Estimates of errors of sunspot number reconstruction from ^{14}C data

Supplementary material to Solanki et al.:
"An unusually active Sun during recent decades compared to the previous 11,000 years"

The reconstruction of the sunspot number consists of a series of steps, which are listed in Table S1, together with the free parameters involved and the independent data used to constrain the reconstruction or to verify the models underlying the individual steps. The various sources of error for each step are evaluated here, starting with the conversion of Δ^{14}C into cosmic-ray flux.

We evaluate here potential sources of error affecting the sunspot number reconstruction from the measured Δ^{14}C for all intermediate steps. This includes the possible systematic error of the calculated ^{14}C production rate calculation related to uncertainties in Δ^{14}C prior to the Holocene and various random errors as described below.

Table S1: Free parameters in the individual steps of the sunspot number reconstruction and independent data used to constrain them.

Step	Freely adjustable parameters	Data used for model constraints and verification
^{14}C –> CR	None	Independent carbon cycle model[a] Geomagnetic data[b]
CR –> Open flux	Proportionality coefficient	Cosmic ray data[c], 1951–2001 Open flux data and calculations[d]
Open flux –> SN	Decay time of the open flux[d]	Direct measurements, 1975–1994 Geomagnetic proxy[e], 1860–2000

[a] see, e.g., (Stuiver & Quay, Science, 207, 11, 1980) or [24].
[b] see [28, 29]
[c] see [14].
[d] see [12, 13].
[e] see (Lockwood et al., Nature, 399, 437, 1999).

Uncertainties in the pre-Holocene ^{14}C production rate

The main source of systematic errors in the reconstruction of SN is introduced by the uncertainties of the ^{14}C production rate in the pre-Holocene period. Owing to the half-life of ^{14}C of 5730 years and the long 'memory' of the carbon cycle, a significantly different production rate even many millennia before the Holocene still affects the Δ^{14}C measured for the Holocene. It is therefore a priori unclear, which fraction of the changes in measured Δ^{14}C are produced by changes in the instantaneous ^{14}C production rate, Q, and which fraction is due to variations of Q occuring prior to the Holocene. In order to estimate how strongly the results are affected by these uncertainties, we have constructed two models based on extreme assumptions for the Δ^{14}C profile prior to the Holocene. The 'true' sunspot number (SN) can then be considered to lie between the limits outlined by the series reconstructed from these models.

The Δ^{14}C time profile is known for several millennia before the Holocene (INTCAL98) and is plotted in panel A of Fig. S1 (solid curve). In this panel we also show the two extreme assumptions. In the first case (solid curve, 'raw') it is assumed that the entire time profile of Δ^{14}C is solely determined by changes in Q, including the pre-Holocene glacial period. This is expected to be an upper limit because it assumes that the properties of the carbon cycle remain unchanged during the glacial and the Holocene periods. The other extreme assumption is to prescribe a constant production rate (steady state, see also [24]) prior to the Holocene (dashed line). We have slightly modified this case by assuming that the 'effective' Δ^{14}C during the glacial period was 70 permille lower than the measured values and that the trend seen in the raw data during 11000–9500 BC is due to the transition of the carbon cycle between the glacial period and the Holocene. The effective Δ^{14}C is here defined as the amount of Δ^{14}C produced by the same value of Q, but assuming a holocene-type carbon cycle. The corresponding curve (dotted, 'corrected') lies close to a steady-state in the beginning of the Holocene and varies in parallel with the measured Δ^{14}C during the ice age. Prior to 13000 BC, a steady state is assumed for all cases. The reconstructed SN series for the two extreme cases ('raw' and 'corrected') are shown in panel B in Fig. S1. The SN series presented in the main paper corresponds to the mean of the two extreme SN reconstructions while the halved difference between them (Δ_{model} shown in panel C of Fig. S1), is considered to be a measure of the systematic error. The systematic error of the SN reaches values up to 15 in the beginning of the Holocene, but decreases rapidly thereafter and becomes almost negligible after 7000 BC. This implies that

