reproduce a full cycle. In such a case, the sinusoidal term is only a supplementary non-physical factor and its period and phase may significantly differ from those of a more physical fundamental harmonic calculated without the second-order polynomial. On the other hand, if the fit interval starts, say, in the middle of an ascending phase, the significance of the parabola in the fit may be greatly diminished. Since Djurovic and Pâquet used strict 11-year fit intervals, they must have started their fit intervals at different phases of the sunspot cycles. Accordingly, in their fit, the second-order polynomial form attains a smaller overall significance than in our second-order polynomial fit. (A more exact comparison is not possible since no results from the individual fits are given in their paper). Thus, their fit gives a residual which is intermediate to our two cases presented above, i.e., our original $P_0 + S$ fit (Figures 3(b) and 4(b)) and our $P_2 + S$ fit (Figures 3(c) and 4(c)). Since our two cases support the above presented interpretation about the spectral peak at 5.5 years, this has to apply to the results of Djurovic and Pâquet as well. However, since the fit by Djurovic and Pâquet treats the various sunspot cycles unequally, their residual does not form a similar homogeneous measure for the asymmetry of the solar cycles as obtained in the present paper.

As a final note we would like to discuss the cross correlation functions that Djurovic and Pâquet calculate between the residuals of the various solar and geophysical parameters. Most of these cross correlation functions show an oscillatory behaviour with a period ranging from 4 to 6−7 years, which Djurovic and Pâquet regard as strong evidence for the existence of a common 5.5-year periodicity. However, it is clear from the above presentation that all parameters that are affected by the fundamental 11-year solar cycle will depict a non-zero residual dominated by the second harmonic. Furthermore, since all parameters are driven by the same mechanism, the residuals of different parameters have a constant relative phase even over a long time, and produce cross correlation functions that are oscillating at the period of half the solar cycle length. Therefore, the oscillation of the cross correlation functions is trivial and does not give support for the existence of a 5.5-year periodicity.

## 5. Concluding Remarks

We have shown that the claimed discovery of a new 5.5-year periodicity in solar activity (Djurovic and Pâquet, 1996) is incorrect. Using a simple nonlinear model (Polygiannakis, Moussas, and Sonnet, 1996), we demonstrated explicitly how the asymmetric form of the sunspot cycle can produce the spectral power at the second harmonic of the fundamental 11-year periodicity. We studied the 21 first sunspot cycles by subtracting the best fitting sinusoid from each cycle separately. Plotting the residual, which mainly consists of the second harmonic, we could straightforwardly demonstrate the Waldmeier effect, i.e., the fact that the more active sunspot cycles are more asymmetric.

Although the present work concentrated on sunspot (Wolf) numbers, it is obvious that a similar analysis as presented here would deny the existence of a 5.5-year periodicity in any other parameter of solar activity which is highly correlated with sunspot numbers. This is true, e.g., for the sunspot area and the 10.7 cm flux, which

Djurovic and Pâquet also claimed to depict a 5.5-year periodicity. Consequently, contrary to the claim of Djurovic and Pâquet (1996), there is no evidence for a direct correlation between solar activity and the possible 4–6 year periodicities found, e.g., in the length-of-day fluctuations (Dickey, Marcus, and Hide, 1994), polar motion (Abarca del Rio and Cazenave, 1994), or some other geophysical parameters. For example, the geomagnetic activity indices are known to have periodicities close to the period range studied here, e.g., about 4 years and 6–7 years (Fraser-Smith, 1972; Delouis and Mayaud, 1975; Currie, 1976; for a recent review see, e.g., Gonzalez *et al.*, 1993). These periodicities are not related to sunspot numbers but rather to the properties of the solar wind, and thus should not be included in the present discussion. Nevertheless, if a connection exists between the various geophysical periodicities around 4–7 years, the geomagnetic activity may be a dominant driving factor.

Nowadays, even more sophisticated techniques are used to study solar activity than those aiming to discover new periodicities. It is well known by now that solar activity time series are significantly stochastic and cannot be considered as a multi-periodical signal (e.g., Kurths and Ruzmaikin, 1990; Ostryakov and Usoskin, 1990; Mundt, Maguire, and Chase, 1991; Kremliovsky, 1994; Rozelot, 1995, etc.). Recent studies of the temporal behaviour of solar activity actively involve nonlinear and stochastic analysis techniques (Rozelot, 1995; Calvo, Ceccatto, and Piacentini, 1995; Qin, 1996). Even when studying a quasi-periodicity, as in the present case, one should check the possibility of an artefact (Alibegov, 1996). Obviously, Djurovic and Pâquet (1996) missed this point, leading them to erroneous results.

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