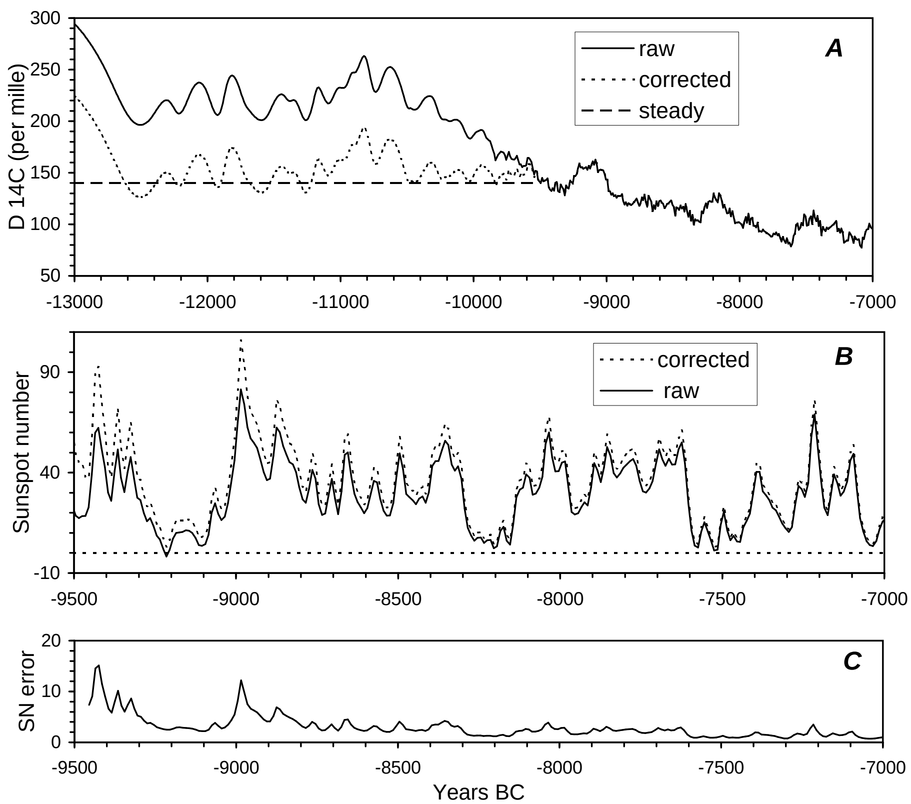

Figure S1: Systematic error in the sunspot number reconstruction due to uncertainties in the carbon cycle and in the ^{14}C production prior to the start of the data set. A: Two extreme cases of the effective Δ^{14}C history. The original ('raw', solid line) and the 'corrected' (dotted line) data , as well as the steady case (dashed line). B: Sunspot number reconstructions (note the different time scale with respect to panel A). C: Halved difference between the curves in panel B as an estimate of the systematic error.

the SN reconstruction during the Holocene is only slightly affected by the uncertain effective Δ^{14}C at earlier times. This conclusion is supported by the excellent agreement between the actual Δ^{14}C data and a reconstruction of Δ^{14}C based on the ^{10}Be concentration measured in ice cores (Muscheler et al., Nature, 408, 567, 2000), since ^{10}Be more clearly reflects the isotope production.

Other sources of error

Δ^{14}C measurement errors

Measurement uncertainties in the Δ^{14}C series are within 2–4 per mille during the Holocene and are higher at earlier times (INTCAL98). Such measurement errors can be regarded as random errors. However, because of the non-linearity in the applied model relating Δ^{14}C to SN, standard formulae describing error propagation cannot be applied. Therefore, we have employed a Monte-Carlo approach. We first construct a number of synthetic Δ^{14}C series, $D_i(t)$, so that

$$D_i(t) = D_0(t) + R_N \cdot \sigma_\Delta(t),$$

where $D_0(t)$ and $\sigma_\Delta(t)$ are the times series of the measured Δ^{14}C and its errors, respectively, and R_N are random numbers from a normal distribution with unit variance. From each series we reconstruct the corresponding series of SN, $S_i(t)$, and use this set of series to estimate the uncertainty of the reconstructed SN. In addition, the reconstruction of the ^{14}C production rate, Q, from Δ^{14}C has an uncertainty of about ± 0.04 atoms$\cdot$cm$^{-2}\cdot$s^{-1} (see [24]), which was also included. The resulting standard deviation of the SN for 20 synthetic reconstructions, $\sigma_{\rm C14}$, is shown in panel A of Fig. S2. The value of $\sigma_{\rm C14}$ generally lies below 5 and rarely exceeds 10 during the Holocene (the mean being 4.3). It increases toward the beginning of the time series because of the larger measurement uncertainties of Δ^{14}C. This is one of the reasons for limiting our SN reconstruction to the last 11,400 years.

Geomagnetic dipole moment

Since the relation between the ^{14}C production rate and the cosmic ray flux depends on the geomagnetic field (expressed through the virtual dipole moment, M), uncertainties in the latter affect the SN reconstruction. We have used values of M and its uncertainties as given by Yang et al. (2000) for the Holocene and by Guyodo and Valet (1996) for earlier times. Again following the Monte-Carlo approach to evaluate the corresponding uncertainties

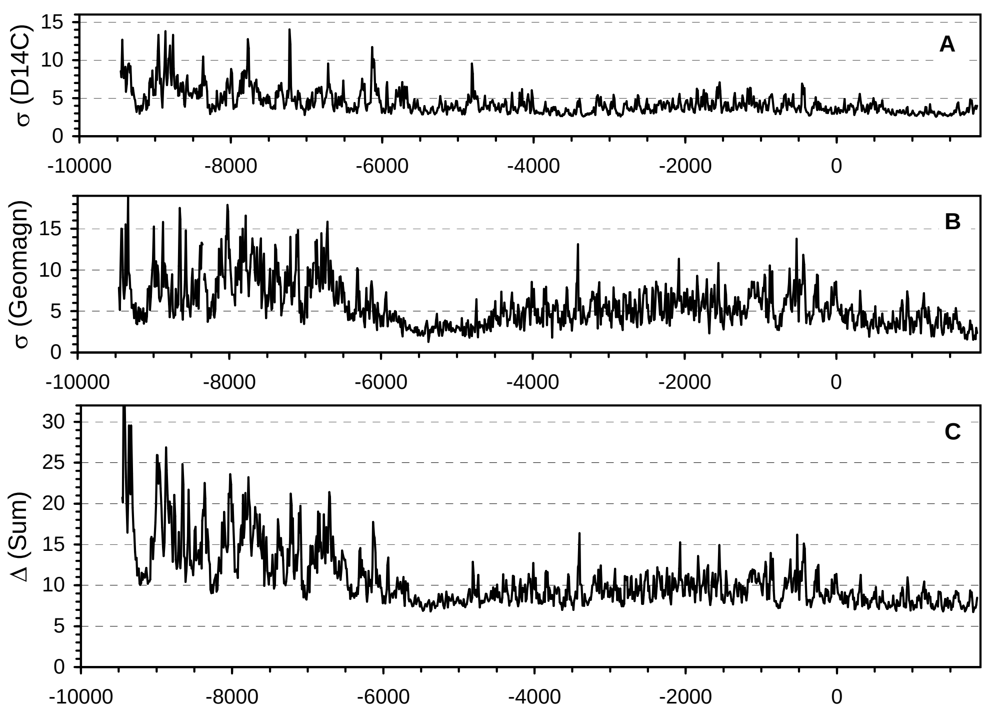

Figure S2: Random and total errors in the SN reconstruction. A: 1σ error resulting from the measurement errors of Δ^{14}C. B: 1σ error due to the uncertainties in the adopted geomagnetic field. C: Total error at the 68% significance level (also shown in panel C of Fig. 3 in the main paper).

in the reconstructed SN, we have performed 20 SN reconstructions using the undisturbed Δ^{14}C series and adding random errors to the magnetic moment at each step, so that

$$M_i(t) = M_0(t) + R_N \cdot \sigma_M(t),$$

where $M_0(t)$ and $\sigma_M(t)$ are the published values of the magnetic dipole moment and its uncertainties, respectively. The resulting errors in the reconstructed SN, σ_{Geo}, are shown in panel B of Fig. S2. Except for the initial phase of the Holocene, the SN error rarely exceeds 10, with the mean being 5.6.

Errors in the conversion of ^{14}C production rate into cosmic ray flux

The conversion from the ^{14}C production rate Q into the cosmic ray flux (parameterized through the modulation strength, Φ) is carried out using a pre-calculated Q-Φ relation. Its accuracy is within ± 50 MV [14], which leads to a corresponding uncertainty $\sigma_{\mathrm{CR}} \approx 4.6$ in the reconstructed SN, estimated again using the Monte-Carlo method.

Errors in the conversion of cosmic-ray flux into open magnetic flux

The conversion from the cosmic ray flux parameter Φ to the open solar magnetic flux is based upon a heliospheric model [15], whose parameters were fixed using data for 1951-2002. The uncertainty in the open magnetic flux is estimated to be about $0.1 \cdot 10^{14}$ Wb. The corresponding error in the SN reconstruction is $\sigma_{\mathrm{OF}} \approx 0.5$.

Errors in the conversion of open magnetic flux into sunspot number

The conversion of open magnetic flux into sunspot number involves the inversion of the non-linear model for computing the open flux from the SN [12,13]. This inversion is identical to the one carried out by Usoskin et al. [16,17] for the reconstruction of the SN from the ^{10}Be concentration. Their tests showed that this step introduces an uncertainty of $\sigma_{\mathrm{NL}} < 2$ in the SN, which is due to the simplifications necessary when inverting the non-linear model. In addition, we have also tested how the uncertainty of ± 0.2 years in the decay time, τ, of the open magnetic flux affects the SN reconstruction. This test gives $\Delta_\tau = \pm 1.2$, which is regarded as a possible systematic error.

Total error

The total error at the 68% confidence level,

$$\Delta_{\rm total} = \Delta_{\rm model} + \Delta_\tau + \sqrt{\sigma^2_{\rm C14} + \sigma^2_{\rm Geo} + \sigma^2_{\rm CR} + \sigma^2_{\rm OF} + \sigma^2_{\rm NL}}\ ,$$

is shown in panel C of Fig. S2. Here $\Delta_{\rm model}$ represents the estimate of the systematic error given in panel C of Fig. S1. The total errors generally lie around 10 after about 6000 BC and below 20 for earlier times in the Holocene. The mean error of the SN reconstruction during the Holocene is about 10 (about 8 for the last millennium), with the main sources of possible errors being related to the geomagnetic dipole moment $\sigma_{\rm Geo}$, to ^{14}C measurement errors $\sigma_{\rm C14}$, and to the conversion of Δ^{14}C into cosmic ray flux $\sigma_{\rm CR}$. Prior to the Holocene, any reconstruction of SN from Δ^{14}C is very uncertain already because of the random uncertainties. We stress that the systematic errors never dominate over the random errors even at the 1σ level (68% confidence limit). The influence of the unknown effective Δ^{14}C prior to the holocene decreases steadily with time and is practically negligible after 7000 BC. Even before that date, at the 95% confidence level, the uncertainties are dominated by random errors, which makes our SN reconstruction quite robust with respect to uncertainties in the carbon cycle prior to the Holocene.

Test of the model

As a test for the reconstruction method we perform a comparison with the actually observed sunspot numbers since 1610 (see Fig. 2 in the main paper). Although this interval constitutes only about 3% of the Holocene, it covers the whole range of possible activity levels including the Maunder minimum and the present period of high activity (although the Suess effect precludes that we carry out the comparison over the last century). We stress here that our model of SN reconstruction does not include any (explicit or implicit) kind of fitting or normalization to the observed SN. All intermediate steps are quantitative physics-based models, whose parameters were fixed independently before the present study was initiated (see Table S1). We stress that none of the time intervals used for constraining or verifying the individual steps coincides with the period of testing the complete model, i.e. 1610–1900. Accordingly, a comparison between the reconstructed and measured sunspot activity levels (see Fig. S3) serves as a direct final test to the method. One can see from Fig. S3a that most of the points (21 out of

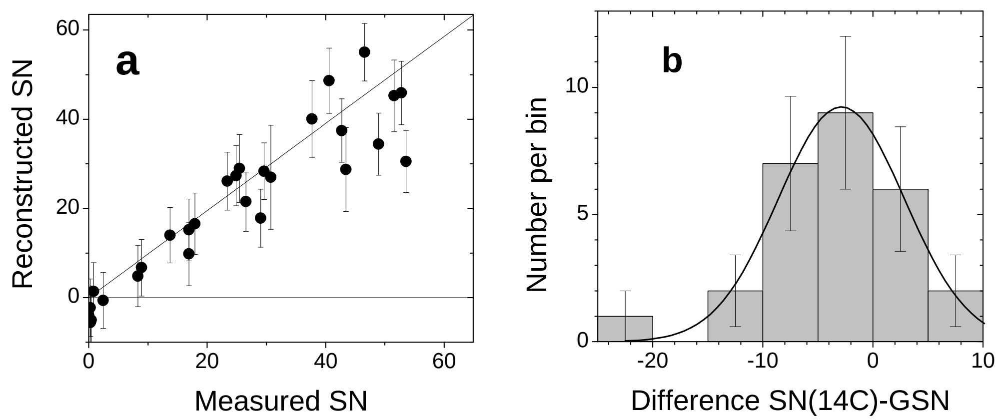

Figure S3: Comparison of the (decadal) reconstruction with the actually measured sunspot numbers for the period of 1610–1900: Scatter plot (panel a), error bars corresponding to Fig. S2c, and the histogram of the difference between reconstructed and measured SN (panel b) together with the best fit Gaussian (mean −2.8 and standard deviation 5.8).

28 or 75%) lie within an interval of $\pm 1\sigma$ from the diagonal while 27 points (96%) are within $\pm 2\sigma$, where σ corresponds to Fig. S2c.

The distribution of the difference between reconstructed and observed SN (Fig. S3b) is approximately Gaussian with the mean lying at -2.8 and a standard deviation of 5.8. The slightly negative mean value implies that our model slightly underestimates SN on average. It also demonstrates that there was no fitting of model values to the actual sunspot numbers (otherwise the average values would be equal). We note that the observed standard deviation of the difference (5.8) is somewhat smaller than the evaluated model uncertainties (about 8 for the last millennium - see Fig. S2c), indicating that the latter are rather conservative. Thus, while a comparison of our reconstruction with the actually measured sunspot numbers is possible only for a relatively short period, it confirms the reliability of both SN reconstruction and error estimate.